Perfect Competition and Profit Maximization

EC 311 - Intermediate Micro

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Perfectly Competitive Markets

- Assumptions of *Perfectly Competitive Markets*:
  1. **Homogeneous goods** (or perfectly substitutable goods). All goods are identical/nearly identical
     - Example: Market for wheat. All bushels of wheat is more or less substitutable for other bushels of wheat
  2. **Price-takers**: firm has no influence over the price of the good. Takes it as given.
     - Agents are aware of the fact that their decisions will not affect prices.

3. **Free-entry/Free-exit:** No special costs that make it difficult to enter or exit an industry
    - Special costs that would limit entry could include R&D and license fees
    - Induces high amounts of competition... consumers can easily switch to another firm if a firm raises its price.

Firms are rarely *PERFECTLY COMPETITIVE*. Usually the real-life examples we can think of are *HIGHLY COMPETITIVE*. In general, however, the analysis is very applicable to these firm behaviors.

Profit Maximization

- One major concern a firm might be considering is how to maximize their **PROFIT**. Define profit of a firm to be revenue net of costs to produce q amount of output
  - Revenue is the amount of money earned from selling q at price P, which is given

$$R(q) = Pq$$

  - Costs are as we discussed in the previous chapter. Denote them as $C(q)$
  - So profits can be written

$$\pi(q) = R(q) - C(q)$$

Marginal Revenues and Marginal Costs

- **Marginal Revenue**: the change in revenue from a one-unit change in output

$$MR(q) = R'(q) = \frac{dR}{dq}$$

- **Marginal Cost**: the change in cost from a one-unit change in output

$$MC(q) = C'(q) = \frac{dC}{dq}$$

- The firm's profit curve increases for some values of q and decreases after some point.
- Call this point at which profit switches from increasing to decreasing as q^*
- So

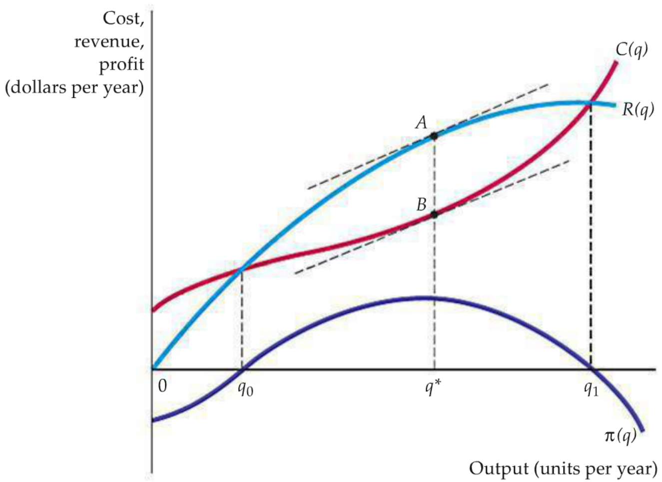
$$\pi'(q) = \begin{cases} > 0 & : q < q^* \\ < 0 & : q > q^* \\ = 0 & : q = q^* \end{cases}$$

- Thus profit is maximized when

$$\pi'(q) = R'(q) - C'(q) = 0$$

$$MC = C'(q) = R'(q) = MR$$

- So marginal costs of production are equal to marginal revenues for a firm, and the firm would want to produce q^* to maximize profit


Cost,
revenue,
profit
(dollars per year)
C(q)
R(q)
A
B
0
q0
q*
q1
π(q)
Output (units per year)

- **Example:** Suppose

$$P = 4$$

$$C(q) = 8 + 2q^2$$

  - What is the firm's revenue?
  - $$R(q) = P * q = 4q$$
  - What is the firm's profit?
  - $$\pi(q) = R(q) - C(q) = 4q - (8 + 2q^2)$$

- **Example, cont.:**
  - What is marginal revenue?
  - $$MR = R'(q) = 4 = P$$
  - What is marginal cost?
  - $$MC = C'(q) = 4q$$
  - What is the profit maximizing output level q^*?
  - $$MC = MR$$
    $$\implies 4q^* = 4 \quad \implies \quad q^* = 1$$

- **Example 2:** Suppose that

$$MC(q) = 3 + 2q$$

$$P = 4$$

- If $P = 4$, then $R(q) = 4q$, so $MR = R'(q) = 4$
- Then the optimal output is

$$MC = MR$$

$$\implies 3 + 2q^* = 4 \quad \implies \quad q^* = \frac{1}{2}$$

- **Example 3:** Suppose

$$C(q) = 50 + 4q + 2q^2$$

- What is the MC function?
-

$$MC(q) = C'(q) = 4 + 4q$$

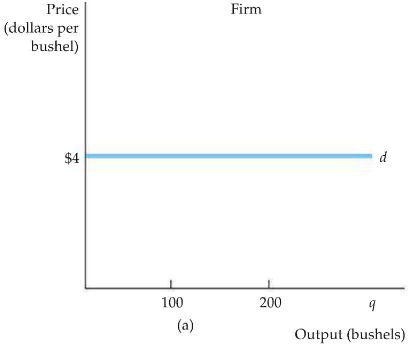
- Suppose $P = 20$. If a firm is currently producing 5 units of output, are they maximizing their profit?
- Firm profit max when

$$MC = MR \quad \Longrightarrow \quad 4 + 4q^* = 20 \quad \Longrightarrow \quad q^* = 4$$

- Is their profit positive, negative, or zero at their current level of output?
- $$\pi(5) = 20(5) - (50 + 4(5) + 2(5)^2) = (100) - (120) = -20$$
- What is their profit at their profit maximizing output?
- $$\pi(4) = 20(4) - (50 + 4(4) + 2(4)^2) = (80) - (98) = -18$$
- Is this firm capable of positive or zero profits in the short-run?
- We might have to consider this for the long-run to. We will get to this.

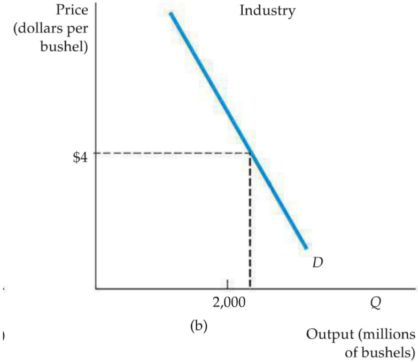
Individual Demand

- Because a firm considers price as given, we know that the demand curve (d) that an individual firm faces is a horizontal line (no actions taken by the firm will affect the price).



Market Demand

- Through interactions between consumers and many firms, prices are determined (no one firm has power, but the interactions of firms and consumers can affect price)

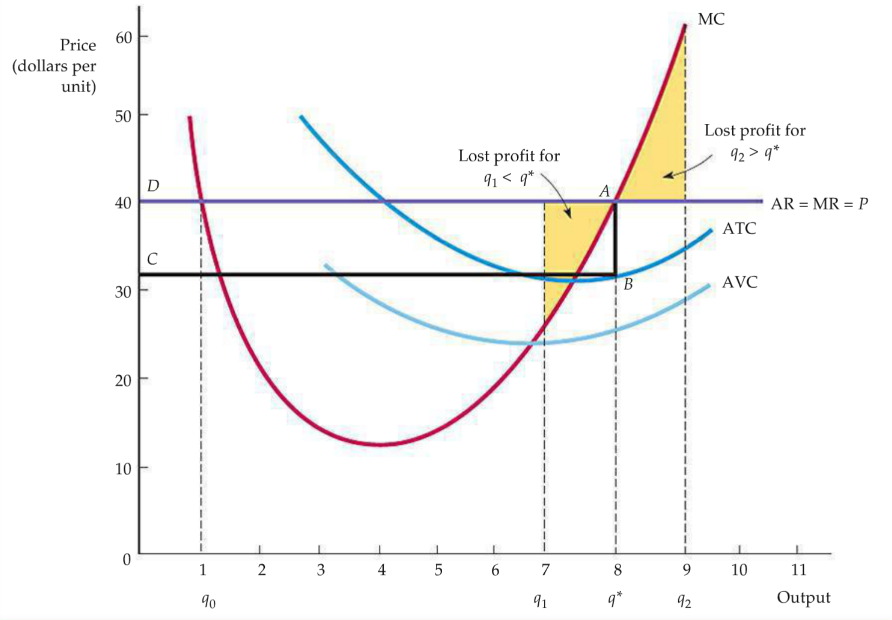


Profit Maximization by a Competative Firm

- Given the price-taker assumption, we can say that

$$MC(q^*) = MR = P$$

- So the MR curve is identical to the demand curve for an individual firm



Price (dollars per unit)
60
50
40
30
20
10
0
MC
Lost profit for
$q_2 > q^*$
Lost profit for
$q_1 < q^*$
D
A
AR = MR = P
ATC
C
B
AVC
1
2
3
4
5
6
7
8
9
10
11
q_0
q_1
q^*
q_2
Output

- At point A, $MR = MC$. and q^* is the profit maximizing level of output
- At q_1, $MR > MC$, so they could earn more than they lose by increasing q
- At q_2, $MC > MR$, so they would lose more than they earn by increasing output
- At q_0, $MC = MR$, but MC is decreasing, and if they increase output they can increase profits.
- So they should *produce at a level when $MC = MR$ and MC is increasing.*

- In the short-run, q^* is the optimal output. Does it have positive profits?
- The distance between A and B gives the distance between price (P) and average total cost (ATC). Also total cost is

$$TC = C(q) = ATC * q$$

If revenue is $R(q) = R * q$, then profits are

$$\pi(q) = Pq - ATCq = (P - ATC)q$$

- Since $P > ATC$, then profits are positive in the short-run
- This can be calcuated as the area of the rectangle ABCD.

- To summarize:
  - Profits are positive in the short-run if $P > ATC$
  - Profits are zero in the short-run if $P = ATC$
  - Profits are negative in the short-run if $P < ATC$
- The firm could still operate in the short-run even if it's profits are negative...
  - Whether or not they choose to do so depends on their expectations on profits in the futures
  - However, in the immediate short-run, fixed costs are irrelevant. If the firm cannot cover variable costs of production, then they cannot produce q^*
  - If they cannot cover variable costs (i.e. if P<AVC), then the firm should **SHUT DOWN** regardess of expectations.

- **Example, cont.**: Recall from the previous example that

$$C(q) = 50 + 4q + 2q^2$$

$$P = 20$$

- We found that the profit maximizing output for this firm was

$$q^* = 4$$

- What is ATC when $q = 4$?

- $$ATC = \frac{C(q)}{q} = \frac{50 + 4(4) + 2(4)^2}{4} = \frac{98}{4} = 29$$
- So, in the short-run, $ATC < P$ so $\pi < 0$
- What is AVC when $q = 4$?
- $$AVC = \frac{VC(q)}{q} = \frac{4(4) + 2(4)^2}{4} = \frac{48}{4} = 12$$
- So $P = 20 > 12 = AVC$, so they can cover their variable costs.
- They can continue to operate in the short-run if they think that future profit will be greater.

- **Example 4:** Suppose that

$$C(q) = q^3 - 8q^2 + 30q + 5$$

  - What is ATC?
  - $$ATC = \frac{C(q)}{q} = \frac{q^3 - 8q^2 + 30q + 5}{q}$$
  - What is MC?
  - $$MC = 2q^2 - 16q + 30 = 2q^2 - 16q + 30$$

- What is AVC?
- $$AVC = \frac{q^3 - 8q^2 + 30q}{q} = q^2 - 8q + 30$$
- Suppose $q^* = 2$. What do we know that market price is?
- $$P = MC \quad \implies \quad P = 2q^2 - 16q + 30 = 2(2)^2 - 16(2) + 30 = 6$$

- Is the firm making positive, negative, or zero short-run profits?
- $$ATC(2) = \frac{q^3 - 8q^2 + 30q + 5}{q} = \frac{2^3 - 8(2)^2 + 30(2) + 5}{2} = 20.5$$
- Since $P = 6 < 20.5 = ATC$, they are making negative profits.
- Should they shut down?
- $$AVC(2) = q^2 - 8q + 30 = (2)^2 - 8(2) + 30 = 18$$
- Since $P = 6 < 18 = AVC$, they should shut down because they cannot cover their variable costs.