ENGS 41

SUSTAINABILITY and NATURAL RESOURCE MANAGEMENT

Non-Renewable Resources

(Lynch book, Chapter 1, with additions)

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If the resource is not sustainable,
we must at least think of the future in some way…

The major questions facing us when dealing with exhaustible resources:

1. Are there any renewable substitutes? Or, do we have the capability of recycling what we have previously extracted?

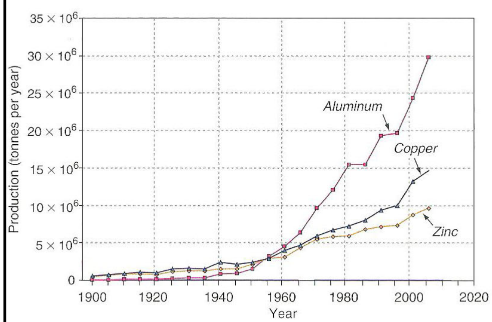
2. How can we stretch the extraction over time in a way that is fair to future generations?

   Anticipated answers:
   - Price equity over time,
   - Stretch for the long run or run out no earlier than when a substitute becomes available.

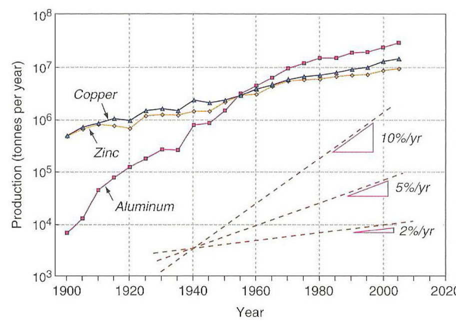
coal

oil

natural gas



Growth of production of three common metals

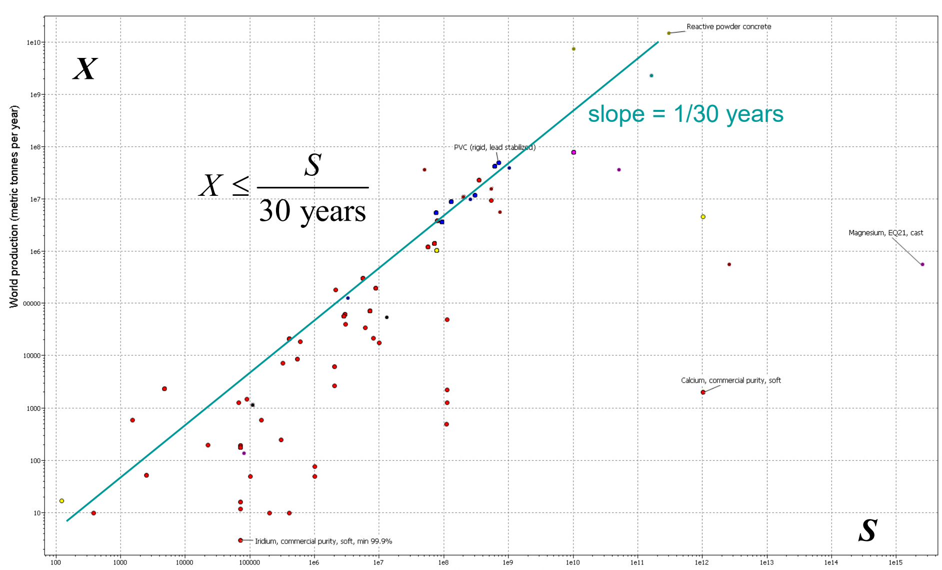


Same displayed with a logarithmic scale

(*Source*: Michael Ashby - 2009 - Fig. 2.9)

We do extract a variety of non-renewable (sterile) resources,
and the annual amount of extraction is increasing (~2%/year).

Rate of extraction (vertical) seems limited by amount of reserves (horizontal).



(*Source*: Ashby - 2009 - CES EduPack)

Note that $X = \dfrac{S}{30 \text{ years}}$ does not mean exhaustion in 30 years.

This is because, as exploitation reduces the remaining stock, the rate of extraction diminishes, too.

Mathematically:

For $X = \dfrac{S}{T}$ with T = 30 years,

the resource is being depleted according to

$$\frac{dS}{dt} = -X = -\frac{S}{T} \quad \Rightarrow \quad S = S_0 e^{-t/T}$$

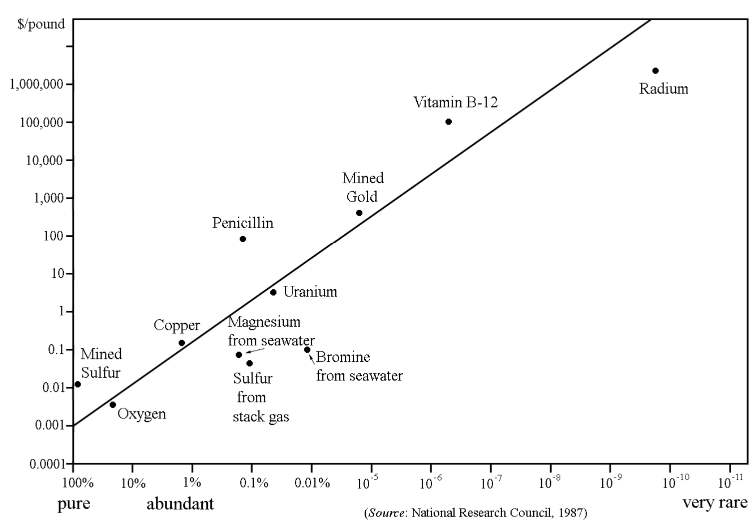
At 30 years ($t = T$), the remaining amount is

$$S(30 \text{ years}) = S_0 e^{-1} = 0.368\, S_0 \approx 40\% \text{ of } S_0$$

Thus, at all times, it remains 40% of what was still there 30 years previously.

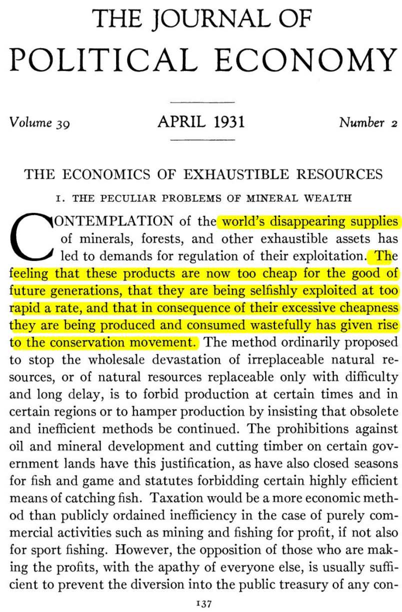
Market price is also related to scarcity (dilution in environment).

Sherwood Plot



(*Source*: National Research Council, 1987)

Harold Hotelling
1895-1973



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THE ECONOMICS OF EXHAUSTIBLE RESOURCES

I. THE PECULIAR PROBLEMS OF MINERAL WEALTH

CONTEMPLATION of the world's disappearing supplies of minerals, forests, and other exhaustible assets has led to demands for regulation of their exploitation. The feeling that these products are now too cheap for the good of future generations, that they are being selfishly exploited at too rapid a rate, and that in consequence of their excessive cheapness they are being produced and consumed wastefully has given rise to the conservation movement. The method ordinarily proposed to stop the wholesale devastation of irreplaceable natural resources, or of natural resources replaceable only with difficulty and long delay, is to forbid production at certain times and in certain regions or to hamper production by insisting that obsolete and inefficient methods be continued. The prohibitions against oil and mineral development and cutting timber on certain government lands have this justification, as have also closed seasons for fish and game and statutes forbidding certain highly efficient means of catching fish. Taxation would be a more economic method than publicly ordained inefficiency in the case of purely commercial activities such as mining and fishing for profit, if not also for sport fishing. However, the opposition of those who are making the profits, with the apathy of everyone else, is usually sufficient to prevent the diversion into the public treasury of any con-

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← Note the concern for future generations well before the word Sustainability was in use.

Hotelling's Rule
(transposed in notation used in Lynch's book)

$X(t)$ = extraction rate (in tonnes per year)
$P(t)$ = market price (in \$/tonne)
} at time t

r = discount rate, dt = 1 year

$X(t)\,dt$ = amount extracted in year t

Rent (= money earned) at time t in the future, discounted to put all years on comparable level, is

$$R(t) = \frac{P(t)\,X(t)\,dt}{(1+r)^t}$$

Question: How to make this equitable across generations?

Hotelling showed that the socially optimal rate of extraction is the one for which the price increases at the discount rate:

$$P(t) = P_0\,(1+r)^t$$

In that case, the rent is the same across years per unit extracted:

$$R(t) = P_0\,X(t)\,dt$$

It can be shown that it also maximizes the total rent over the entire extraction time.

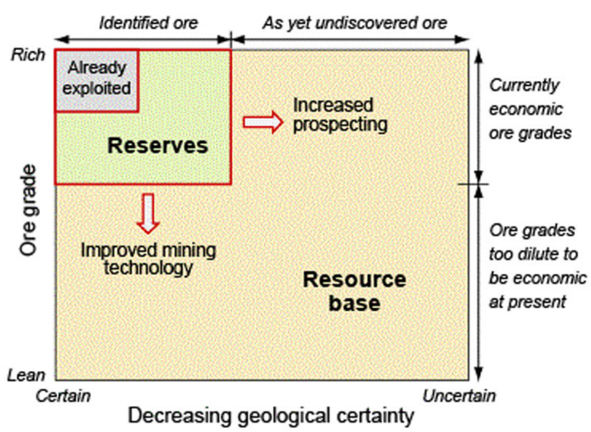
Hotelling's Rule is valid only under the following restrictions:

- No technological development over the years,
- Fixed stock of the exhaustible resource (no new discoveries),
- Constant market conditions,
- Totally competitive market (actors adjust their production until their marginal production cost + their opportunity cost* reaches the market price).

Obviously, many events and disruptions occur at various times, sometimes quite frequently (ex. volatility of oil price), but we may assume that Hotelling's Rule holds during the stretches between consecutive events.

* Opportunity cost = loss of future earning due to extracting that portion today, also called "scarcity rent".

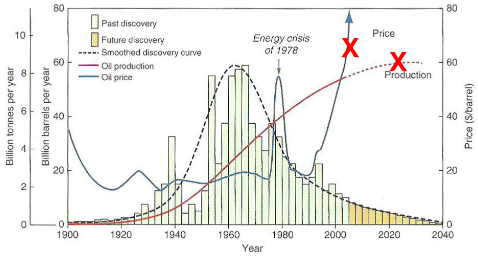
What is meant by the word "Reserves".



(*Source*: Michael Ashby - 2009 - Figure 2.10)

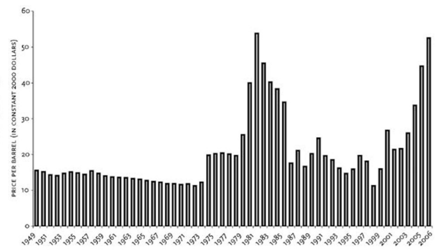
Distinction between *Reserves* and *Resource Base*:
Resource Base = All that there ever was minus what has been exploited to date
Reserves = Part of Resource Base that has been discovered and established as economically viable for extraction.

Well… Let's admit that the situation is a little more complicated.



The evolution of the rates of discovery, the rate of production, and the price of oil.

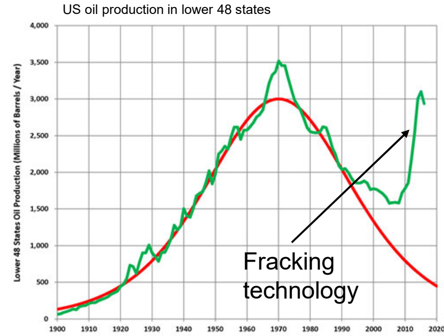
(*Source*: Ashby, 2009, Figure 2.14)



Inflation-adjusted Petroleum Prices, 1949-2006

Crude Oil Prices - 35 Years

US oil production in lower 48 states



Simple Modeling (Lynch, Section 1.1.1)

Variables: $S(t)$ = stock of the resource remaining
$X(t)$ = extraction rate
$P(t)$ = market price per unit production
at time t

Mass balance: $\dfrac{dS}{dt} = -X$ (resource dwindles as it is being extracted)

Price-sensitive demand: $X = \dfrac{a}{P^\beta}$ (high price, low demand, low extraction)

Price evolution: $\dfrac{dP}{dt} = rP$ (Hotelling's Rule with continuous time, as default choice)

Solution:

$$\frac{dP}{dt} = rP \quad \rightarrow \quad P(t) = P_0\, e^{rt}$$

$$X = \frac{a}{P^\beta} \quad \rightarrow \quad X(t) = \frac{a}{P_0^\beta} e^{-\beta r t}$$

$$\frac{dS}{dt} = -X \quad \rightarrow \quad S(t) = S_0 - \frac{a}{\beta r P_0^\beta}\left(1 - e^{-\beta r t}\right)$$

If you don't like using differential equations, use discrete math on an Excel spreadsheet.

$$P_{n+1} = (1+r)P_n \,, \quad X_n = \frac{a}{P_n^{\beta}} \,, \quad S_{n+1} = S_n - X_n$$

Plotting of solution: $X(t) = \frac{a}{P_0^{\beta}} e^{-\beta r t}, \quad S(t) = S_0 - \frac{a}{\beta r P_0^{\beta}}\left(1 - e^{-\beta r t}\right)$

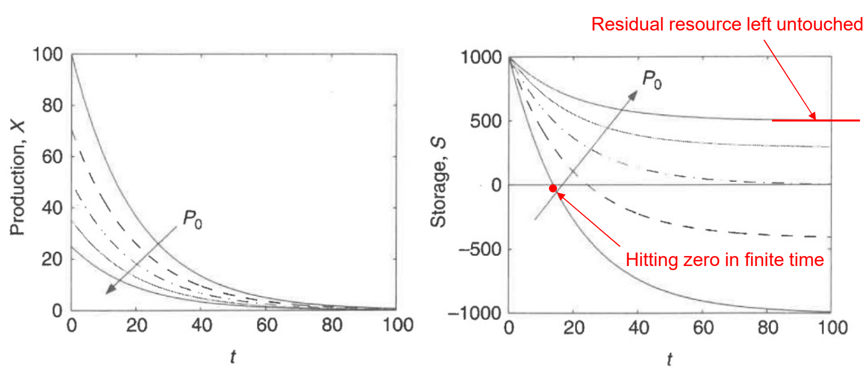


Figure 1.1. Five different depletion histories, identical except for initial price. P_0 increases by factors of 2 in the direction of the arrows.

We note that, depending on the rate of extraction (and thus on the market price), the amount of resources can reach zero in a finite time or level off at a residual positive value.

Terminal condition

$$S(t) = S_0 - \frac{a}{\beta r P_0^\beta}\left(1 - e^{-\beta r t}\right) \quad \rightarrow \quad S(\infty) = S_0 - \frac{a}{\beta r P_0^\beta} \qquad > 0 \text{ or } < 0?$$

If end value is < 0, it means that the resource is exhausted in a finite time T

$$S(T) = S_0 - \frac{a}{\beta r P_0^\beta}\left(1 - e^{-\beta r T}\right) = 0 \quad \rightarrow \quad T = \frac{1}{\beta r}\ln\left(\frac{a}{a - \beta r P_0^\beta S_0}\right)$$

If end value is > 0, it means that there is a leftover that will never be extracted:

$$S(\infty) = S_0 - \frac{a}{\beta r P_0^\beta}$$

The optimal use of the resource is $S(\infty) = 0 \quad \rightarrow \quad S_0 = \dfrac{a}{\beta r P_0^\beta} \quad \rightarrow \quad P_0 = \left(\dfrac{a}{\beta r S_0}\right)^{\frac{1}{\beta}}$
because, if so, we use it all across all generations.

This occurs for the initial production rate $\boxed{X_0 = \dfrac{a}{P_0^\beta} = \beta r S_0}$ $\qquad \beta r \leq \dfrac{1}{30 \text{ years}}$

Revenue

Rate of revenue = unit price times the number of units produced: $P\,X$

Cumulated over time, with proper discounting:

$$R = \int_0^\infty e^{-rt} P\, X\, dt$$ This is called the **Rent**.

Value:

$$R = \int_0^\infty \overbrace{e^{-rt} P}^{P_0} X\, dt = P_0 \int_0^\infty X\, dt = P_0 (S_0 - S_\infty)$$

(initial price times total amount extracted)

$R_{\max} = P_0 S_0$ when nothing left at the end

Of course, it is hard to achieve this optimal situation because
1. We don't know how much is there to begin with, and
2. We don't control the price; the market does.

So, we very much act in the dark.

In many situations, an external pressure not related to near exhaustion comes to change the dynamics.

Examples:
- Coal extraction much decreased when petroleum became available; it was just a matter of convenience (a liquid is more convenient than a solid);
- We are about to switch away from petroleum not because we are running out but because of the climate impact of carbon dioxide.
- We may be running of rare-earth minerals (needed for electronics) but, if we do, we can switch from mining new minerals to recycling our old electronics.

<u>Effect of a maximum price</u>: (Lynch, Section 1.1.2)

Consider the case when a high price causes buyers to switch to an alternative.

Extraction X ceases when the price P reaches a maximum $P_{\max}$, which is the price of the alternative.

Assume that this maximum price is known all along.
Thus, to optimize rent (income), producers seek to exhaust the resource by the time T when the price P reaches the maximum $P_{\max}$:

$$\left.\begin{array}{l} P_{\max} = P_0\, e^{rT} \quad \rightarrow \quad P_0^{\beta} = P_{\max}^{\beta}\, e^{-\beta r T} \\ 0 = S(T) = S_0 - \dfrac{a}{\beta r P_0^{\beta}}\left(1 - e^{-\beta r T}\right) \end{array}\right\} \rightarrow \quad \frac{\beta r S_0 P_{\max}^{\beta}}{a} = e^{\beta r T} - 1$$

$$T = \frac{1}{\beta r} \ln\left(\frac{\beta r S_0 P_{\max}^{\beta}}{a} + 1 \right)$$

Corresponding initial extraction rate:

$$X_0 = \frac{a}{P_0^{\beta}} = \frac{a}{P_{\max}^{\beta}} e^{\beta r T} = \beta r S_0 + \frac{a}{P_{\max}^{\beta}}$$

Hence, we extract more now if we know that we will have to stop at some point.

The preceding expressions were under the assumption that the maximum price was known since the beginning of the exploitation.

Should this not be the case, that is, if an alternative emerges at one point along the way, then simply consider the current values as new starting values.

Thus,

$$X = \beta r S + \frac{a}{P_{\max}^{\beta}} \qquad \text{adjusted optimal rate of extraction}$$

$$P = \left(\frac{a}{\beta r S + \frac{a}{P_{\max}^{\beta}}} \right)^{\frac{1}{\beta}} \qquad \text{adjusted current price}$$

$$T = \frac{1}{\beta r} \ln\left(\frac{\beta r S P_{\max}^{\beta}}{a} + 1 \right) \qquad \text{time remaining}$$

Under these conditions, the rent is:

$$R = \int_0^T e^{-rt} XP\, dt = P_0 \int_0^T X\, dt = P_0 (S_0 - 0) = P_0 S_0$$

unchanged since the entire resource is still produced equitably across the now-finite number of generations (until time T).

End of Part 1

Consumer's Surplus: (Lynch, pages 7-8)

The selling price is P, but consumers may value the resource at value V, higher than P : $V > P$, up to P_{max} the price of an alternative.

Now, V would be set by a supply-demand curve, as on the graph.

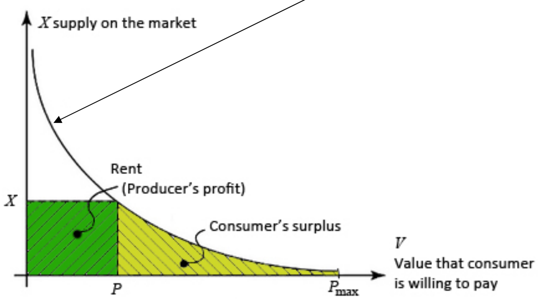


Figure 1.7. Demand curve as in Figure 1.6, adding the actual price P. The area to the right of the price line is the consumers' surplus; that to the left is the rent transferred to the seller.

The consumer pays P but was ready to pay up to P_{max}. Thus, the consumer's surplus is $P_{max} - P$ at any given time.

The cumulated surplus is obtained by integration of sale over price (yellow area).

Effect of discovery (Lynch, Section 1.3)

Imagine that the market prompts an effort to discover additional reserves.

Denote the rate of discovery by D and the remaining still undiscovered resources by U. The pertinent mass-conservation equations are:

$$\frac{dS}{dt} = -X + D$$
$$\frac{dU}{dt} = -D$$

These equations state that D takes away from U and adds to S, the known amount resource (zero-sum because of no creation).

Now, we need to say what controls the rate of discovery.
The reasoning is that it is easier to discover the resource if there is still more of it "out there." Thus, the rate D may be taken proportional to the undiscovered amount U:

$$D = \rho U$$

The coefficient ρ represents the ease (high ρ) or difficulty (low ρ) with which new discoveries occur for a given undiscovered stock. It is a function of existing technology and proclivity to invest in a discovery effort.

With $D = \rho U$, the remaining undiscovered amount U is dwindling according to:

$$\frac{dU}{dt} = -D = -\rho U \quad \rightarrow \quad U = U_0\, e^{-\rho t}$$

Since $D = \rho U \quad \rightarrow \quad \frac{dD}{dt} = -\rho D$

tracking $D(t)$ over time permits the estimation of the parameter ρ.

Obviously, U_0 remains a major unknown because we still don't know how much was "out there" as we are still in the process of discovering some of it.

To $D = \rho U = \rho U_0\, e^{-\rho t}$, we now add the remaining dynamics with the existence of a maximum price at which production will cease:

$$X = \frac{a}{P^{\beta}} = \beta r S + \frac{a}{P_{\max}^{\beta}}$$

$$\frac{dS}{dt} = -X + D \quad \rightarrow \quad \frac{dS}{dt} = -\beta r S - \frac{a}{P_{\max}^{\beta}} + \rho U_0 e^{-\rho t}$$

The algebra to obtain the analytical solution is very cumbersome, but clearly the solution should look like:

$$S = (...) + (...)\, e^{-\rho t} - (...)\, e^{-\beta r t}$$

positive contribution due to discovery, but slowing down over time as it gets harder to discover more

negative contribution due to gradual exhaustion caused by extraction, like in basic model

$S_0 = 10$
$U_0 = 10$
$\rho = 0.10$

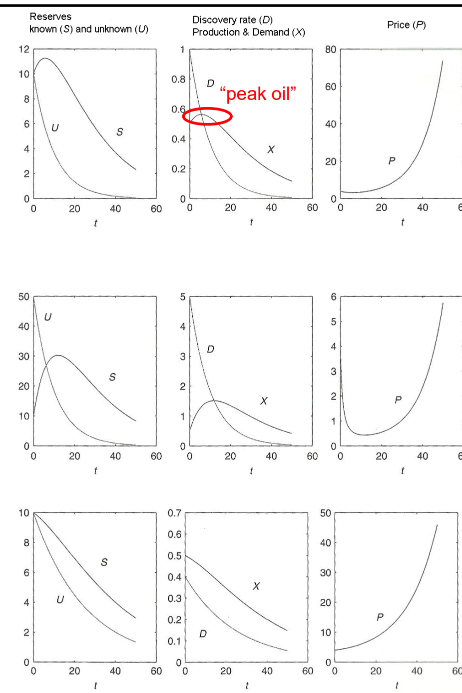
peak production at $t = 5.75$
with $S_{max} = 11.25$

$S_0 = 10$
$U_0 = 50$
$\rho = 0.10$

peak production at $t = 11.96$
with $S_{max} = 30.26$

Same as above but with
slower rate of discovery
$\rho = 0.04$

No peak production



For all cases:
$r = 0.10$
$\beta = 0.5$
No price limit

(*Source*: Lynch, 2009 - Figures 1.14 & 1.15, page 26)

Production with discovery and effort
(Lynch, Section 1.3.4)

Rationale:
Income attracts effort to discover

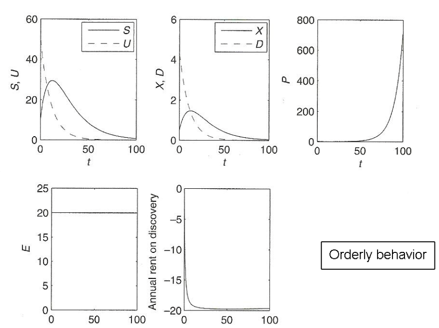


Figure 1.19. Time history with discovery adjusted as in Equations 1.142–1.148, with constant effort. Parameters: $r = .10$; $a = 1$; $\beta = .5$; $\bar{P} = \infty$; $\epsilon = .1$; $E^* = 10$; $p_d = P(t)$; $w = 1$; $\nu = 0$.

Effort of exploration E is modeled as

$$D = \frac{E}{E^*} U$$

Discovery D proportional to effort E

$$\frac{dE}{dt} = \nu \, \pi \qquad \pi = (\ldots) D - (\ldots) E$$

Rate of discovery

Effort

Coefficient of proportionality

Profit earned on production

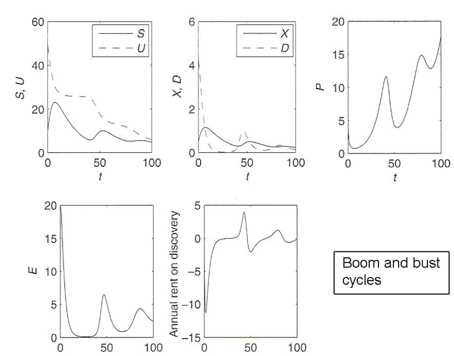


Figure 1.20. Same as in Figure 1.19, except rent-sensitive discovery $\nu = 0.3$.

Varying Demand (Lynch, Section 1.1.4)

We now consider the fact that the production rate X may be a function of more than only price P. In particular, it may also depend on customer factors unrelated to price, such as availability of alternatives, changing tastes, or new technologies (*ex*. less need for gasoline when electric cars get on the market).

The way to model this is to include a time variation in the numerator of the expression linking price to production:

$$X = \frac{a(t)}{P^{\beta}}$$

With price as before: $P = P_0\, e^{rt} \rightarrow \quad X = \frac{a(t)}{P_0^{\beta}} e^{-\beta r t}$

An interesting case is that of linear growth in demand (Lynch, pages 12-14).

In this case, we take: $a(t) = a_0 + a_1 t$

The mathematics give sequentially:

$$X(t) = \frac{a_0 + a_1 t}{P_0^{\beta}} e^{-\beta r t}$$

$$\frac{dS}{dt} = -X = -\frac{a_0 + a_1 t}{P_0^{\beta}} e^{-\beta r t} \quad \rightarrow \quad S(t) = S_0 - \frac{1}{\beta r P_0^{\beta}} \left[(a_0 + a_1)\left(1 - e^{-\beta r t}\right) - a_1 t e^{-\beta r t} \right]$$

The interesting aspect is the initial behavior of the production:

$$\frac{dX}{dt} = \frac{a_1 - \beta r \left(a_0 + a_1 t\right)}{P_0^{\beta}} e^{-\beta r t}$$

$a_1 > \beta r\, a_0 \quad \rightarrow \quad \left.\frac{dX}{dt}\right|_{t=0} > 0, \quad X$ grows initially

$a_1 < \beta r\, a_0 \quad \rightarrow \quad \left.\frac{dX}{dt}\right|_{t=0} < 0, \quad X$ always decays

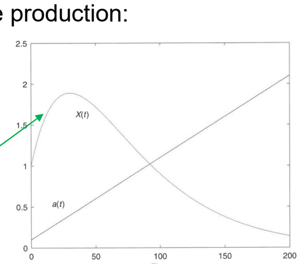


Figure 1.10. Time history of production with linear growth in demand. Parameters: $a_0 = 0.1$; $a_1 = 0.01$; $X_0 = 1.$; $\beta = 0.5$; $r = .05$. Production peaks at $t^* = 30$.

Costly Production (Lynch, Section 1.2.2)

If we denote the cost of extraction per unit produced as C, then the profit to the producer is $P - C$ per unit produced.

The rent is likewise adjusted downward:

$$R(t) = \int_0^t e^{-rt}\left(P - C\right) X \, dt$$

Generally, the cost of extraction is related to the abundance of the resource: The more there is (larger S), the easier and cheaper it is to extract some of it, and the cost increases as the resource becomes more scarce ($C\uparrow$ when $S\downarrow$). Thus, $C = C(S)$, a decreasing function.

Consider a time interval Δt, such as one quarter (3 months) or one year. The present contribution to rent plus that of the next consecutive period Δt is:

$$R_{12} = R_1 + R_2 = \left(P_1 - C(S_1)\right) X_1 \, \Delta t + \frac{\left(P_2 - C(S_2)\right)}{1 + r\Delta t} X_2 \, \Delta t$$

The producer may ask:
What if I anticipate a small portion ΔX of production from period 2 to period 1?

To answer this question, we consider the change in the above income when

$$X_1 \to X_1 + \Delta X, \quad X_2 \to X_2 - \Delta X, \quad S_2 \to S_2 - \Delta X \, \Delta t$$

$$C(S_2) \to C(S_2) - \frac{dC}{dS} \Delta X \, \Delta t$$

Some algebra shows that the change to the two-period rent above is:

$$\Delta R_{12} = \left(P_1 - C(S_1)\right) \Delta X \, \Delta t - \frac{\left(P_2 - C(S_2)\right)}{1 + r\Delta t} \Delta X \, \Delta t + \frac{1}{1 + r\Delta t} \frac{dC}{dS} \Delta X \left(X_2 - \Delta X\right) \Delta t^2$$

The point of indifference corresponds to no net change in rent:

Higher-order differential

$$\Delta R_{12} = 0 \quad \rightarrow$$

$$\left(P_1 - C(S_1)\right) - \frac{\left(P_2 - C(S_2)\right)}{1 + r\Delta t} + \frac{1}{1 + r\Delta t}\frac{dC}{dS}(X_2 - \cancel{\Delta X})\,\Delta t = 0$$

Re-arranging, we obtain sequentially:

$$\left(P_2 - C(S_2)\right) = (1 + r\Delta t)\left(P_1 - C(S_1)\right) + \frac{dC}{dS} X_2\,\Delta t$$

$$\frac{\left(P_2 - C(S_2)\right) - \left(P_1 - C(S_1)\right)}{\Delta t} = r\left(P_1 - C(S_1)\right) + \frac{dC}{dS} X_2$$

In the limit of a short time interval:

Time discounting (inflation)

called “stock” effect

$$\frac{d\left(P - C(S)\right)}{dt} = r\left(P - C(S)\right) + \frac{dC}{dS} X$$

Finally, recalling the mass balance $\frac{dS}{dt} = -X$

we have $\frac{dC}{dS} X = -\frac{dC}{dS}\frac{dS}{dt} = -\frac{dC}{dt}$

and the point of indifference corresponds to:

$$\frac{d\left(P - \cancel{C(S)}\right)}{dt} = r\left(P - C(S)\right) - \cancel{\frac{dC}{dt}}$$

$$\frac{dP}{dt} = r\left(P - C(S)\right)$$

This is the decision rule under costly production.

We note that the cost of production C slows down the rate of price increase.

Closing remarks
(Lynch, page 42)

The only possible fate of a non-renewable resource is EXHAUSTION.

But exhaustion may take three different forms:

1. Physical exhaustion – We just run out of the resource.

2. Economic exhaustion – The resource can no longer be produced economically.

3. Political exhaustion – Extraction of the resource has been made illegal.

More closing remarks for Chapter 1
(Lynch, page 32)

A sterile resource, like petroleum, has only one fate, exhaustion.

Time to exhaustion is likely to be finite (as when there is a maximum price above which demand vanishes).

Implication for sustainability:

- A substitute needs to be found during the extraction process before exhaustion.

- Some of the accumulated income (rent) needs to be invested in the search for the alternative, or one has to learn "how to do without." Either way, there is a learning curve.

- The cumulative learning ought to be the sustainable aspect.

Ultimately, however, substitution alone is inadequate if it progresses through a series of finite resources exhaustions.
There must be a "Sustainable Finale"!