

Beauty Bare: The Sonnet Form, Geometry and Aesthetics

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Beauty is the first test: there is no permanent place in the world for ugly mathematics.¹

Euclid alone has looked on Beauty bare...²

The appeal of the sonnet form over centuries is both unquestionable and curious. Most poets attempt a sonnet at one time or another in their career, and many readers in English, as well as other Western languages, have encountered a sonnet or two. While some might see the sonnet as an old-fashioned form, poets continue to produce them in great quantities, and new collections and studies of them continue to appear. Zachariah Wells, editor of one sonnet anthology, attributes the form's longevity to its "adaptability, flexibility, plasticity" through history.³ Others are more mystical in their ideas about the persistence of the sonnet: "Poets," Don Paterson reflects, "write sonnets because it makes poems easier to write. Readers read them because it makes their lives easier to bear".⁴ There is a long-held perception that sonnets teach us something about the very nature of poetry, especially in relation to poetic form. This is frequently cited as the justification for insisting that creative writing students try to write them, and that literature students learn to read them. But aside from the structural rigour of the form – with the attendant assumption that the more restrictive the form, the greater the poetic challenge (haikus have a similar status in this respect) – the actual reasoning behind the idea of the sonnet's edifying qualities remains vague.

Similarly, the sonnet's aesthetic qualities are frequently proclaimed, but slippery and difficult to articulate. Paul Oppenheimer writes that the sonnet has a "mysterious aesthetic" that reveals "a psychological, as well as an aesthetic, law, or equation, or archetype" that makes it one of the most "secure and enduring forms in poetry", but he remains vague as to what that is, precisely.⁵ Paterson connects the aesthetic and the psychological appeal of the sonnet: "a miraculous little form in which our human need for unity and discontinuity, repetition and variation, tension and resolution, symmetry and asymmetry, lyric inspiration and argumentative rigour, are all held in near-perfect oppositional balance" (xxvi-xxvii). It is frequently seen as having a meditative quality (xvi) – an interiority, which Oppenheimer suggests is due to its non-musical, purely literary, history of circulation. He provides convincing proof that despite the popular idea of *sonnet* meaning "little song," it was always

¹ G.H. Hardy, *A Mathematician's Apology*. Cambridge: Cambridge University Press, 1941, p.25. All subsequent references are to this edition and are given in the text.

² Edna St. Vincent Millay, 'Euclid Alone has Looked on Beauty Bare,' in *The Harp-Weaver and Other Poems*. 1920. New York: Harper, 1923, 74. All subsequent references are to this edition and are given in the text.

³ Zachariah Wells, *Jailbreaks: 99 Canadian Sonnets*. Toronto: Biblioasis, 2008, p.11. All subsequent references are to this edition and are given in the text.

⁴ Don Paterson, 'Introduction' in *101 Sonnets*. ed. Don Paterson. London: Faber, 1999, xi-xxvii., pp.xxvii. All subsequent references are to this edition and are given in the text.

⁵ Paul Oppenheimer, 'The Origin of the Sonnet.' *Comparative Literature* 34 (1982), 289-304., p.290. All subsequent references are to this edition and are given in the text.

intended for the written page.⁶ This textual presentation history is part of the aesthetic power for Paterson:

As poetry moved slowly off the tongue and onto the page, the visual appeal of an approximately square field on a sheet of white paper must have been impossible to resist. Which is what a sonnet *is*, first and foremost: a small square poem. It presents both poet and the reader with a vivid symmetry that is the perfect emblem of the unity of meaning a sonnet seeks to employ. (xvi)

Phrases such as "vivid symmetries" suggest that the sonnet's beauty might be connected to other fields in which symmetry is also frequently cited as an aesthetic component; in, for example, science and mathematics. Indeed one sonnet – Edna St. Vincent Millay's "Euclid Alone Has Looked on Beauty Bare" – seems to take as its topic the very idea of an aesthetic link between mathematics (and by extension science) and the sonnet form:

Euclid alone has looked on Beauty bare.
 Let all who prate of Beauty hold their peace,
 And lay them prone upon the earth and cease
 To ponder on themselves, the while they stare
 At nothing, intricately drawn nowhere
 In shapes of shifting lineage; let geese
 Gabble and hiss, but heroes seek release
 From dusty bondage into luminous air.
 O blinding hour, O holy, terrible day,
 When first the shaft into his vision shone
 Of light anatomized! Euclid alone
 Has looked on Beauty bare. Fortunate they
 Who, though once only and then but far away,
 Have heard her massive sandal set on stone. (74)

Millay's sonnet has been cited by scientists, mathematicians and literary critics alike to articulate a definition of beauty. The poem is included in mathematical textbooks and specialized mathematical studies.⁷ It is also found in histories of mathematics and books that explore mathematical intersections with history and culture.⁸ The sonnet – especially the first line – occasionally appears as an epigram.⁹ It is also found in biographies of mathematicians, like Paul Hoffman's biography of Paul Erdős, *The*

⁶ See: Paul Oppenheimer, 'The Origin of the Sonnet.'

⁷ See: Robin Hartshorne, *Geometry and Beyond*. New York: Springer, 2000, p.50; William McGown Priestley and F. W. Gehring, *Calculus: A Liberal Art*. 2nd edn. New York: Springer, 1998, pp.78-9; Philip J. Davis and Reuben Hersh, *The Mathematical Experience*. Boston: Birkhäuser, 1987, p.150.

⁸ See: William P. Berlinghoff and Fernando Quadros Gouvêa, *Math Through the Ages: A Gentle History for Teachers and Others*. Expanded Edition. New York: Mathematical Association of America, 2004, p.155; Robert Osserman, *Poetry of the Universe: A Mathematical Exploration of the Cosmos*. New York: Anchor-Random, 1995, p.6; Mario Livio, *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number*. New York: Broadway, 2002, p.3.

⁹ See: H. E. Huntley, *The Divine Proportion: A Study in Mathematical Beauty*. New York: Dover, 1970, p.xiii.

Man Who Loved Only Numbers.¹⁰ From a literary perspective the entire sonnet is a frequent inclusion in poetry anthologies and sonnet collections, and it has the honour of being the first text presented in Denis Donoghue's appendix on notable quotations on beauty in *Speaking of Beauty*.¹¹ Finally, the poem is frequently cited in online sources related to both mathematics and literary studies.

The sonnet's first line is often isolated as some sort of definitive statement about the relationship between science and beauty. The phrase "beauty bare," in particular, seems to resonate within different reading communities, but the meaning of this phrase is not really unpacked. The word "bare" seems to be uncomplicatedly self-evident and accessible. An indicator of plainness, it is apparently resistant to exegesis. This ironically produces a barrenness of meaning; the word is unanalysable because of its apparent semantic obviousness. This initiates a series of contradictory attributes concerning the nature of beauty. The light imagery that Millay attaches to the idea of Beauty, for example (Beauty is "luminous air" and "light anatomized" that "shone" for Euclid, in a "blinding hour") does not illuminate, but obscures – endarkens – our understanding. The repetition of the light images without specific detail as to *what* is shining results in an impenetrable glare of imagery: an interpretive snow-blindness. The impenetrability of Millay's imagery provoked a protracted debate in *Explicator* from 1942 to 1948 over whether or not the poem referred to Euclid's theory of optics; no satisfactory conclusions were reached, and the matter was quietly dropped after 1948.¹² Similarly, Euclid's "bare" vision is a "shifting lineage" that is "intricately drawn", yet is also "nothing," "nowhere." It is "massive": it is physically huge, cosmic, and beyond the scope of human grasp, but it also *possesses mass*, in that it is the indefinite and intangible embodied. It is obvious and overt, but also intimate; "bare" in the sense of naked and revealed, which places Euclid in the role of both a lover and a voyeur. Even syntactically, the word "bare" performs double duty; it could modify either Beauty herself or Euclid's own vision: "bare Beauty" or "looked upon . . . bare," as in a gaze undertaken without mediation.

Despite the abstraction and impenetrability of the central adjective of the first line, the *idea* of beauty in this sonnet retains its association with transparency, crispness and clarity. These qualities are connected to the study of geometry, which is probably the discipline that most readers imagine as the subject of the poem. The reader who first raised the problem of the sonnet's subject matter in *Explicator* noted that "generally the sonnet is read as if concerned with geometrical discoveries."¹³ The geometric principles of the "small square poem" are especially clear in Millay's text; the rhyme is simple and rigorous; a perfect, four-rhyme Petrarchan (or Italian) sonnet, with an octave and a sestet that can be further subdivided into quatrains and tercets. Therefore the idea of three and four pointed figures – triangles and squares – is implied in the structure of the poem. Like Euclid's ideal geometric figures, however, they are without dimension – they are not "real" aside from the quadrangle of the single stanza itself.

¹⁰ Paul Hoffman, *The Man Who Loved Only Numbers: The Story of Paul Erdős and the Search for Mathematical Truth*. New York: Hyperion, 1998, p.33.

¹¹ Denis Donoghue, *Speaking of Beauty*. New Haven: Yale University Press, 2003, p.179. All subsequent references are to this edition and are given in the text.

¹² See the first entry in this exchange: 'Q9: Millay's 'Euclid Alone has Looked on Beauty Bare', *Explicator* 1.1 (1942) n. pag. The ensuing debate can be followed by consulting the index of subsequent volumes of the journal.

¹³ See: 'Q9: Millay's 'Euclid,' n. pag.

True beauty, Millay suggests, belongs to the intangible absolutes of the mathematical visionary; the rest of us can only "gabble and hiss" about beauty. But reading a poetic text on the subject of mathematical beauty begs the question as to what extent mathematical beauty transfers to literary beauty? Certainly, the readership of this poem indicates that the text is meaningful to both scientists and literary scholars alike. But how strong is the relationship between scientific and poetic beauty generally, and is that relationship embodied in this poem? Beyond that, to what extent can we see scientific and poetic aesthetics sharing an interpretative space in other poems or in poetry generally?

Gillian Beer has noted that "stories of culture" tend to "go largely *undescribed*: symmetry, simplicity, development, hierarchy, chance, provide models, ideals, and implied narratives in science as much as literature." We might regard symmetry and simplicity as "ideals of scientific elegance," Beer observes, but the underlying aesthetic narratives or models of symmetry and simplicity – the idea of elegance itself – for example, tend to be "sequestered" from debate.¹⁴ Elegance, or beauty, simply *is*: it is just *bare*. Yet there are a number of important studies on the subject of science and aesthetic theory.¹⁵ For example, mathematicians and scientists like Paul Dirac, Roger Penrose, Jacques Mandelbrojt and Steven Weinberg have meditated on beauty in their professions. Commentaries on mathematical biographies and testimonies of particular scientific experiences frequently segue to meditations on beauty, as seen in G.H. Hardy's *A Mathematician's Apology* (1941), Henri Poincaré *Science and Method* (1952), Werner Heisenberg's *Across the Frontiers* (1974), and J.W.N. Sullivan's *The Limitation of Science* (1933).¹⁶ More general studies of aesthetics occasionally engage the issue of beauty in science; an early example is Francis Hutcheson's 1725 analysis *An Inquiry Concerning Beauty, Order, Harmony, Design*. In the twentieth century, Roger Fry's canonical *Vision and Design* (1920) also considered scientific aesthetics.

Ideas of mathematical and scientific beauty are frequently collapsed in these studies; indeed, mathematical aesthetics are frequently read as a foundation for scientific aesthetics, especially in our mathematically-determined scientific methods of today. For the mathematically inclined the idea of mathematical beauty is so self-evident as to be irreducible to analysis, and for the non-mathematically inclined it is so distant as to be almost incomprehensible: heard "once only and then but far away." While Millay's poem seems to be as much about the inexpressibility of aesthetic visions as it is about aesthetics – we are not permitted to see what precisely it is that Euclid is seeing, for example – it is still clearly meaningful to mathematicians, readers of literature and others. As an aesthetic object, the poem is self-reflective; embedded

¹⁴ Gillian Beer, *Open Fields: Science in Cultural Encounter*. Oxford: Clarendon, 1996, p.161.

¹⁵ The fullest treatments are found in the following book-length studies: Subramanyan Chandrasekhar, *Truth and Beauty: Aesthetic Motivations in Science*. Chicago: University of Chicago Press, 1987; Ernst Peter Fischer, *Beauty and the Beast: The Aesthetic Moment in Science*. trans. Elizabeth Oehlkers. New York: Plenum, 1999; James McAllister, *Beauty and Revolution in Science*. Ithaca: Cornell University Press, 1996; Arthur I. Miller, *Insights of Genius: Imagery and Creativity in Science and Art*. New York: Springer, 1996, as well as the proceedings of the 1980 Nobel Conference edited by Deane W. Curtin: Deane W. Curtin, ed. *The Aesthetic Dimension of Science: 1980 Nobel Conference*. New York: Philosophical Library, 1982.

¹⁶ See also chapters by Poincaré and Morse in Raymond G. Ayoub, ed. *Musings of the Masters: An Anthology of Mathematical Reflections*. Washington, DC: Mathematical Association of America, 2004, pp.17-30, 81-96; and Paul Hoffman's biography of Paul Erdős: Paul Hoffman, *The Man Who Loved Only Numbers*.

in its meditation on mathematical beauty is an interrogation of poetic beauty, and the relationship between the two. It asks us to consider if the text can possess the same transcendent aesthetic qualities that the mathematical visionary engages in, and, more subtly, it interrogates the aesthetics of the sonnet form specifically – arguably, the most significant single poetic form in our literary inheritance in English, which acts as an archetype for the idea of poetic form generally.¹⁷

So how does this sonnet – “Euclid Alone has Looked on Beauty Bare” – work as a link between literary and mathematical aesthetics? The meaning of the poem is found not in its literal applications, but in its actions, which are experienced through its form of the English-language version of the Petrarchan, or Italian, sonnet form. The form of the sonnet itself is inherently mathematical, and the aesthetic functions of the mathematical elements of the sonnet are responsible for much of the form’s beauty, as well as its deeper meanings.

Shifting Lineages: Sonnet Form and Mathematics

Previous scholarship has explored numbers and numeric symbolism in poetic forms, including sonnets, within the broader context of numerical mysticism of classical, medieval and early modern cultures. S.K. Heninger and Alastair Fowler have contributed substantial studies of Platonic and Pythagorean numerology in English literature.¹⁸ More recently, Marcia Birken and Anne C. Coon’s *Discovering Patterns in Mathematics and Poetry* established a broad range of connections between the two fields, with a special focus on using these common grounds as a productive teaching strategy. There has even been a single-author study on the subject: William Goldbloom Bloch’s *The Unimaginable Mathematics of Borges’ Library of Babel*.

The sonnet form is usually of interest in such studies of mathematical-literary relationships, but many of their engagements are under-theorized; Birken and Coon provide several examples of the various patterning of the fourteen-line stanza (including “Euclid Alone has Looked on Beauty Bare”) although they do not extend their discussion to mathematical formulae or theories. Fowler’s detailed analysis of numerology and Elizabethan poetry includes a chapter on the sonnet sequence, although he does not discuss the sonnet as a single stanza. Paul Oppenheimer and Don Paterson have provided the most sophisticated analyses of the aesthetic relationship between mathematics and poetry; both discuss the sonnet’s relationship to the proportions of the Golden Section, and Paterson has also drawn connections to the Fibonacci sequence. In order to make these theories work, however, both Oppenheimer and Paterson have to tweak the sonnet structure slightly. To reconstruct the Golden Ratio, for example, Oppenheimer suggests that the last two lines of the stanza be seen as connected to, but distinct from, the previous twelve (302-3). This constructs a numerical idea within the core of the sonnet of 6:8:12, which is the harmonic proportion in the Renaissance architecture of rooms (303). Similarly, Paterson also explores the sonnet’s connection to the Fibonacci sequence

¹⁷ Obviously this may be challenged, but insofar as the sonnet has remained emblematic of poetry generally, and it has not, like the verse romance, merged with prose forms over the last 500 years, it is a fair assessment of the form’s importance. One is hard pressed, for example, to find significant numbers of whole print collections devoted only to other single poetic forms.

¹⁸ See: S.K. Heninger Jr., *Touches of Sweet Harmony: Pythagorean Cosmology and Renaissance Poetics*. San Marino, California: Huntington Library, 1974; Alastair Fowler, *Triumphal Forms: Structural Patterns in Elizabethan Poetry*. Cambridge: Cambridge University Press, 1970. See also Marianne Shapiro’s study of number symbolism in the sestina form: Marianne Shapiro, *Hieroglyph of Time: The Petrarchan Sestina*. Minneapolis, MN: University of Minnesota Press, 1980, pp.10-13.

(1,1,2,3,5,8,13, 21, etc), which itself is related to the Golden Mean and which is patterned in the sonnet up to 13; the extra 1, he suggests should be taken in the mathematical equation from the repeated 1 that starts the sequence, so the sonnet form adds to 14 (xviii-xix).

Both of these theories are workable and probably accurate in a general respect; there is no question that the Golden Ratio was a dominant force in Renaissance aesthetics and that it would have been read into poetic forms including the sonnet.¹⁹ Furthermore, the Fibonacci numbers and the Golden Ratio have mathematical connections to each other, so it makes sense that they can both be seen in the sonnet.²⁰ But although creative, both Oppenheimer's and Paterson's theories lack a certain mathematical elegance, since they require us to rework the basic fourteen line form in some respect – with the result that the mathematical figures are more *alluded to* than *represented in* the text. While these allusions are important to the meaning of the sonnet form, the significance of numbers and their behaviours is more than allusive in relation to mathematical constructs in the sonnet form (and in Millay's sonnet specifically). The sonnet form contains within it a mathematical theorem that is very beautiful – and quite literal: the Petrarchan English sonnet, with the octave/sestet/iambic pentameter stanza, embodies two geometrical constructs exactly: the Pythagorean Theorem and the Primitive Pythagorean Triple.

We can construct the Pythagorean Theorem out of the three primary numeric components of the sonnet; 8 (the octave), 6 (the sestet) and 10 (the number of syllables in each line): $8^2+6^2=10^2$; $64+36=100$. In this respect, the sonnet form does not merely *represent* the Pythagorean Theorem, it both *is* it and it *does* it. The form is symbolic but it also *enacts* the elegant mathematical form. But not only does the Italian sonnet form embody – or perform – one of the classically beautiful mathematical theorems, but it divides down to another essential mathematical beauty: the Primitive Pythagorean Triple.

If we take any triangle and multiply all of its sides by 2, we arrive at a scaled-up version of the original triangle; it has all of the same angles as the original triangle and looks exactly the same (except enlarged). In mathematics, these triangles are called "similar triangles." The 6, 8, and 10 triangle of the Italian sonnet is simply a scaled up version of 3, 4 and 5; this is also true for 9, 12, and 15; 12, 16 and 20, as well as 15, 20, and 25. In this sense, 3, 4, and 5 is the parent to all of these other triples and it is the most primitive of all of them since it cannot be subdivided any further, as long as we want to work in integers.

This idea of the Primitive Pythagorean Triple is analogous to irreducible fractions; mathematicians always write fractions in their lowest terms (instead of writing 12/8 or 6/4 they would write 3/2). Therefore, the sequence 3, 4, and 5 has special significance in this respect, since it characterizes an entire family of solutions to the Pythagorean Theorem. While the 3/4/5 triple isn't the only Primitive Pythagorean Triple,²¹ it is significant that all Primitive Pythagorean Triples can be generated from the 3/4/5 triangle by use of three relatively simple algorithms. This

¹⁹ On the cultural influence of the Golden Ratio see: H. E. Huntley, *The Divine Proportion*; and Mario Livio, *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number*. New York: Broadway, 2002.

²⁰ Richard Dunlap explores these connections extensively. See: Richard A. Dunlap, *The Golden Ratio and the Fibonacci Numbers*. New Jersey: World Scientific, 1997.

²¹ 5, 12, and 13; 7, 24, and 25, etc. are also primitive solutions. In fact, there are infinitely many Primitive Pythagorean triples, each characterizing their own separate family of solutions to the Pythagorean Theorem. See Eves 45-6 and 80-2.

means that 3, 4, and 5 is the *most* primitive of all Primitive Pythagorean Triples; it can be used to generate all of the others. The 3/4/5/ triple may be regarded, therefore, as the mother of all solutions, which captures perfectly both the centrality and the generative function of the sequence. Furthermore, in addition to being the smallest Primitive Pythagorean Triple that can generate all other Primitive Pythagorean Triples by a simple application, it also has the important feature that 3, 4, and 5 are consecutive numbers. For these reasons the 3/4/5 Primitive Pythagorean Triple holds much mathematical fascination, and is considered especially elegant. The Italian sonnet in English possesses this same reduction: sestet, octet and iambic pentameter can be subdivided into tercet, quatrain and pentameter.

One circumstance of the historical context of the invention of the sonnet is that it coincided with an era of mathematical innovation. The sonnet was invented in the court of Emperor Frederick II, probably by a courtier and notary named Giacomo da Lentino (or Lentini), whose fellow courtiers included Leonardo "Fibonacci" Pisano himself.²² Shortly before that time, Euclid's *Elements* was translated from Arabic into Latin by Gherardo of Cremona, making it available to European scholars at the end of the twelfth century.²³ It was this text that contained the first widely circulated formal statement and proof of the Pythagorean Theorem (Proposition 47 of Book 1). Therefore, the poetic innovators of the early thirteenth century that produced the sonnet, as well as other number-based forms like the sestina, the strambotto, and terza rima forms, were working within a mathematical renaissance of sorts, alongside mathematicians like Fibonacci who were interrogating the very nature of number, and therefore the very nature of space, time, nature and beauty.²⁴ The circulation history of Euclid's *Elements* corresponds with important moments in English literary history as well; it was first printed in 1482, just before the start of the Tudor dynasty that would produce the first English sonneteers.²⁵ Although manuscript evidence of the direct mathematical influence on literature is lost to history and must remain speculative, it would be reasonable to expect that the *Elements* would have contributed to the intellectual milieu of the Tudor court as it did in the court of Frederick II, and that it may have reinforced the links between mathematics and poetry as Sir Thomas Wyatt and the Earl of Surrey continued to experiment with number and pattern in the sonnet.

²² See: Paul Oppenheimer, 'The Origin of the Sonnet,' p.289; Don Paterson, 'Introduction,' p.xiii; Michael R. G. Spiller, *The Development of the Sonnet: An Introduction*. London: Twayne, 1992, p. 13; Ernest H. Wilkins, 'The Invention of the Sonnet.' *Modern Philology* 13 (1915), 463-94., p.463. While Paterson points out the connection between the Fibonacci sequence and the Golden Mean, which he feels is a significant aspect of the sonnet's construction, he fails to note the historical connection between Fibonacci and da Lentino (xviii). Regarding Fibonacci's name, see: Alfred S. Posamentier, and Ingmar Lehmann, *The Fabulous Fibonacci Numbers*. Amherst: NY: Prometheus, 2007, pp.17-18. On Da Lentino's poetry see: Ernest F. Langley, *The Poetry of Giacomo da Lentino: Sicilian Poet of the Thirteenth Century*. Cambridge, M: Harvard University Press, 1915.

²³ Howard Eves, *An Introduction to the History of Mathematics*. 6th edn. Philadelphia: Saunders College, 1990, p.261.

²⁴ On mathematicians and poets in the court of Frederick II see: David Abulafia, *Frederick II: A Medieval Emperor*. 1988. New York: Oxford, 1992, pp.251-89; Paul Oppenheimer, 'The Origin of the Sonnet,' pp.300-1; Michael R. G. Spiller, *The Development of the Sonnet*, pp.1-27; Georgina Masson, *Frederick II of Hohenstaufen: A Life*. 1957. London: Secker and Warburg, 1973, pp.112, 223-43.

²⁵ Eli Maor, *The Pythagorean Theorem: A 4,000 Year History*. Princeton: Princeton University Press, 2007, pp.72-3.

Beauty, Meaning, and the Sonnet's Mathematics

Both the Pythagorean Theorem and the Primitive Pythagorean Triple embedded in the English Petrarchan sonnet form are considered beautiful by mathematicians for several reasons. Most obviously, they both express ideas of symmetry and proportion. Of the Pythagorean Theorem, Johannes Kepler said: "Geometry has two great treasures: one is the theorem of Pythagoras, the other [the Golden Ratio]. The first we may compare to a measure of gold; the second to a precious jewel".²⁶ Centuries later, the mathematician and writer Charles Dodgson (Lewis Carroll) insisted the Theorem was "as dazzlingly beautiful now as it was in the day when Pythagoras first discovered it".²⁷ But there are two aesthetic qualities of the Pythagorean Theorem and Pythagorean Triples that should be of special interest to readers of the sonnet: both theorems are *generative* and *spatial* in nature: they are generative in the sense that they are procreative and prolific, and they are spatial in the sense that they interrogate the meaning of physical and intellectual space.

The Primitive Pythagorean Triple of 3/4/5 produces replicas of itself by being scaled up. The generative aspect of the Pythagorean Theorem is its applicability and connectivity to other ideas, mathematical and otherwise. In fact, Francis Hutcheson used the Pythagorean Theorem as his ideal example of beauty in theorems generally, which he defined as possessing "uniformity in variety", wherein the theorem offers a model that demonstrates the essence of a thing, which can in turn be generalized to express a multitude of examples, however various they may seem (such as right-angled triangles of different sizes).²⁸ Hutcheson's requirement is fulfilled by the Pythagorean Theorem in that it reveals a universal truth of every possible manifestation of right-angled triangles according to the axioms of Euclidean geometry. Its application to a theoretical infinity of examples makes it especially beautiful. More recently, in his definition of mathematical aesthetics, G.H. Hardy required "significance" of an elegant or beautiful mathematical statement. The significant (or "serious") theorem connects in a "natural and illuminating way" to things outside itself, so as to shed light on our larger understanding of the nature of number, and perhaps on human understanding in general (29).

These qualities of the Theorem and the Triple can be seen to share functions with the sonnet form, which itself is generative. Sonnet experimentation and innovation produced new forms and new understandings of the nature of literary form, especially in English. Wyatt's import into the young English literary culture put new demands on the artistic possibilities of the language.²⁹ Not only were new literary forms spawned through the experimental energy of such project, but the language itself flexed and grew to accommodate the structural rigour of the form. Commentators have noted that the sonnet, while providing a structural model that is rigorous enough to determine the form, is nevertheless wondrously adaptable, and is thus both beautiful and enduring. As Wordsworth reflected, the sonnet, though

²⁶ Quoted in Eli Maor, *The Pythagorean Theorem*, p.47.

²⁷ Charles L. Dodgson, *Curiosa Mathematica: Part 1, A New Theory of Parallels*. London: MacMillan, 1888, p.x.

²⁸ Francis Hutcheson, *An Inquiry into the Origin of Our Ideas of Beauty and Virtue in Two Treatises*. ed. Wolfgang Leidhold. Indianapolis, IN: Liberty Fund, 2004, p.36.

²⁹ See: Michael R. G. Spiller, *The Development of the Sonnet*, pp.83-101; Elizabeth Heale, *Wyatt, Surrey and Early Tudor Poetry*. London: Longman, 1998, p.118; F. W. Bateson, *English Poetry and English Language*. New York: Russell, 1961, pp.26-34.

structured, is not a restrictive "narrow room."³⁰ Instead, the fact that the sonnet is "adept at daring escapes and covert crossings" of its own form is integral to that form.³¹ In such work, the sonnet form fulfills Hardy's requirement of being significant and illuminating, and, as Wordsworth's metaphor suggests, the form is implicitly interrogating physical space itself, and this is where it connects most directly with mathematics.

Spatially, both the sonnet and the Pythagorean Theorem also have something to teach us, and their engagement with physical space contributes directly to the mathematical aesthetics that underlie their respective forms. The fact that the Pythagorean Theorem extends beyond any specific example to illustrate the nature of all Euclidean right angles connects us to the issue of space. Jacob Bronowski called the Pythagorean Theorem "the most important single theorem in the whole of mathematics," noting that "what Pythagoras established is a fundamental characterisation of the space in which we move".³² This is true most especially in the architectural applications of the theorem: "the Pythagorean Theorem," Michio Kaku writes, "is the foundation of all architecture; every structure built on this planet is based on it".³³ The relationship between the Pythagorean Theorem and the Primitive Pythagorean Triple is part of architectural history; ancient builders knew that ropes composing the 3/4/5 triangle could be used to form right angles long before mathematicians like Pythagoras stated the more general theorem that characterized *all* right-angled triangles. In other words, the builders knew that 3/4/5 was an extremely useful and beautiful relationship, but they did not actually know *why* that was the case until the theorem.

The Pythagorean Theorem articulates spatial integrity; its essential beauty is its articulation of the relationships of wholes and parts, and its demonstration of how they are unified: "the result is magical and of immense usefulness" Bryon E. Wall concludes.³⁴ This spatial significance of Euclid's visionary mathematics is captured by Millay in the phrase "light anatomized", which constructs light as a physical body that can be dissected, deconstructed and even reshaped by the geometer. The true resonance of Millay's meaning is actually a sort of "active pun": breaking the word "anatomized" into its semantic parts – anatomizing it – releases "atom", which is Greek for "indivisible", implicitly, then, a whole. While we associate anatomy with dissection, its more general meaning is simply to divide the indivisible, and the mystical associations with this idea play into our feelings about the human body, the earth, and light itself. Dividing the indivisible (anatomy literally means "un-dividing"³⁵) is what a geometer (earth-measurer) does, and what Euclid could do to an extent that was impossible for average thinkers: his dimensionless lines and points divided the entirety of light.

³⁰ William Wordsworth, 'Nuns Fret Not at their Convent's Narrow Room' in *Wordsworth: Poetical Works*. ed. Thomas Hutchinson. Oxford: Oxford University Press, 1973, p.199.

³¹ Zachariah Wells, *Jailbreaks*, p.11.

³² Jacob Bronowski, *The Ascent of Man*. London: BBC, 1974, pp.160-1.

³³ Michio Kaku, *Hyperspace, A Scientific Odyssey through Parallel Universes, Time Warps, and the 10th Dimension*. New York: Oxford University Press, 1994, p.37.

³⁴ Byron E. Wall, 'Proposition I.47 of Euclid: The Pythagorean Theorem,' in *Science in Society: Classical and Contemporary Readings*. ed. Byron E. Wall. Toronto: Wall and Thompson, 1989, 55-8., p58. On the study of parts and whole – or "mereology" – in literature see: Amanda Jernigan, 'Wholes and Parts (All Puns Intended): The Mereological Vision of Richard Outram's Poetic Sequences.' MA thesis. Memorial University of Newfoundland, 2007. Especially pp.1-14.

³⁵ StevenConner, *Fly*. London: Reaktion Books, 2006, p.90.

Similarly, the beauty of the sonnet is often read as the way in which it provides form for another indivisible space – mental space: the sonnet is, according to Paterson, “one of the most characteristic shapes human thought can take” (xxvii). Wells writes that sonnets “are built the way that people think and speak and argue” (11); the containment on thinking imposed by the rigour of the form, particularly the octave-sestet construction, imparts a holism of identity. In Millay’s text the subject (Euclid) and the object (Beauty) of the poem are joined as the mathematician “seeks release” in the “luminous air” of Beauty’s light, which in turn penetrates his vision with “light anatomized.” In his history of the form, Oppenheimer reads its structure as possessing a similar type of investment in the unification of psychological spaces that have previously been perceived as discrete. The sonnet form, specifically the way in which the octave and sestet sections are distinct but also irrevocably part of the sonnet unit, is meant to reconcile the split between the poet and his conventional courtly-lover personae that had been the dominant model for lyric poetry into the 13th century. It is one of the sonnet’s marks of modernity, he suggests, that the form “will solve the problem” of the “*persona* split into rival *personae*” (299). This is what he calls the psychological work of the sonnet; the poet, he writes, “addresses himself not to any outsider but to the form itself” (299). The sestet (6) and the octave (8) are like the two perpendicular legs of the right angled triangle, representing the distinct poetic split or fork. The thing that structurally binds them and reconciles the split is the iambic pentameter (10), which persists through the entire poem and would represent the hypotenuse of a right-angled triangle; ultimately, the hypotenuse completes the triangle and makes it a closed geometrical figure.

The ultimate experience of Millay’s sonnet is, then, the experience of meditating on the relationship between its parts and the whole. Just as Euclid meditated on dimensionless lines of anatomized light and saw their relationship to each other, the reader of this sonnet – and of sonnets in general – experiences a beauty of the “conformity of the parts to one another, and to the whole,” which Werner Heisenberg declares to be the best definition of mathematical beauty.³⁶ The Pythagorean Theorem articulates an underlying structure or relationship that hitherto had gone unexpressed, except in discrete, disconnected instances: like the ancient builders, the mystical persistence of the sonnet indicates that we recognize the usefulness and beauty of the sonnet form without really understanding *why* it is so. There is an underlying structure or relationship that has gone unnoticed (in both cases, the Pythagorean Theorem). The unveiling of this structure may be the “bare beauty” experienced by visionaries like Pythagoras or Euclid; it is the difference between “prating” about a single instance or physical example of beauty – like a single 3/4/5 triangle – and knowing the pure and basic form of that beauty that cannot be intricately drawn.

The closed form of the right-angled triangle determined by its own dimensions mimics the psychological, meditative work of the form, which is to demonstrate its own integrity. Perhaps this is the psychological and aesthetic law, equation, or archetype that Oppenheimer senses as the mystery of the sonnet form (290). For Paterson, the sonnet presents a “unity of meaning”:

³⁶ Werner Heisenberg, *Across the Frontiers*. trans. Peter Heath. New York: Harper, 1974, p.167.

something that is impossible to represent in any sustained, linear, complex utterance – but it's crazily, what our human poetry tries to do. So a sonnet is a paradox, a little squared circle, a mandala that invites our meditation." (xvi).

This experience of integrity (from the Latin wholeness, soundness, uprightness, honesty, as well as "integer" – untouched) may fulfil a deeper psychological desire: "The urge to understand is the urge to embrace the world as a unity," the mathematician Marston Morse wrote, "to be a man of integrity in the Latin meaning of the word".³⁷ This extends the meaning of the Pythagorean form of the sonnet beyond the original Renaissance interest in Platonic ideals that may have motivated the mathematical experimentation with poetic forms. We do not have to subscribe to archaic philosophical perspectives to appreciate the beauty, however contingent, of expressions of integrity. The sonnet, with its Pythagorean structure, allows us to experience even briefly Virginia Woolf's "triumph" and "consolation" of seeing the "perfect dwelling-place" of ideas: "The structure is now visible; what is inchoate is here stated; we are not so various or so mean".³⁸ We may still have some cultural nostalgia for such an integral form of Beauty, even if culturally it is "but far away," that brings us back to the sonnet form again and again.

Conclusion

It is sensible to assume that Edna St. Vincent Millay was aware of the Pythagorean perfection of her favoured poetic form, and that "Euclid Alone has Looked on Beauty Bare" is her meditation on the synthesis between mathematical and literary aesthetics. She was, after all, a consummate sonneteer and had an intimacy with the form that few others would understand.³⁹ Similarly, the early Renaissance poets – and perhaps many who came later – likely recognized the mathematical structures within the form as being so obvious as to not merit comment; such structures were endemic to their intellectual and artistic worlds, and as such, they became one of those undescribed stories of culture that we have since forgotten. In rediscovering them we can gain knowledge about the original creative impulses of our poetic inheritances, as well as finding new significance for these structures in our own literary worlds.

Employing Millay's sonnet as a case study illuminates larger conclusions that go beyond the issue of the mathematical significance within this one poem, or even within the tradition of one poetic form. Mathematical forms – numbers and number theorems – are not inert, passive or purely descriptive structures. Literary form, in that it has a spatial and temporal presence in poetry (it takes up space on the page, it involves measurements of time in language) is not merely symbolic – representation is not its only level of participation. Denis Donoghue considers form to be intrinsic to

³⁷ Marston Morse, 'Mathematics and the Arts' in *Musings of the Masters: An Anthology of Mathematical Reflections*. ed. Raymond G. Ayoub. Washington, DC: Mathematical Association of America, 2004, 81-96., p.90.

³⁸ Virginia Woolf, *The Waves*. ed. Gillian Beer. Oxford: Oxford University Press, 1992, p.134.

³⁹ On Millay's sonnets see: Cedric Barfoot, 'Edna St. Vincent Millay's Sonnets: Putting 'Chaos into Fourteen Lines,' in *Uneasy Alliance: Twentieth Century American Literature, Culture and Biography*. ed. Hans Bak. Costerus New Series 150. Ropodi: Amsterdam, 2004, 81-100; Judith Farr, 'Elinor Wylie, Edna St. Vincent Millay, and the Elizabethan Sonnet Tradition,' in *Poetic Traditions of the English Renaissance*. ed. Maynard Mack and George de Forest Lord. New Haven: Yale University Press, 1982, 287-305; Debra Fried, 'Andromeda Unbound: Gender and Genre in Millay's Sonnets.' *Twentieth Century Literature* 32 (1986), 1-22; Michael R. G. Spiller, *The Development of the Sonnet: An Introduction*. London: Twayne, 1992 68-76.

and inseparable from the meaning of the text ("meaning" as opposed to "content" or even "subject"); "Form is the achieved, purposed deployment of energy, energy available on need and not there till looked for" (123). The idea of the text's *meaning*, which is additional to its subject, might be aligned with Hardy's idea of mathematical *significance*. In its *meaning* the text is active, it performs its own subject in a way that pushes the reader beyond the literal interpretation to a more emotional and aesthetic experience. It connects to things (readers, other texts, larger ideas) outside of itself that were not consciously built into its narrative – hence it can connect to a future. Michael Wood has discussed this as the inherent knowledge of form, placing an autonomous interpretative level within form that precedes the reader's interpretation, which we experience emotively, sensually, physically, as well as intellectually.⁴⁰ We recognise this formal knowledge in our experience of reading the poems, and we respond emotionally and aesthetically. A keystone form like a Primitive Pythagorean Triple, or a sonnet, can support whole new systems of knowledge, and contain a generative energy that ripples outward from its core, seeking "release / From dusty bondage into luminous air". Of course, not all sonnets discuss mathematical or scientific subjects directly: very few of them do, in fact. But certainly all sonnets engage aesthetics generally and an aesthetics of form specifically, and on both those levels they are connected historically and structurally to principles of mathematical and scientific beauty, and so on those levels it is fair to say that the *meaning* of the sonnet form can be connected to scientific aesthetics. In an era where science is profoundly mathematical, and mathematics is a language that most non-scientists don't speak, it is a beautiful idea that poetics may have the capacity to silently and covertly "speak" beauty mathematically, bringing us back to the shared intellectual heritage of science and literature.

Mathematics in poetry is always a creative application, and therefore any sort of absolutist assertion of a mathematical "formula" for the sonnet runs counter to the poetic impulse from *any* era. Poets, early or late, who would pick up such a project must do it with the spirit of experimentation and play. In this case of the mathematics in the sonnet, the synthesis of form and formula must be opened up to the more extended meanings of both the sonnet form and the Pythagorean structures. Ultimately, their true common ground (and true common beauty) is what they mean to us *beyond* their rote formulations: and these ideas (space, integrity, generation) could be transferred to other ideas and forms, poetic and mathematic, far beyond those examined in this essay. We can recognize, without being reductive, that mathematical and poetic forms sometimes come from very similar intellectual and creative places. Our reconnections of these shared heritages between the disciplines must always be modest, however; mathematics and poetry must retain their distinct qualities; words and numbers have a different resonance, and these differences must also be recognized.

These differences are acknowledged in Millay's sonnet through her handling of the gender conventions of the sonnet tradition, especially in relation to the traditional lyric subjectivity of the sonneteers. Constructing Beauty as female retains the convention of the chilly erotic power of the courtly lady (such as the hind – *noli me tangere* – in Wyatt's "Whoso List to Hunt"). But in Beauty's "massive" size and dominance over the lover-mathematician we also see a loosely Freudian maternity; Beauty is the mother of all solutions in an expanded sense, a sort of sandaled lover-

⁴⁰ See: Michael Wood, *Literature and the Taste of Knowledge*. Cambridge: Cambridge University Press, 2005, p.136.

mother goddess of the classical pantheon, like Juno. Beauty holds the mathematician in thrall from all her sources of feminine power. Similarly, the conventional poet-lover-first person subject of the traditional sonneteer is loosened by Millay's division between the (male) mathematician and the (female) poet; they remain analogous, but not identical. As a woman writing about a man, and in eschewing the first-person lyric voice of the traditional courtly sonneteer, Millay puts distance between herself and the poetic subject, Euclid. She lives in the realm of those who never really connect with the level of beauty that Euclid does. Euclid's singularity – his aloneness – adds a modern dimension to the alienation of the conventional courtly lover from the beloved; the subject of the poem is also separated from the poet and the expected total identification between the audience and the speaker is frustrated; we can hear Beauty, but we cannot see her.

Our limited access to Euclid's experience is due to our dependence on language itself. In Millay's sonnet, Beauty is a feminized version of the common mathematical expression "God is a mathematician". This formulation is more philosophical than theological in its meaning; the divinity of mathematics is usually presented (although not always) as a shorthand for the issue of mathematical realism or Platonism. This idea of an ultimate mathematical reality and/or divinity has a long tradition and is still in play today.⁴¹ The issue of whether or not mathematics should be considered the basis of reality is contentious, and probably irresolvable. What is important is that Millay, the early sonneteers, and many other poets, are interested in the aesthetic possibilities of mathematical realism. Obviously, for a poet working in words, this is a challenge. Millay seems to want to address the issue from the mathematician's perspective, which reserves the ultimate aesthetic experience for the geometer, but to represent that experience in language. This is an enterprise that is obviously doomed to fail. Her use of the blinding light imagery calls our attention to this problem, and she instead locates the idea of the beauty of the poem – which is the object of inquiry here – in the form, not the descriptive language. It reminds us that poetry works on levels beyond language, and that something like form may be more connected to other levels of meaning – in this case mathematics – than linguistic meaning.

Thinking about science and literary form in this way involves two complementary methodologies: the investigation of the direct, causal relationship between literature and science, and a second approach, which examines indirect and non-casual relationships between the two disciplines. In this study, the first method is found in the historical linkages between mathematical and poetic innovators like Fibonacci and da Lentino, as well as the cultural context of Pythagorean principles of Renaissance poetics. But the true resonances in thinking about relationships between science, literature, form and aesthetics are not found in the causal arguments, but in a more fluid realm of ideas, where we may obtain a "sense of the movement of ideas from context to context" in order to "emphasize the accidental, the partial, and the metaphorical".⁴² Recognizing the aesthetic function of the sonnet form in relation to the Pythagorean Theorem is part of the expanded mandate of science and literature studies, as determined by scholars like Gillian Beer and N. Katherine Hayles, who challenge us to "avoid stabilizing the argument so that one form of knowledge

⁴¹ A recent and very accessible history of mathematical realism is found in Mario Livio's *Is God a Mathematician?*.

⁴² Alice Jenkins, *Space and the "March of the Mind": Literature and the Physical Sciences in Britain 1815-1850*. Oxford: Oxford University Press, 2007, p.142.

becomes again the origin of all others".⁴³ We can recognize the influence of Pythagorean numerology on poetics, but we should avoid viewing this as a strictly causal and static relationship. In the realm of aesthetics, the mathematical and literary forms work together and in fact exemplify ideas of unity and integrity in space and consciousness. This is not an issue of precedence of knowledge: the Pythagorean Theorem and the Primitive Pythagorean Triple do not tell the sonnet what to think. Rather, they are part of the shared knowledge of beauty in form. The knowledge that is shared between the Pythagorean Theorem and the sonnet is the idea of integral quality of the forms themselves – the principle of unification that is transmitted by the structural qualities and their attendant meanings.

⁴³ See: Gillian Beer, *Open Fields*, p.177. See also Hayles, who poses the challenges the dominant methodology of establishing scientific knowledge as the casual force on literary knowledge: Katherine N. Hayles, 'Turbulence in Literature and Science: Questions of Influence,' in *American Literature and Science*. ed. Robert J. Scholnick. Lexington, KT: University of Kentucky Press, 1992, 299-50., p.229.

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