The Supply of Non-Renewable Resources*

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Abstract

There exists no formal treatment of non-renewable resource (NRR) supply, systematically deriving quantity as function of price. We establish instantaneous restricted (fixed reserves) and unrestricted NRR supply functions. The supply of a NRR at any date not only depends on the contemporary price of the resource but also on prices at all other dates. Besides the usual law of supply, which characterizes the own-price effect, cross-price effects have their own law. They can be decomposed into an intertemporal substitution effect and a stock compensation effect. We show that the substitution effect always dominates: a price increase at some date causes NRR supply to decrease at all other dates. This new but orthodox supply setting extends to NRR the partial equilibrium analysis of demand and supply policies. The properties of restricted and unrestricted supply functions are characterized for Hotelling (homogenous) as well as Ricardian (non homogenous) resources, for a single deposit as well as for several deposits that endogenously come into production or cease to be active.

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1. Introduction

In this note, we formulate a simple theory of resource supply that extends the standard microeconomic modeling of supply to non-renewable resources. A basic tenet of the theory of supply is the exogeneity of price; we assume the path of producer prices to be given and we study the effect of a price change at any date of the path. This highly orthodox approach is new: it extends to non-renewable resources the time-honored partial equilibrium method of shifting demand and supply curves. It has immediate applications to analyze the green paradox, spatial carbon leakages or other policy-induced changes in extraction.

We are following up on a task first addressed by Gray (1914) and formally undertaken by Burness (1976). They specifically inquired about the effect on the extraction path of changing the exogenous price, assumed to be constant throughout the extraction period. Sweeney (1993) later attempted to reconcile the economics of non-renewable resources with conventional supply theory, by deriving the static resource supply as a function of the contemporary producer price. As he put it, "static supply functions, so typical in most economic analysis, are inconsistent with optimal extraction of depletable resources." (p. 780). In the textbook formulation of a supply function, producers take price as given and choose production to maximize profits. In the discrete formulation adopted here, supply depends on a vector of exogenous prices applying throughout the relevant period rather than on a single price. We distinguish short-run (restricted) and long-run non-renewable resource – hereafter NRR – supply functions, and we study the fundamental properties of these functions.

When total cumulative extraction is taken as given, an exogenous price change occurring at any date modifies the relative marginal profit from extracting at that date relative to other dates and thus entails a *pure intertemporal substitution* effect; this is the essential cause of the green paradox as initially formulated (Long and Sinn, 1985; Sinn, 2008). A change in the resource price path faced by producers may also affect ultimately exploited reserves, both because some existing reserves may become economic or cease to be economic as a result of the price change, and because exploration and discoveries as well as reserve development are affected by the price change. We call this the *stock-compensation effect* and we show that the substitution effect always dominates the compensation effect. Consider an increase in price at some particular date leaving prices at all other dates unchanged; the pure intertemporal substitution effect increases supply at that date and reduces supply at all other dates; the stock effect results in an increase in ultimately extracted reserves. It follows that supply at the date of the price rise increases as the stock effect and the substitution effect work in the same direction; this is the NRR version of the law of supply. At all other dates, since the substitution effect dominates the stock effect, it follows that supply diminishes; this is the law of intertemporal substitution in NRR supply.

When the time dimension is combined with a space dimension as in Fischer and Salant (2013), the same result applies to a price change occurring at a point in space and time. Thus the intertemporal substitution effect is accompanied by an analogous spatial substitution effect: a price rise at any point in space and time reduces supply at all other points.

These results are simple; yet the literature abounds in papers addressing variations on their theme. For example, much theoretical and policy research involves the green paradox, which is rightly presented as a current supply reaction to policies affecting future demand for fossil fuels. Similarly many environmental policy issues revolve around the leakage of resource flows across regional markets. In most of these problems, demand is just ordinary and it is supply that needs to be explained.

Besides prices, the cost of production is the basic element underlying supply. Resource extraction technologies are often peculiar; extraction costs may depend on past extraction and may reflect spatial or geological constraints. We give due consideration to these issues in the final part of the paper where our results are shown to be robust to the peculiarities of extraction technologies.

Nonetheless, an even more crucial difference between a NRR and a conventional good arises from the resource rent. The supply of a conventional good or service is independent from the price of another good or service as long as their production costs are independent; the supply of a NRR at one point in time and space is affected by its price at other dates and locations even when production costs, by which we mean extraction, transformation, transportation, etc, are separable across dates and locations. This is because the so-called "augmented" marginal production cost includes a resource scarcity rent, the opportunity cost of extracting the scarce resource, that connects economic costs at all dates and locations to each other: a change in the price of the resource at any date or outlet affects the rent at all dates and outlets, which in turn affects supply even though the pure accounting components of resource production costs are typically independent.

The resource rent drives the substitution and compensation effects identified above. These effects are reminiscent of demand theory: resource producers allocate a stock of resource to different dates and outlets in a way that is comparable to the way consumers allocate their income to different expenditures on different goods. The time space or spatial space play a similar role as the good space in static demand. The stock of reserves is not unlike the budget constraint in demand theory, as both set a cap on quantities that can be allocated to alternative supplies or to expenditures on alternative goods; furthermore, both constraints are affected by prices, although by different channels.[1] Unlike the Giffen paradox, the supply of a NRR always increases if its price rises: the law of supply always holds. Similarly, inferior goods have no counterpart in NRR supply: given a price vector, supply does not diminish at any date if reserves are exogenously increased.

Reserves to be used for extraction are produced by costly exploration and development technologies. This is an important aspect of resource supply. We assume that the stock of reserves is produced prior to extraction via exploration and development. Reserves are

[1] As we make clear further below, the analogy is not an isomorphism. For one thing preferences are price independent unlike profits.

developed and then completely exhausted, as in Gaudet and Lasserre (1988) and Fischer and Laxminarayan (2005). In the words of Gordon (1967), "Properties are explored only because firms want to exploit them." (p. 277). This does not mean that all resources in the earth crust are exhausted; but resources not to be exploited are left undeveloped until, if ever, it becomes rational to sink costs into them.[2]

Exploration and development are sensitive to the rent that accrues to the extractor during the exploitation of the resource. This rent is affected by future supply prices and in turn determines the stock compensation effect mentioned earlier. Furthermore, the constitution of reserves is necessarily subject to decreasing returns to scale as exploration prospects are finite. If such was not the case, NRRs would be indefinitely reproducible like conventional commodities in the long run; the resource rent would reduce to the constant quasi-rent associated with expenditures in exploration and reserve development; the intertemporal substitution effect of NRR supply would not materialize.

For ease of exposition, we initially focus in Section 2 on a NRR from a single Hotelling type deposit with a single outlet per date, postponing until sections 3 and 4 a general interpretation with the time space combined with a spatial dimension, and until Section 5 formulations involving multiple deposits or stock dependent extraction costs. In Section 2, a parsimonious model of NRR supply illustrates how the effect of a price change occurring over part of the extraction period can be decomposed into a substitution effect and a stock compensation effect and allows us to show that the substitution effect dominates the compensation effect. Section 2 also draws the distinction between short term and long term, and adapts the well-known concepts of restricted (McFadden, 1978) cost and supply functions to the case of NRRs.

It is customary to use the apparatus of supply and demand to study policies. This

[2]Following Gordon (1967), Gerlagh (2011), Hoel (2012), van der Ploeg and Withagen (2012) and Grafton, Kompas and Long (2012) in their analysis of the green paradox, consider that some part of the resource may be left unexploited; the same is true of the Herfindahl-type model of Fischer and Salant (2013). This can be rational if no costs are experienced to develop the resource, or if an unexpected change in the economics of extraction, such as a drop in the price, makes the extraction of developed reserves uneconomic despite the costs already sunk in them.

is the realm of partial equilibrium analysis. Applying this apparatus to NRR markets requires taking into account the intertemporal nature of resource supply. The supply analysis of Section 2 allows us to do that. This is outlined in Section 3 under perfect competition or in presence of market power; technical details are relegated to the Appendix. As an example, we show that the green paradox holds under very general conditions. As a second example we reestablish some results on spatial leakages and we show how to address resource taxation.

NRRs are heterogenous. In Section 5, with details in the Appendix, we consider the two major alternative approaches that have been used in the literature to deal with resource heterogeneity, each corresponding to a particular cost structure. One is the model initiated by Gordon (1967), where the cost of extraction increases with cumulative extraction, under the assumption that resources of easier access or higher quality are exploited first. This model has been further explored and perfected among others by Weitzman (1976) and Salant, Eswaran and Lewis (1983) and is being used in a recent literature that investigates the green paradox (e.g. Gerlagh, 2011; Hoel, 2012; van der Ploeg and Withagen, 2012; Grafton, Kompas and Long, 2012). The second approach, which may be attributed to Herfindahl (1967), considers a multiplicity of different deposits; it has given rise to a series of papers that study the optimum sequence and possible overlap of deposit exploitation (e.g. Amigues, Favard, Gaudet and Moreaux, 1998; Gaudet and Lasserre, 2011; Salant, 2012). In the version analyzed here, deposits have different costs of extraction and different costs of exploration and development; the timing of deposit development and exploitation is endogenous and is part of the producer's supply problem. We show that the substitution effect dominates the compensation effect under both approaches, but we also argue that the second approach is a better way to model supply as it does not rely on implicit assumptions about the optimal extraction sequence and as it avoids the issue of resource supply aggregation.

2. A Synthetic Theory of Non-Renewable Resource Supply

A quantity $x_t \geq 0$ of a NRR is supplied at each of a countable set of dates $t = 0, 1, 2, ...$ The initial stock $X > 0$ of the resource is finite and treated as exogenous at this stage, with $\sum_{t \geq 0} x_t \leq X$. The present-value producer price is denoted by $p_t \geq 0$. The stream of prices $p \equiv (p_t)_{t \geq 0}$ is taken as given by the producers and treated as exogenous at this stage.[3] The present-value cost of producing a quantity x_t is denoted by $C_t(x_t)$,[4] where the function C_t may be time varying, is increasing and twice differentiable, and satisfies $C_t''(x_t) > 0$. For simplicity, we also assume[5] $C_t(0) = 0$ for all dates $t \geq 0$ and $C_t'(x_t) > p_t$ for at least one date so that some exploitation is warranted.

The stock of reserves to be exploited by a mine does not become available without some prior exploration and development efforts. Although exploration and exploitation often take place simultaneously at the aggregate level (e.g. Pindyck, 1978, and Quyen, 1988; see Cairns, 1990, for a comprehensive survey of related contributions), at the microeconomic level of a deposit they occur in a sequence, as in Gaudet and Lasserre (1988) and Fischer and Laxminarayan (2005). This way to model the supply of reserves is particularly adapted to the problem under study because it provides a simple and natural way to isolate the effect of an anticipated price change on the size of the exploited stock at the firm level. Specifically, assume that the present-value cost[6] $E(X)$ of developing an initial, exploitable stock X at date 0 is twice differentiable, increasing, strictly convex, and satisfies $E(0) = 0$ and $E'(0) = 0$. The property $E'(0) = 0$ that the marginal cost of reserves development is zero at the origin is introduced because it is sufficient to ensure

[3] We will show how the results carry over to a partial-equilibrium setting where prices are endogenously determined on markets.

[4] The cost function C_t results from the minimization of the cost of producing x_t given factor prices, with factor prices omitted from the notation (e.g. Varian, 1992, p. 26). Some authors model extraction costs as dependent on the stock of reserves to reflect the fact that increasing scarcity justifies incurring higher costs to reach and extract the resource. This would imply here to define the cost function as a restricted or short-run cost function $C_t(x_t, X_t)$, treating the remaining stock of reserves X_t as a quasi-fixed input (p. 26). We discuss this modeling and deal more explicitly with resource heterogeneity in Section (**5.**).

[5] The qualitative results follow through in presence of a fixed cost or lump sum tax.

[6] In this cost function also, input prices are omitted from the notation.

that a positive amount of reserves is developed. It thus rules out uninteresting situations where resource prices do not warrant the production of any reserves.

Since the development of reserves is costly, the optimum plans of the producers will always bind the exhaustibility constraint. In other words, leaving part of the developed stock ultimately unexploited does not maximize cumulative net discounted revenues. For a given price sequence p, the cumulative value function corresponding to a producer's optimum is

$$\max_{(x_t)_{t\geq 0}, X} \sum_{t\geq 0} (p_t x_t - C_t(x_t)) - E(X) \tag{1}$$

subject to

$$\sum_{t\geq 0} x_t = X. \tag{2}$$

Denoting by λ the Lagrange multiplier associated with constraint (2), the necessary first-order conditions characterizing the optimum extraction path are

$$p_t - C_t'(x_t) \leq \lambda \text{ with } (p_t - C_t'(x_t) - \lambda)\, x_t = 0,\ \forall t \geq 0, \tag{3}$$

i.e. at dates where extraction is strictly positive[7]

$$p_t - C_t'(x_t) = \lambda,\ \forall t \geq 0,\ x_t > 0. \tag{4}$$

For the choice of initial reserves, the first-order condition is

$$E'(X) = \lambda. \tag{5}$$

Expression (4) is the Hotelling rule stating that the marginal profit from extraction must be constant in present value over the period of active exploitation, equal to λ, the unit present value of reserves underground, called the Hotelling scarcity rent. (5) is a standard supply relationship that sets marginal cost equal to price. The price in this case is the unit scarcity rent and is defined implicitly; in other words reserves are the output of

[7] If the price is too low at some date, production may be interrupted before exhaustion, and start again once prices are high enough. The first-order condition during production interruptions (it must also hold after exhaustion) is $p_t - C_t'(x_t) \leq \lambda$ for all $x_t \geq 0$, that is equivalently $p_t - C_t'(0) \leq \lambda$.

a production process whose technology is described by the cost function E. However reserves are not like conventional goods that can be produced under constant returns to scale, because of the scarcity of exploration prospects. The supply of reserves is thus a strictly increasing function of the rent:[8]

$$X = X(\lambda) \equiv E'^{-1}(\lambda). \tag{6}$$

The Hotelling rule (4) implicitly defines the solution of Problem (16)-(2) as a series of functions x_t giving extraction at each date.[9] Each function is increasing in the current price p_t and decreasing in the rent λ:

$$x_t = x_t(p_t, \lambda), \ \forall t \geq 0. \tag{7}$$

As Sweeney (1993) noted, functions like (7) can be interpreted as conventional static supply functions, whose only arguments are the price p_t of the extracted resource and the reserve price λ. However λ is not a conventional price; unlike standard price parameters, it corresponds to the "shadow" or implicit valuation of reserve units and thus is endogenous to the resource producer problem. Formulating regular supply functions thus further requires expressing the rent λ as a function of the vector of exogenous prices p.

Treating the stock of initial reserves as given at this stage and combining all relations (7) into (2), we obtain that the rent is a function increasing in all prices in $p \equiv (p_t)_{t \geq 0}$ and decreasing in the stock X; we denote that function with a tilde, as all functions of given reserves:

$$\lambda = \widetilde{\lambda}(p, X). \tag{8}$$

Substituting (8) into (7) gives the restricted supply functions, one at each date:[10]

$$x_t = \widetilde{x}_t(p, X) \equiv x_t \left(p_t, \widetilde{\lambda}(p, X) \right), \ \forall t \geq 0. \tag{9}$$

[8]The finiteness of exploration prospects amounts to a fixed factor being imposed on the production process. Hence reserves are produced under rising marginal costs.

[9]During extraction interruptions or after exhaustion, the function takes a null value.

[10]A standard restricted supply function depends on the output price, on the prices of variable factors, and on the quantity of at least one restricted factor. Here variable-factor prices are the prices of the factors entering the extraction technology, omitted from the notation for simplicity, and the restricted factor is the initial stock of reserves.

These functions do not make use of the first-order condition for initial reserves. Conditional on the initial reserve stock X and given the sequence p of prices, they determine how the suppliers allocate extraction from the stock to different dates. Unlike the restricted supply of a conventional good which only depends on its own price and on the quantity of some factor, the restricted NRR supply function at t further depends on resource prices at all other dates. This is so despite the fact that the same standard technological assumptions hold: the extraction cost at one date does not depend here on the extraction cost at another date, just as when the cost of producing a conventional good is independent of the cost of producing another good.

Hotelling's lemma is obtained from the optimized value function by use of the envelope theorem for constrained problems. That is, substituting (9) and (8) into the Lagrangian function associated with Problem (16)-(2) and differentiating with respect to p_t, while holding the restricted level of X and its multiplier as well as all extraction rates constant, gives the restricted supply at t. [11]

The restricted NRR supply function $\widetilde{x}_t$ for any date $t \geq 0$ is strictly increasing in X. Holding the reserve level unchanged, consider the partial effects of prices, that is the *direct price effects*. $\widetilde{x}_t(p, X)$ is strictly increasing in p_t and strictly decreasing in any p_T, $T \neq t$. This can be shown as follows. By (9), $\frac{\partial \widetilde{x}_t(p,X)}{\partial p_T} = \frac{\partial x_t\left(p_t, \widetilde{\lambda}(p,X)\right)}{\partial p_T} + \frac{\partial x_t}{\partial \lambda} \frac{\partial \widetilde{\lambda}(p,X)}{\partial p_T}$, where the first term on the right is zero unless $T = t$, as $x_t(p_t, \lambda)$ is not directly dependent on prices other than the contemporary price. The second term is clearly negative whether $T = t$ or $T \neq t$ since x_t decreases in λ while $\frac{\partial \widetilde{\lambda}(p,X)}{\partial p_T}$ is clearly positive since a rise in the resource price at any date cannot reduce the rent. It follows that $\frac{\partial \widetilde{x}_t(p,X)}{\partial p_T}$ is negative for $T \neq t$ while a contemporary rise in price involves two effects working in opposite directions. However, if extraction diminishes at all dates $t \neq T$, it must increase at $t = T$ for otherwise reserves would not be exhausted, which would be suboptimal as already discussed: the law of supply also applies to restricted supply. Consequently, in case of a

[11] Hotelling's lemma is obtained similarly in the case of non-restricted supply functions defined further below. The non-restricted value function is obtained by replacing the restricted level of X and the rent λ by their optimized values $X^*(p)$ and $\lambda^*(p)$ defined shortly below.

contemporary price rise, the direct price effect given by the first term must dominate the second term that operates via the resource rent.

Consider the choice of initial reserves. While (6) is a standard stock supply relation, the price λ is not a standard exogenous price but an endogenous variable. The supply of reserves at the producer's optimum in Problem (16)-(2) can be expressed as a function of exogenous prices. The value of the unit rent at the producer's optimum satisfies $\lambda = \widetilde{\lambda}(p, X)$. By (6), the optimum amount of reserves satisfies $X = X(\lambda) = X\left(\widetilde{\lambda}(p, X)\right)$, which implicitly defines X and λ as functions of p:

$$X = X^*(p) \text{ and } \lambda = \lambda^*(p) \equiv \widetilde{\lambda}\left(p, X^*(p)\right). \tag{10}$$

Thus the supply of reserves depends on the whole sequence of resource prices, although this can be summarized into one single rent.[12]

Restricted supply or factor demand as well as restricted cost or profit functions are usually interpreted as representations of the short run. In the long run, the restricted factor is variable. This interpretation is adequate here, exploration and reserve development being analogous to capital investment. Just as capital goods are produced, reserves in (10) are the outcome of a production process. Then they are used as a factor of production in the resource production process that generates the restricted supply (9).[13]

The optimal (unrestricted) NRR supply functions are defined as

$$x_t^*(p) \equiv \widetilde{x}_t\big(p, X^*(p)\big),\ \forall t \geq 0. \tag{11}$$

Like the restricted supply, the (unrestricted) supply of a NRR differs from a conventional supply function under identical standard technological assumptions in that it not only depends on its own price, the current price, but also on the prices at all other dates.

[12]As before, factor prices are omitted for notational simplicity from the reserve supply function. They are the prices of the factors entering the extraction process because they affect the optimum rent, but also the prices of the factors entering the exploration and development process which are omitted arguments of the E cost function.

[13]Although this is not usually modeled, capital does get depleted (worn out) by production at a rate that depends on the rate of production. However conventional capital can be replenished in a plant while this is not, or only partially, true of the reserves of a mine. On the related subjects of resource substitution and sustainability, see the huge literature initiated with the 1974 Symposium of the Review of Economic Studies.

The comparative statics properties of NRR supply are thus defined over a wider set of variables than those of a conventional supply function. With conventional supply functions, attention is usually limited to the law of supply, the effect of a change in the price of the good supplied.[14] With NRR, supply cross-price elasticities, the effect on supply at t of changes in prices at other dates, are also of theoretical interest: as we shall see, they obey their own law.

The resource literature has seldom considered such exogenous price changes, and never in a systematic treatment of supply. One exception is Burness (1976) who forced prices to be constant in current value and investigated the effect on production of a simultaneous change in all prices. Another exercise frequent in the resource taxation literature (e.g. Dasgupta, Heal and Stiglitz, 1981) has been to ask what time profile of taxes would be neutral. Both Burness' and Dasgupta, Heal and Stiglitz's findings result from the following property of the restricted NRR supply function. Let $A \equiv (a)_{t\geq 0}$ be a vector of equal constants a with the same dimension as p. Assume that $a > -\widetilde{\lambda}(p, X)$ for any given level X. Then it is easily shown that $\widetilde{\lambda}(p + A, X) = \widetilde{\lambda}(p, X) + a$ and $\widetilde{x}_t(p + A, X) = \widetilde{x}_t(p, X)$; if a is a unit tax such as a unit severance tax, this shows that the tax is neutral if it is constant in present value. This result does not extend to the unrestricted supply function, when initial reserves are allowed to adjust (Gaudet and Lasserre, 2013). Indeed, if $a < 0$ so that $\widetilde{\lambda}(p + A, X) < \widetilde{\lambda}(p, X)$, the initial reserve level is lower at prices $p + a$ by (6) so that the NRR supply at all dates is reduced.

Let us now turn to standard comparative supply analysis: what is the effect of a change in price at date T on supply at date t. One must distinguish between a change at the same date $T = t$ and a change at $T \neq t$. From (11), this decomposes into a *direct price effect* and a *stock compensation effect*:

$$\frac{\partial x_t^*(p)}{\partial p_T} = \frac{\partial \widetilde{x}_t(p, X^*(p))}{\partial p_T} + \frac{\partial \widetilde{x}_t(p, X^*(p))}{\partial X}\frac{\partial X^*(p)}{\partial p_T} . \tag{12}$$

When $T = t$, the total price effect may be called the *own price effect*; since $\widetilde{x}_t$ is increasing

[14] We ignore factor prices for simplicity.

in both p_t and X, and as resource prices always affect developed reserves positively, the own price effect is positive. Expression (12) when $T = t$ indicates that the law of supply holds and illustrates the Le Châtelier principle, which says that the long-run (unrestricted) elasticity is higher than the short-run (restricted) elasticity.

When $T \neq t$, the direct price effect in (12) may be called the *pure substitution effect* as it reflects the reallocation of an unchanged reserve stock to extraction at a date different from T; (9) makes clear that this substitution effect only arises via the effect of the rent on the $\widetilde{x}_t$ function: $\frac{\partial \widetilde{x}_t(p,X)}{\partial p_T} = \frac{\partial x_t(p_t,\lambda)}{\partial \lambda} \frac{\partial \widetilde{\lambda}(p,X)}{\partial p_T}$. Also by (9), the *stock compensation effect* of opposite direction can be itself decomposed into $\frac{\partial \widetilde{x}_t(p,X)}{\partial X} \frac{\partial X^*(p)}{\partial p_T} = \frac{\partial x_t(p_t,\lambda)}{\partial \lambda} \frac{\partial \widetilde{\lambda}(p,X)}{\partial X} \frac{\partial X^*(p)}{\partial p_T}$ so that the total cross-price effect can be factorized as follows:

$$\frac{\partial x_t^*(p)}{\partial p_T} = \frac{\partial x_t\left(p_t, \lambda^*(p)\right)}{\partial \lambda} \left[\frac{\partial \widetilde{\lambda}\left(p, X^*(p)\right)}{\partial p_T} + \frac{\partial \widetilde{\lambda}\left(p, X^*(p)\right)}{\partial X} \frac{\partial X^*(p)}{\partial p_T} \right], \ T \neq t, \tag{13}$$

where the term between brackets turns out to be the total derivative of $\widetilde{\lambda}(p,X)$ with respect to p_T, decomposed into a direct price effect at constant initial reserves, and the effect on the rent of the change in initial reserves induced by the price change. Resource prices at all dates affect the rent positively, i.e. $\frac{\partial \lambda^*(p)}{\partial p_T} \geq 0$, $\forall T$.[15] Consequently,

$$\frac{\partial x_t^*(p)}{\partial p_T} = \frac{\partial x_t\left(p, \lambda^*(p)\right)}{\partial \lambda} \frac{\partial \lambda^*(p)}{\partial p_T} \leq 0, \ \forall t \neq T, \tag{14}$$

implying that the stock compensation effect is never high enough to offset the pure substitution effect.[16]

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[15]Formally, the definition of $X^*(p) = X\left(\widetilde{\lambda}\left(p, X^*(p)\right)\right)$ yields $\frac{\partial X^*(p)}{\partial p_T} = \frac{X'(\lambda)\frac{\partial \widetilde{\lambda}(p,X^*(p))}{\partial p_T}}{1 - \frac{\partial \widetilde{\lambda}(p,X^*(p))}{\partial X} X'(\lambda^*(p))}$, implying that the term between brackets in (13) can be factorized as $\frac{\partial \lambda^*(p)}{\partial p_T} = \frac{\partial \widetilde{\lambda}(p,X^*(p))}{\partial p_T}\left(\frac{1}{1 - \frac{\partial \widetilde{\lambda}(p,X^*(p))}{\partial X} X'(\lambda^*(p))}\right)$, which is positive since $\frac{\partial \widetilde{\lambda}(p,X)}{\partial X}$ is negative. By (10), it also follows that $\frac{\partial X^*(p)}{\partial p_T}$ is positive.

[16]We have assumed decreasing returns to the development of reserves – increasing marginal cost of development, i.e. strict convexity of the cost function E. This assumption reflects the finiteness of extraction and exploration prospects and is essential to the result.

Suppose on the contrary that the development of reserves were subject to constant returns to scale: $E(X) = eX$. As before, λ would give the present value of each reserve unit so that $\lambda = e$. The rent, thus determined by the technology, would then be insensitive to variations in prices p, and resource supply at t would only depend on current resource price by (9). Constant returns to scale in the development of X make all cross-price effects on extraction vanish, just like in the classical theory of supply under separable costs.

In the simple context of this section, our analysis has established two new properties of NRR supply functions; Sections 5 will show how those results carry over to more complex settings.

Proposition 1 (Supply from a homogeneous NRR deposit)

- *Stock effect on restricted supply: An exogenous rise in exploitable reserves increases (restricted) supply at all dates.*
- *Cross-price effects: A price rise at any date T reduces short-run (restricted) supply and long-run (unrestricted) supply at all dates $t \neq T$.*

Although reminiscent of the decomposition of Marshallian demand, the decomposition of the change in NRR supply at t following a price change at $T \neq t$ into a pure substitution effect and a stock compensation effect is not isomorphic to the Slutsky decomposition. The substitution effect and the stock compensation effect of a resource price change are illustrated in Figure 1 for the case of two periods, which corresponds to the two-good representation of demand theory. Assuming prices p_0 and p_1, point $O = (x_0, x_1)$ in Figure 1 depicts the producer optimum. Given a stock of reserves X, periods 0 and 1 extraction levels are chosen such that producers reach the highest possible two-period iso-extraction-profit curve for prices (p_0, p_1) (of level $\overline{\pi}$).[17] The optimum allocation (x_0, x_1) is thus at the point of tangency between the $\overline{\pi}$ iso-profit curve and the exhaustibility constraint, the -45 degree line which expresses the trade-off between quantities extracted in Period 1 and quantities extracted in Period 2 in such a way that $x_0 + x_1 = X$. Unlike the case of Marshallian demand, this linear constraint is not affected by changes in prices.

[17] In Figure 1, the iso-profit curves correspond to the two-period extraction profit, conditional on X and before deduction of the sunk exploration cost $E(X)$: $\overline{\pi} = (p_0 x_0 - C_0(x_0)) + (p_1 x_1 - C_1(x_1))$. By (4), any optimum extraction is such that $p_t - C_t'(x_t) = \lambda$. Thus in a neighborhood of any optimum, $p_t - C_t'(x_t) > 0$. In the neighborhood of an optimum, it follows from the convexity of C_t that the slope $-\frac{p_0 - C_0'(x_0)}{p_1 - C_1'(x_1)}$ of any iso-profit curve at prices (p_0, p_1) is negative, increasing in x_0 and decreasing in x_1. In Figure 1, we focus on the relevant convex parts of the iso-profit curves. On other parts, they need not be convex.

Also, while prices do not affect iso-utility curves, they affect the slope of iso-profit curves: iso-profit curves may cross at different prices.

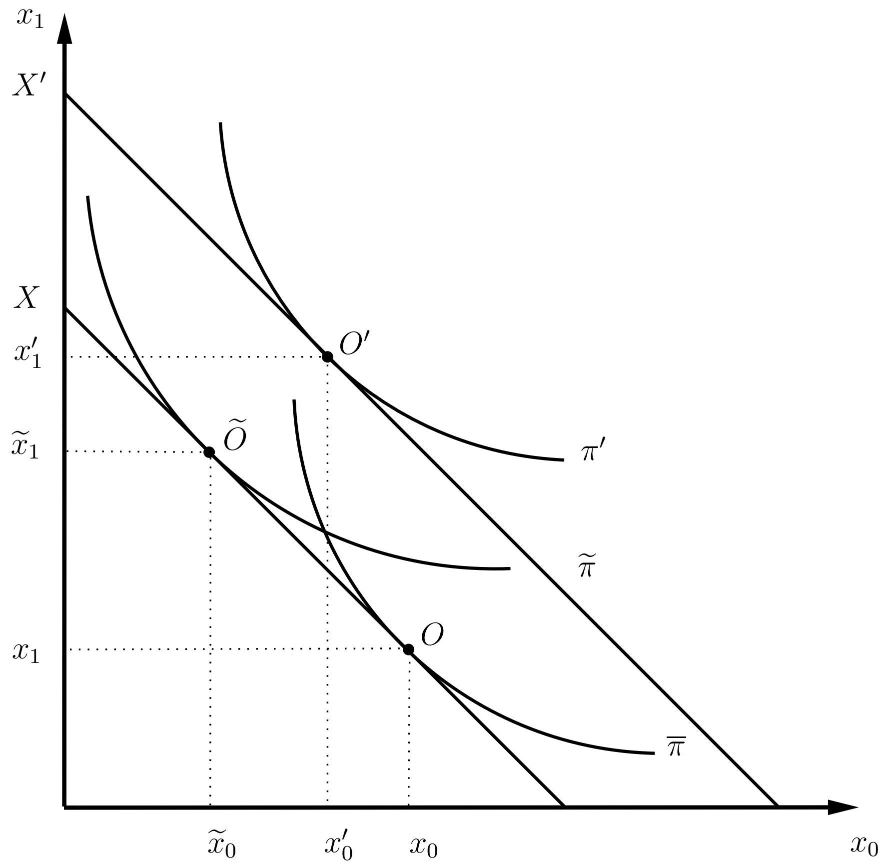

Figure 1: Price effect decomposition with $p_1' > p_1$

Consider a rise in p_1 to $p_1' > p_1$. The price change implies that all iso-profit curves become flatter at any given feasible level of x_0. If the stock of reserves remains unchanged at X, the new tangency point is along the same exhaustibility constraint and along the iso-profit curve of level $\widetilde{\pi} > \overline{\pi}$, at point $\widetilde{O}$ above O, so that $\widetilde{x}_0 < x_0$ and $\widetilde{x}_1 > x_1$. The move from O to $\widetilde{O}$ represents the substitution effect.

However the rise in price leads producers to increase reserve development to X'. Taking this stock effect into account brings the new optimum to O'. It is clear that $x_1' > \widetilde{x}_1 > x_1$. Unlike the Slutsky decomposition, there is no possibility of a commodity analogous to a Giffen good, whose supply would diminish as a result of a rise in its price. Moreover, in the case of NRR supply, the substitution effect always dominates the compensation effect, so that, by (14), x_0' must be lower than x_0 following the rise in p_1. There is no such thing as NRR supply complements; quantities extracted at different dates are always substitutes.

3. Partial Equilibrium and Policy Analysis

Having defined and characterized NRR supply functions in the standard fashion opens the field of all applications that rely on the demand-supply schedule, in particular the partial-equilibrium analysis of economic policies. Policy-induced changes are more complex than the above analysis of supply for two main reasons. First, policy-related price changes usually take place over an extended period rather than at a single date; second, the policy often affects prices indirectly, because it affects the demand for the NRR. Three examples are provided below as illustrations: one on supply extraction taxation; one of a reserve-reduction policy; and one of a policy affecting demand and raising the issue of the green paradox.

3.a. Supply Extraction Taxation

Suppose that prices are determined by the equilibrium of NRR supply and demand and suppose that the supply side is taxed, so that net after-tax spot extraction revenues are now $\Pi_t(x_t, p_t) = \pi_t(x_t, p_t) - G_t(x_t, p_t)$ where $G_t(x_t, p_t)$ is the tax function (Gaudet and Lasserre, 2013), expressed in present-value terms. When the tax is a unit or specific severance tax, $G_t(x_t, p_t) = \alpha_t x_t$; when it is an ad valorem severance tax, $G_t(x_t, p_t) = \gamma_t p_t x_t$. When policies combine those taxes, $G_t(x_t, p_t) = (\gamma_t p_t + \alpha_t)x_t$ and after-tax extraction revenues may be rewritten $\Pi_t(x_t, p_t) = \pi_t(x_t, p_t(1-\gamma_t) - \alpha_t)$. Therefore, the analysis of Section 2 is only modified to the extent that the producer price in absence of supply taxation should be replaced by the after-tax price $p_t(1-\gamma_t) - \alpha_t$. Supply functions at all dates are $x_t^*(q)$, where the vector $q \equiv (p_t(1-\gamma_t) - \alpha_t)_{t \geq 0}$ and the function x_t^* is identical to (11). This implies that increasing a severance tax at T while keeping the tax unchanged at other dates increases production at all dates $t \neq T$ and reduces production at T. Assume an exogenous demand for the resource at date $t \geq 0$, $x_t^D(p_t)$ that depends on the date, and that is decreasing on the resource price at that date p_t: the immediate partial-equilibrium implications of a rise in $p_T(1-\gamma_T)-\alpha_T$ are depicted in Figure 2. [**GRAPH TO BE INSERTED**] Furthermore, as already mentioned in Section 2, a specific severance tax of constant present value at all dates $\alpha_t = -a$ is neutral as it amounts to reducing all prices by the same present-value amount.

When the tax is a lump-sum tax, $G_t(x_t, p_t) = \beta_t$; obviously such tax has no effect as is well known of a lump-sum tax. However such is not the analysis found in the resource literature: a lump-sum tax on a mine has been found to reduce the length of the extraction period. The reason for this apparent discrepancy is that the resource literature does not assume the tax to be exogenous. Instead, the tax is implicitly defined as applying only during the extraction period so that its cumulative amount can be reduced by shortening the extraction period. [18]

3.b. Reserve-Reducing Policy

Consider a supply policy that aims at reducing exploitable reserves, in the spirit of Harstad (2012). If initial exploitable reserves were fixed, resource supply would be given by short-run restricted functions $\widetilde{x}_t(p, X)$ defined by (9). The property that such functions are all increasing in exploitable reserves X immediately implies that any policy that reduces those reserves causes production to diminish at all dates.

When reserves are endogenous, reserves reduction may be partly compensated by the development of new reserves. In that case, the supply of exploitable reserves $X(\lambda)$ in (33), giving the amount of reserves produced as a function of their unitary value λ, is reduced. Assume a market for developed reserves on which the unit reserve price μ is determined. Equilibrium requires that the price μ equal the producer's valuation λ. Assume that exploitable reserves are reduced by an exogenous demand for reserves X^D, that may be decreasing in $\mu = \lambda$ and increasing in the index Ω, that denotes the stringency of the

[18] Besides the endogeneity of the tax, the case of the lump-sum tax also violates the assumption that π_t is differentiable at $x_t = 0$. The model of Section **2. can easily be adapted to that situation as it endogenously determines periods of active extraction and of extraction interruptions.**

reserve-reducing policy.[19] Then, $X(\lambda) \equiv E'^{-1}(\lambda) - X^D(\lambda) \leq E'^{-1}$. It can easily be shown that an increase in Ω results in lower reserves produced $X^*(p)$, in a greater rent $\lambda^*(p)$ and in lower instantaneous supplies $x_t^*(p)$ for all $t \geq 0$; partial-equilibrium implications are illustrated in Figure 3. (TO BE ADDED)

3.c. Demand-Reducing Policy

Consider now a policy that reduces the demand for the NRR during the periods over which it is implemented. Assume that the demand for the resource at date $t \geq 0$ does not only depend on the date, on the resource price at that date p_t, but also on the stringency at date t of NRR demand-reducing policies, synthesized by the index θ_t. NRR demand may thus be written[20] $x_t^D(p_t, \theta_t)$ and assumed continuously differentiable and decreasing in both arguments. The path of the policy stringency index $\Theta \equiv (\theta_t)_{t \geq 0}$ is exogenously given. The equilibrium effect of changing θ_t and date T is illustrated in Figure 4; it can be formally established as follows.

The inverse demand function $P_t(x_t, \theta_t)$ gives the price of the NRR at t in present value; it is continuously differentiable and decreasing in its two arguments. Taking prices $p \equiv (p_t)_{t \geq 0}$ as given, producers solve (16) subject to (2). Equations (4)–(7) hold as before. Substituting $P_t(x_t, \theta_t)$ for p_t in (7) and modifying the rest of the derivation in Section 2 accordingly gives the equilibrium quantity supplied at t as function of the policy path rather than the price path. As explained in detail in the appendix, we redefine $\widetilde{x}_t(\Theta, X)$ to represent the equilibrium restricted NRR quantity rather than the restricted NRR supply function. Similar redefinitions are applied to reserves and unrestricted supply, that respectively become $X^e(\Theta)$ and $x_t^e(\Theta)$ in equilibrium. These equilibrium functions have the same comparative-static properties with respect to the exogenous demand policy

[19] How the demand X^D for reserves by non-producers is determined is out of the scope of the present paper.

[20] A less synthetic demand function could be $D_t(p_t) \equiv f_t(\phi_t) d_t(p_t + \psi_t, p_t^s - \chi_t)$, where f_t is decreasing, d_t is decreasing in its two arguments, and where $(\phi_t)_{t \geq 0}$, $(\psi_t)_{t \geq 0}$, $(p_t^s)_{t \geq 0}$, $(\chi_t)_{t \geq 0}$ respectively denote the given time paths of an index of demand-reducing technical progress, of a tax on resource extraction, of the price of a substitute, and of a subsidy to this substitute. An increase in θ_t at any date t may then reflect technical change, an increase in resource taxation, an increase in subsidies to substitutes, or any combination of such resource demand-reducing policies.

parameters θ_t, *mutatis mutandis*, as their NRR supply counterparts have with respect to the exogenous prices.

As an immediate application, the green paradox is often described as the effect on current or near-future equilibrium NRR supply of policies reducing NRR demand over some extended future period via various forms of assistance to alternative energy sources. Suppose that the policy reduces NRR demand at all dates $T \in \Delta$, where Δ is a strict subset of dates, and remains unchanged otherwise. Equilibrium NRR supply at $t \notin \Delta$ is negatively affected by the demand reducing policy parameter at $T \in \Delta$. When demand reductions are unanticipated and not accompanied by any adjustment in the stock of reserves, they affect the restricted equilibrium supply $\widetilde{x}_t(\Theta, X)$; when they are anticipated and associated with a drop in developed reserves, they affect the regular, unrestricted, supply $x_t^e(\Theta)$. In either case, reductions in demand at $T \in \Delta$ increase supply at all $t \notin \Delta$, confirming the validity of the green paradox.[21]

This formalization can also be adapted to situations such as monopoly extraction not strictly involving supply functions but amenable to the formal treatment of Section 2, some of which are briefly discussed in the next section.

4. Extensions: Market Power, Spatial Supply

4.a. Market Power

Although convention has it that a monopoly has no supply curve, the production decision analysis approach of Section 2 applies to the extractive monopoly just as conventional supply analysis is often adjusted to the case of a monopoly and to some imperfect-competition contexts.

A firm that enjoys market power on the market for the extracted resource solves

[21] If the change in demand affects a single date as in the analysis of Section 2, the drop in price unambiguously causes a drop in supply at that date. More generally if Δ contains more than a single date, the reaction of NRR supply at $T \in \Delta$ depends on the magnitude of the price change occurring at that date relative to the changes occurring at other dates $T' \in \Delta$. However, the analysis of Section 2 indicates that cumulative extraction over Δ is reduced. This is because X decreases while cumulative supply at all dates $t \notin \Delta$ increases.

Problem (16) subject to (2) except that it is aware of the incidence on prices of its quantity decisions: marginal profits in (4) no longer reflect the partial effect of quantities on revenues $\frac{\partial(p_t x_t - C_t(x_t))}{\partial x_t} = p_t - C_t'(x_t)$ as under competition, but their total effect $\frac{\partial(p_t x_t - C_t(x_t))}{\partial x_t} + \frac{\partial(p_t x_t - C_t(x_t))}{\partial p_t}\frac{dp_t}{dx_t} = p_t - C_t'(x_t) - x_t\frac{dp_t}{dx_t}$. Although prices are no longer exogenous, they may be affected by exogenous parameters possibly corresponding to policies. Define net spot extraction revenues under monopoly as $\pi_t^m\,(x_t, \theta_t) \equiv p_t(x_t, \theta_t)x_t - C - t(x_t)$ where θ_t is any exogenous vector of parameters monotonically affecting price at t. Thus θ_t may be a parameter reflecting the stringency of a NRR demand-reducing policy as defined in the previous section, or it may correspond to the tax regime affecting NRR supply as also explained in the previous section with a different notation. As long as π_t^m is concave in x_t, conditions similar to (4) govern extraction with θ_t taking the role played by prices in the competitive case:

$$\frac{\partial \pi_t^m(x_t^m, \theta_t)}{\partial x_t} = p_t - C_t'(x_t^m) - x_t^m \frac{\partial p_t(x_t^m, \theta_t)}{\partial x_t} \lambda^m, \ \forall t \geq 0, \ x_t^m > 0, \tag{15}$$

where the superscript m refers to a variable or function under monopoly.

The solution described in Section 2 starting at expression (4) is then easily adapted throughout the analysis with the values of the θ_t parameters replacing prices. All results survive: the substitution effect dominates the stock effect, and policy implications such as the green paradox or the neutrality of a constant present-value severance tax remain true.

The analysis also applies if the extractive firm is not the owner of the raw resource but has to buy the stock of initial reserves X at a unit supply price μ. At the producer's optimum, the extraction rent λ then must equal the reserve supply price, which is given by the inverse reserve supply function $\mu = E'(X)$. The producer's objective does not directly internalize the exploration and development cost $E(X)$, but does so via the expenditure μX. Under perfect competition, this problem is isomorphic to the problem treated in Section 2. If the resource extractor holds market power in the acquisition of initial reserves, the price parameter μ in μX must be replaced with the relevant inverse residual supply function – in the monopsony case, this is $E'(X)X$, and X is chosen in such a way that the marginal revenue $E'(X)+E''(X)X$ equals the shadow value λ. There still exists a relation like (5) with the same properties as in Section 2; the analysis follows through and the results are unchanged as long as the extractor's problem is well-behaved, i.e. as long as the marginal expenditure on resource acquisition is increasing.

4.b. Spatial Leakage

Policies are often local or regional. For simplicity, we have focused so far on the dynamic interpretation of our model. However, to the countable set of dates $t = 0, 1, 2, ...$ introduced in Section 2 we may add a spatial dimension indexed by $l = 0, 1, 2, ..., \bar{l}$; in that formulation, the price p_{tl} is the present-value producer price at date t and location l. Location may then refer to a particular country or jurisdiction characterized by a particular price sequence, or an outlet commanding particular marketing efforts or transportation costs. The net spot revenue from selling the amount x_{tl} at location l at date t is $p_{tl}x_{tl} - c_{tl}(x_{tl})$, where the function $c_{tl}(x_{tl})$ gives the cost of selling specifically in location l at t; it may be a transportation cost, a marketing cost, etc. It is assumed that c_{tl} is increasing and strictly convex. The net spot revenue from serving all locations $l = 0, ..., \bar{l}$ at date t is $\sum_{l=0,...,\bar{l}} (p_{tl}x_{tl} - c_{tl}(x_{tl})) - C_t(x_t)$, where $x_t = \sum_{l=0,...,\bar{l}} x_{tl}$ is the quantity of NRR extracted from the deposit at date t and the function $C_t(x_t)$ gives the cost of extracting and otherwise processing this quantity before dispatching it to all locations $l = 0, ..., \bar{l}$. The cumulative value function corresponding to a producer's optimum allocation of the production for a given matrix of prices is

$$\max_{(x_t)_{t\geq 0},(x_{tl})_{t\geq 0,l=0,...,\bar{l}},X} \sum_{t\geq 0}\left(\sum_{l=0,...,\bar{l}} (p_{tl}x_{tl} - c_{tl}(x_{tl})) - C_t(x_t)\right) - E(X) \tag{16}$$

not only subject to the exhaustibility constraint (2) as in Section 2, but also to the constraint

$$\sum_{l=0,...,\bar{l}} x_{tl} = x_t,\ \forall t \geq 0. \tag{17}$$

Defining λ_t as the Lagrange multiplier associated with the new constraint (17) and reminding the λ is that of the exhaustibility constraint (2), the necessary first-order conditions characterizing the producer's optimum are

$$p_{tl} - c'_{tl}(x_{tl}) = \lambda_t, \ \forall t \geq 0, \ \forall l = 0, ..., \bar{l}, \tag{18}$$

for the allocation of the production at any date t to all locations l,

$$\lambda_t - C'_t(x_t) = \lambda, \ \forall t \geq 0, \tag{19}$$

for the choice of the extraction of date t. For simplicity, and with no consequence on the sign of the price effects to be established, we focus here on interior resource allocations where $x_{tl} > 0$. The choice of initial reserves is determined by the same condition (5) as in Section 2. The obtained conditions are reminiscent of those of Section 2, except that the implicit value λ_t plays the role of the single date-t price of Section 2. Thus the analysis then unfolds as in Section 2, but in two stages rather than one.

The allocation rule (18) implicitly defines the solution as a series of functions

$$x_{tl} = x_{tl}(p_{tl}, \lambda_t), \tag{20}$$

with properties similar to those of (7).

The rule (19) similarly defines functions

$$x_t = \hat{x}_t(\lambda_t, \lambda), \tag{21}$$

with properties analogous to those of (7): $\hat{x}_t$ is increasing in the implicitly price λ_t and decreasing in the rent λ.

Treating date-t extraction x_t to be allocated to locations as given, in the same way as we took reserves X to be allocated across dates as given in Section 2, combining all relations (20) into (17), we obtain the new multiplier λ_t as a function increasing in all prices of date t $p_t \equiv (p_{tl})_{l=0,...,\bar{l}}$ and decreasing in x_t:

$$\lambda_t = \tilde{\lambda}_t(p_t, x_t). \tag{22}$$

Substituting (22) into (21) implicitly yields a function identical to (7)

$$x_t = x_t(p_t, \lambda), \tag{23}$$

except that p_t is a vector rather than a single price.

Like in Section 2, a price rise at any date $T \geq 0$ and any location $L \in [0, ..., \bar{l}]$ causes the rent λ to increase. This is true whether the size of the exploitable reserves X is restricted or not. When reserves X are endogenous, they always increase as a result of the price rise. Thus, by (23), the extraction decreases at all dates t other than T; it follows from the fact that X, if not fixed, may increase, that x_T increases.

One can further show that all λ_t, $t \geq 0$, increase as a consequence of the price rise.[22] In turn, it implies by (20) that supply diminishes at any date t other than T or all locations l other than L. At date T, since x_T increases and x_{tl} decreases for all $l \neq L$, this is consistent with the law of supply that x_{TL} increases after a rise in p_{TL}.

The results also extend in the directions of Section 3. In particular, a reduction in price at any strict subset of dates and regions increases supply at all other dates and regions. As a result, if a reduction in demand affects one or more regions at identical dates, as with Fischer and Salant (2013) technology-oriented policies, NRR supply increases at all locations and dates where demand has not changed; this means that extraction leakages always work in the same direction spacewise, or timewise, in the short run or in the long run, and are a phenomenon of exactly the same nature as the green paradox.

5. Heterogenous Resources and Aggregation

Non-renewable resources are notoriously heterogenous. This heterogeneity is mostly manifest in the costs involved in discovering, developing, and extracting the resource, as well as transforming it into a homogenous commodity. The seminal intuition of Herfindahl (1967) known as the "least-cost-first principle" has given rise to a substantial, sometimes

[22] We have shown that x_T was increasing by the decrease, established with (23), of all x_t at dates t other than that of the price rise T. Using that λ increases, it is clear by (21) that only a rise in λ_T is consistent with the the increase in x_T.

controversial, literature. .As Slade (1988) puts it "The idea that the least-cost deposits will be extracted first is so firmly embedded in our minds that it is an often-made but rarely tested assumption underlying the construction of theoretical exhaustible-resource models." (p. 189). The particular notion of cost involved and context of analysis are further subject to appreciation (Kemp and Long, 1980). Are the costs inclusive of resource rents or not (Hartwick, 1978; Amigues and Moreaux, 2001)? What variables, extraction rates and reserve levels most importantly, determine extraction costs? Are the relevant costs short-run cost or long-run costs (Hartwick et al., 1986)? Are capacity limitations determined by reserves or capital investment? Does an industry cost function exist or should costs be measured at the deposit level?

One strand of literature considers that industry NRR supply results from contributions from individual deposits and studies the sequence of their exploitation. It stems directly from Herfindahl's initial contribution showing that deposits of constant but different unit costs of extraction were to be exploited in a strict sequence. As was later established, simultaneous exploitation of different deposits may be optimal when marginal costs depend on extraction rates or capacities are limited or costly to acquire (see, e.g. Amigues et al., 1998; Gaudet and Lasserre, 2011; Salant, 2012). The least-cost-first principle may further fail, most notably when deposits differ in size. In this multi-deposit approach, resources are homogenous within a deposit and heterogeneity results from differences between deposits.

An alternative way to model NRR heterogeneity is due to Gordon (1967), has been further discussed or refined by Weitzman (1976), Levhari and Liviatan (1977), Pindyck (1978) and Salant et al. (1983), and was used recently by Hoel (2010), Gerlagh (2011), van der Ploeg and Withagen (2012) and Grafton et al. (2012) in works related to the green paradox. By assumption, the cost of extraction increases with cumulative extraction under the rationale that least-cost resources are used first irrespective of the deposit from which they are extracted. Reserves may be left underground if and when the unit extraction cost is not met by the price, which implies that prices affect the level of economic reserves. Although the original paper of Gordon argues that the increase in extraction cost with cumulative extraction occurs at the firm's level as well as in the aggregate, Gordon's postulate is often implicitly assumed to apply across deposits at the industry level implying that the relevant stock of reserves is the industry aggregate stock and that the corresponding cost function is an aggregate industry cost function.

We show that the properties of NRR supply established in Section 2 carry over almost unchanged when the resource is heterogenous, whether heterogeneity takes the form of inter-deposit heterogeneity as in the branch of literature that stems from Herfindahl's initial paper, or takes the form of intra-deposit heterogeneity as in the tradition initiated by Gordon. Consider the latter first.

Let X_t represent the stock of reserves remaining at t, and relabel the stock of initial reserves X_0 rather than X. Cumulative extraction between dates 0 and t is equal to $X_0 - X_t$, with $X_{t+1} = X_t - x_t$. Given X_0 the dependency of extraction cost on cumulative extraction implies that the cost at t depends on X_t negatively: higher current reserves imply lower cumulative extraction, hence a lower cost. Thus we write the extraction cost function as $C_t\left(x_t, X_t;\ X_0\right)$ with $\frac{\partial C_t(x,X;\ X_0)}{\partial X} < 0$. The parameter X_0 will be dropped from the notation; it is introduced here to insist on the fact that X_0 represents some measure of geological reserves, a data that cannot be affected by any economic decision, including the decision to explore and to develop reserves for exploitation. This will be discussed shortly.

The net present-value extraction revenue at t is $p_t x_t - C_t\left(x_t, X_t\right)$. Assume that $C_t\left(x, X\right)$ is strictly convex [23] and that $C_t\left(0, X\right) = 0 \ \forall\ X$. We owe James Sweeney (1993) a thorough investigation of the discrete version of Gordon's model; as he showed, the existence of an underlying continuous-time representation of the technology implies restrictions on the partial derivatives of allowable discrete-time cost functions; precisely, it

[23] Under the maintained assumption that the cost function is weakly convex, the objective function for this problem is weakly concave and the feasible set is convex. Thus the first-order necessary conditions for optimality are sufficient for optimality and multiple local unconnected optima cannot exist (Sweeney, 1993, p. 771). We assume strict convexity in order to avoid having to deal with supply correspondences rather than supply functions.

must be true that $\frac{\partial^2 C_t(x,X)}{\partial x^2} + \frac{\partial^2 C_t(x,X)}{\partial x \partial X} > 0$, a property called the dominance of extraction rate on marginal cost[24] which will also be assumed to hold here.[25] Under such a techno-geological constraint on extraction, the net extraction revenue not only depends on x_t and p_t as in Section 2, but also on remaining reserves X_t.

As in the simple model of Section 2, the producer must identify and develop the reserves to be exploited before extraction begins. A portion $X_0 - X^F$ is chosen within the initial stock X_0 of geological reserves and undergoes a costly exploration and development process that makes it suitable for exploitation. No development expenditure is applied to reserves that are not deemed economical, implying that the stock $X^F \geq 0$ of geological reserves left undeveloped will be left unexploited at the end of the extraction process. The amount $X_0 - X^F$ defines economic reserves. Economic reserves can be increased at date zero by reducing X^F. It is sensible to assume decreasing returns to exploration and development on the ground, as argued before, that the best prospects are developed first. Redefining the function E, we thus assume that the cost of developing an initial stock of exploitable economic reserves at date zero is $E\left(X_0 - X^F\right)$,[26] with $E(0) = 0$ and $E'(0) = 0$.[27]

[24]Salant et al. 1983) also impose this assumption, although not in reference with any underlying continuous time technology.

[25]As a simple way to focus on the rise in extraction cost with cumulative extraction, the extraction cost of the underlying continuous-time cost function is sometimes assumed to depend on the remaining-reserve stock, but not on the extraction rate; i.e. total cost is assumed linear in extraction rate. However, this linearity assumption in a continuous-time model implies a discrete-time representation in which marginal extraction cost is a strictly increasing function of extraction rate and a decreasing function of the remaining stock (Sweeney, 1993). Thus the discrete-time model presented here encompasses both continuous-time versions of Gordon's model that treat the marginal cost of extraction as constant or as strictly rising.

[26]We write the amount of reserves developed for exploitation as $X_0 - X^F$ rather than merely X as in Section 2 in order to convey an important property of the model. The marginal unit of reserves being developed at date zero is the unit that will be extracted last, not first. In models of homogenous resources, this does not matter; in Gordon's model, the sequence of reserve development and extraction is a geological and technological assumption. Increasing the stock of developed reserves at date zero means reducing the amount of geological reserves X^F to be left unexploited at closure date S. As a result the cost of extraction at date zero is the same whatever the amount of developed reserves, so that supply at date zero can be affected only by price signals.

[27]As in Section 2, the property $E'(0) = 0$ that the marginal cost of reserves development is zero at the origin is introduced because it is sufficient to ensure that a positive amount of reserves is developed. It thus rules out uninteresting situations where resource prices do not warrant the production of any reserves.

The problem faced by a NRR producer under such conditions is: (see the appendix for details of the resolution)[28]

$$\max_{(x_t)_{t\geq 0},\ S,\ X^F} \sum_{t=0}^{S} (p_t x_t - C_t(x_t, X_t)) - E(X_0 - X^F) \tag{24}$$

subject to

$$X_{t+1} = X_t - x_t,\ t = 0, ..., \bar{S} \tag{25}$$

$$x_t \geq 0,\ t = 0, ..., \bar{S} \tag{26}$$

$$X_t \geq X^F \geq 0,\ t = 0, ..., \bar{S} \tag{27}$$

where S is the last date at which strictly positive extraction occurs, so that $X_t = X_{t+1}$ for all $t \geq S+1$, and $\bar{S} > S+1$ is a positive date sufficiently big for the endogeneity of S.

Because of resource heterogeneity, the present-value resource rent measured by the Lagrangian variable associated with (25) is not constant over time. It is sometimes called a Ricardian resource rent and has been shown to diminish as reserves diminish; we denote it μ_t rather than λ, the Lagrangian multiplier associated with the same constraint in Section 2, to emphasize that this rent is different from a pure Hotelling scarcity rent.

Pure scarcity arises in this model at two levels, associated with the two inequality constraints in (27). The right-hand side inequality addresses the possibility that the totality of geological reserves be worth exploiting. It has been well investigated (e.g. Levhari and Liviatan, 1977) and will be neglected here with no significant effects on results. The left-hand side inequality recognizes the assumption that only those reserves that have been prealably discovered and developed at date zero may be exploited. As already argued, once the costs of exploration and reserve development are sunk, variable costs are covered so that this constraint is binding. The Lagrangian multiplier and

[28] In Problem (16)-(2) of Section 2 we do not consider the mine closure date as a choice variable explicitly, as it coincides with the first occurrence of a reserve stock of zero and is consequently implicit in the choice of extraction. Here the amount of (geological) reserves left at closure date is endogenous and is determined jointly with the closure date. Thus we introduce the latter among the decision variables explicitly.

rent associated with constraint $X_t \geq X^F$ is denoted λ, is strictly positive, and may be interpreted as a pure Hotelling rent in a short-run perspective, or a quasi-rent in a long-run perspective.

The first-order condition for strictly positive extraction at date t is

$$p_t - \frac{\partial C_t(x_t, X_t)}{\partial x} = \mu_t, \; x_t > 0, \; t = 0, ..., S; \tag{28}$$

the evolution of the resource rent is

$$\begin{aligned} \mu_t &= \mu_{t-1} + \frac{\partial C_t(x_t, X_t)}{\partial X}, \; t = 0, ..., S \\ & \text{with} \\ \mu_S &= \lambda = E' \left(X_0 - X^F \right). \end{aligned} \tag{29}$$

The present-value rent diminishes as the stock of reserves diminishes and extraction cost increases. At the end of operations all pre-developed reserves are exhausted so that remaining geological reserves equal X^F and the resource rent equals the cost of finding and developing the marginal unit of economic reserves, as stated by the right-hand equality in (29).

The same basic steps as in Section 2, (28) define the restricted supply functions, one at each date, as functions $\widetilde{x}_t(p, X_0 - X^F)$ of the price vector and the amount of reserves coined for exploitation. Similarly the (unrestricted) supply function $x_t^*(p)$ is obtained by allowing X^F to be endogenous. The following proposition and its corollary applying at date zero characterize the effect at any date on restricted supply of an exogenous change in the restricted factor (reserves) and the effect on both the restricted and the unrestricted supply at any date of a change in price at any other date before the terminal date. They are the counterpart of Proposition 1.

Proposition 2 (Supply from a heterogeneous NRR deposit)

- *Stock effect on restricted supply: An exogenous rise in exploitable reserves increases (restricted)* cumulative *supply at all dates.*

- *Cross-price effects: A price rise at any date $T \leq S$ reduces short-run (restricted)* cumulative *supply and long-run (unrestricted)* cumulative *supply at all dates $t \neq T$, where cumulative supply if $t > T$ is defined to exclude the supply at date T*

Corollary 1 (Supply from a heterogeneous NRR deposit)

- *Stock effect on restricted date-*0 *supply: An exogenous rise in exploitable reserves increases (restricted) date-*0 *supply.*

- *Cross-price effect on date* 0*: A price rise at any date $T \leq S$ reduces short-run (restricted) date-*0 *supply and long-run (unrestricted) date-*0 *supply.*

When possible, a much better approach is to construct aggregate NRR supply as the sum of supplies from distinct individual deposits. We do so in the Appendix, where we consider a multiplicity of different deposits indexed by j, developed at endogenous dates τ^j, and contributing to the supply of a unique resource whose price is p_t. Each deposit is similar to the single deposit considered in Section 2. However deposits differ by their size X^j, their geology, location and depth or quality, as reflected in the technologies underlying both extraction $\pi_t^j(x_t^j, p_t)$ and exploration $E_t^j(X^j)$ as well as the evolution of these technologies over time. In this highly general setup, the supply from all deposits already developed at t diminishes as a result of a rise in price at any date $T > t$. The intertemporal substitution effect dominates the reserve compensation effect for each active deposit, hence at the aggregate level. This reduction in supply may be sharpened by the postponement of some deposit developments. The green paradox is observed.

Proposition 2

Lemma 1 (Supply from a heterogeneous NRR deposit with endogenous opening dates)

- *Cross-price effects: A price rise at any date postpones the opening date of a deposit.*

Proposition 3 (Supply from multiple heterogeneous NRR deposits with endogenous opening dates)

- *Stock effect on restricted supply: An exogenous rise in exploitable reserves increases (restricted)* aggregate *supply at all dates.*

- *Cross-price effects: A price rise at any date reduces short-run (restricted)* aggregate *supply and long-run (unrestricted)* aggregate *supply.*

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APPENDIX

Partial equilibrium analysis

In equilibrium, it must be that $x_t = x_t\big(P_t(x_t,\theta_t),\lambda\big)$, where the function x_t is defined by (7). This implicitly defines the equilibrium quantity as a function ϕ_t which is decreasing in its two arguments:

$$x_t = \phi_t(\theta_t,\lambda),\ \forall t \geq 0. \tag{30}$$

The rest of the analysis follows the same steps as in Section 2, with the exogenous index levels θ_t replacing the prices p_t. Combining (2) with (30) gives the resource rent as a function that is decreasing in all indices Θ and in the stock X; to simplify notation we redefine $\widetilde{\lambda}$ to be that equilibrium rent function:

$$\widetilde{\lambda} = \widetilde{\lambda}(\Theta, X). \tag{31}$$

Substituting (31) into (30) yields the equilibrium extraction flows, conditional on the stock of reserves; redefining $\widetilde{x}_t$ to denote that function, we have the restricted NRR equilibrium supply functions, one at each date:

$$\widetilde{x}_t = \widetilde{x}_t(\Theta, X) \equiv \phi_t\big(\theta_t, \widetilde{\lambda}(\Theta, X)\big),\ \forall t \geq 0. \tag{32}$$

Equilibrium extraction quantities are increasing in X and in $\theta_{t'}$, $t' \neq t$. The partial derivative of $\widetilde{x}_t(.)$ with respect to θ_t is negative, for the same reasons that explain the restricted supply to be increasing in the current price in Section **2.**.

Denote by X^e the equilibrium amount of reserves. The unit rent in equilibrium is thus $\lambda^e = \lambda(\Theta, X^e)$. By (6), the equilibrium stock of reserves satisfies $X^e = X(\lambda^e) = X\left(\widetilde{\lambda}(\Theta, X^e)\right)$, which implicitly defines X^e as a function of Θ only:

$$X^e = X^e(\Theta). \tag{33}$$

The equilibrium stock of initial reserves is decreasing in all θ_t, $t \geq 0$. Finally, the unrestricted equilibrium extraction flows x_t^e are determined by

$$x_t^e = x_t^e(\Theta) \equiv \widetilde{x}_t\big(\Theta, X^e(\Theta)\big),\ \forall t \geq 0. \tag{34}$$

By (32), the effect of a change in the stringency of policies at T, on the equilibrium extraction quantities x_t^e at all dates $t \neq T$, can be decomposed into a substitution effect and a reserve compensation effect as in (12). Obviously the substitution effect dominates as in Section 2.. Although demand-reducing policies at some dates result in lower total cumulative extraction (the stock effect), they always lead to increased equilibrium extraction flows at all dates where they have no effect on demand (the substitution effect dominates).

As argued in the main text, this is also true if the resource is supplied by a monopoly or if the exploration sector is a monopsony.

Resource heterogeneity: Gordon's approach

$$\max_{(x_t)_{t\geq 0},\ S,\ X^F} \sum_{t=0}^{S} (p_t x_t - C_t(x_t, X_t)) - E(X_0 - X^F) \tag{35}$$

subject to

$$X_{t+1} = X_t - x_t \tag{36}$$

$$X_{t+1} \leq X_t \tag{37}$$

$$x_t \geq 0$$

$$X_t \geq X^F \geq 0 \tag{38}$$

Note that the constraint $X_{t+1} \leq X_t$ is implied by $X_{t+1} = X_t - x_t$ and $x_t \geq 0$, so we drop it. The Lagrangian function associated with Problem $(24) - (27)$ is

$$\begin{aligned}
\mathcal{L} &= \sum_{t=0}^{S} (p_t x_t - C_t(x_t, X_t)) - E(X_0 - X^F) \\
&\quad + \sum_{t=0}^{\bar{S}} \mu_t (X_t - x_t - X_{t+1}) - \sum_{t=0}^{\bar{S}} \eta_t x_t \\
&\quad + \lambda \left(X_{S+1} - X^F\right) + \xi X^F
\end{aligned}$$

where S is the last date at which strictly positive extraction occurs, so that $X_t = X_{t+1}\ \forall\ t \geq S+1$, and $\bar{S} > S+1$ is finite. Note that $X_t \geq X^F$ is imposed only at $S+1$ because the non negativity of x_t at $t \leq S$ implies that it is satisfied at all $t \leq S$ if it holds at $S+1$, and $x_t = 0$ at and after $S+1$ implies that $X_t \geq X^F$ remains satisfied beyond $S+1$ if it is satisfied at $S+1$. Assume (Ricardian; the mixed Ricardian Hotelling with $\xi > 0$ can be handled as well) that $X^F > 0$ so that $\xi = 0$. Assume for simplicity that no occurrence of $x_t = 0$ happens before S. The Lagrangian may be written as

$$\begin{aligned}
\mathcal{L} &= \sum_{t=0}^{S} (p_t x_t - C_t(x_t, X_t)) - E(X_0 - X^F) \\
&\quad + \sum_{t=0}^{S} \mu_t (X_t - x_t - X_{t+1}) - \sum_{S+1}^{\bar{S}} \eta_t x_t \\
&\quad + \lambda \left(X_{S+1} - X^F\right)
\end{aligned}$$

The foc are:
with respect to x:

$$\begin{aligned}
p_t - \frac{\partial C_t(x_t, X_t)}{\partial x} &= \mu_t,\ x_t > 0,\ t = 0, ..., S \\
p_t - \frac{\partial C_t(0, X_t)}{\partial x} &= \eta_t,\ x_t = 0,\ t = S+1, ..., \bar{S}
\end{aligned}$$

with respect to X

$$\begin{aligned} \mu_{t-1} &= \mu_t - \frac{\partial C_t(x_t, X_t)}{\partial X}, \ t = 0, ..., S \\ \mu_S &= \lambda \end{aligned}$$

with respect to X^F

$$E'\left(X_0 - X^F\right) = \lambda$$

Thus μ_t diminishes from date zero to a minimum of $\lambda > 0$ at date S. From $S+1$ to $\bar{S}$, $X_t = X^F$ and $p_t - \frac{\partial C_t(0,X^F)}{\partial x} = \eta_t < \mu_S$.

Consider a rise in p_T where $T < S$. It follows that $dX^F \leq 0$ (sort of law of supply). It follows that $d\lambda \geq 0$. The change in price may affect the closure date. Let S' denote the new closure date. Hence we have $\mu_{S'} \geq \mu_S$. Either S is affected ($S' \neq S$); or S is unchanged ($S' = S$). Because time is discrete we cannot consider small variations dS.

1. **Suppose that S is unchanged. Then we have $\mu_{S'} \geq \mu_S$ with $S' = S$ and we can write $d\mu_S \geq 0$. the recurrence p. 3 of Julien follows.**

2. **Suppose $S' < S$, then $p_S - \frac{\partial C_S(0,X^F)}{\partial x} < \mu_{S'}$, which is not a contradiction. Also $X_{S'}(p') < X^F(p)$ where p' denotes the new price sequence and p the old sequence; hence we can adapt the last line of p. 4 and go on with $dX_{S'} < 0$.**

   Suppose $S' > S$, $p_S - \frac{\partial C_S(0,,X^F)}{\partial x} > \mu_{S'} > \mu_S$ contradiction with $x_S > 0$ before the rise in p_T.

Resource heterogeneity: multiple deposits

Let the various possible supply sources (deposits, developed or not) be identified by j, $j = 1, ..., J$, and let resource supply at date t be

$$S_t = \sum_{j=1}^{J} x_t^j, \tag{39}$$

where $x_t^j \geq 0$ is the quantity of resource j supplied at date t. Net spot extraction revenues are $\pi_t^j = \pi_t^j(x_t^j, p_t)$, with the same properties as in the single source case of Section 2.. Since x_t^j may be zero, there is no loss of generality in assuming that the same countable set of dates applies for all sources. Each source is constrained by its own finite reserve stock. As the marginal reserve unit of any deposit will only be developed if it is to be exploited, that constraint binds:[29]

$$\sum_{t \geq 0} x_t^j = X^j, \ j = 1, ..., J. \tag{40}$$

[29] If deposit j is never developed, $\sum_{t \geq 0} x_t^j = 0 = X^j$.

Each source is characterized by its own exploration and development cost $E_t^j(X^j)$ whose qualitative properties are the same as in the case of a single resource, with the following minor difference. The property $E^{j\prime}(0) = 0$ is replaced with $E_t^{j\prime}(0) \geq 0$, so that a resource whose marginal exploration and development cost is too high for profitability need not be developed. However the same deposit that is not economic at date zero may be developed when prices become higher and/or when the technologies encompassed in the functions π_t^j and E_t^j justify it.[30] We assume that technological progress on exploration and development is such that, for any date $t' > t$ and initial reserves $X^{j\prime} > X^j$,

$$E_t^j\left(X^j\right) \geq E_{t'}^j\left(X^j\right) \text{ and } E_t^j\left(X^{j'}\right) - E_{t'}^j\left(X^{j'}\right) \geq E_t^j\left(X^j\right) - E_{t'}^j\left(X^j\right),\ \forall\ j. \tag{41}$$

As before, it is supposed that exploration and development are instantaneous and undertaken only once for each deposit; extraction may take place only after deposit development. All potential producers face the same given known price stream. For source j, the producer solves

$$\max_{(x_t^j)_{t\geq 0}, X^j, \tau^j} \sum_{t\geq 0} \pi_t^j(x_t^j, p_t)(1+r)^{-t} - (1+r)^{-\tau^j} E_{\tau^j}^j(X^j) \tag{42}$$

subject to (40) and to

$$x_t^j = 0,\ t < \tau^j,$$

where $\tau^j \geq 0$ is the development date for deposit j. There is one specific resource rent λ^j associated with each deposit. Suppose that $\tau^j > 0$. No production occurs before that date, so that $x_t^j = 0,\ t < \tau^j$. We further assume that the combination of resource price changes and technological change affecting π_t^j is such that, once initiated, production is not interrupted until exhaustion.[31] We also assume that the problem is well behaved in the sense that the optima being characterized are global rather than local, at least in the neighborhood of the price vector under consideration. This rules out jumps from one local maximum to another local maximum as a result of a change in the price vector. Clearly, the problems to be solved for each source are independent of each other. Thus the sole difference with the one-resource case analyzed earlier is the fact that all resources need not be developed at date zero if at all. Roughly, given a price path, resources whose extraction cost is higher and/or whose cost of exploration and development is higher will be developed later. We are interested in aggregate resource supply at dates $t \geq 0$; in particular we want to determine how supply S_t reacts to a change in

[30] It is well-known that, for some price and technology combinations, development occurs only at $\tau = 0$ if at all. Such is the case, for example, if prices are constant while extraction and development technologies are stationary.

[31] This assumption greatly facilitates the analysis while it eliminates situations of only minor economic interest. It is satisfied if prices do not diminish too fast and technological change is such that extraction costs do not increase too fast over any part of the exploitation period.

price at $T \geq t$. Since each component x_t^j of S_t is determined independently of the others, consider deposit j in particular.

Suppose at this stage that the development date τ^j is given. Then the derivations and properties established in Section 2. for Problem (16) can be readily adapted to Problem (42). If $\tau^j = 0$ the solution is identical to that of Section 2.; if $\tau^j > 0$, $x_t^j = 0$ when $t < \tau^j$ and, for $t \geq \tau^j$, the first-order conditions for the choice of the optimum extraction path and initial reserves are, instead of (4) and (5),

$$\frac{\partial \pi_t^j(x_t^j, p_t)}{\partial x_t^j}(1+r)^{-t} = \lambda^j, \ \forall \ t \geq \tau^j, \tag{43}$$

and

$$E_{\tau^j}^j{}'(X^j) = (1+r)^{\tau^j} \lambda^j, \tag{44}$$

where all values, including the unit Hotelling scarcity rent λ^j, are evaluated at date zero, although development occurs at τ^j rather than at zero.

All properties of the supply functions established in Section 2. apply for each deposit, provided it is active at t. Still holding τ^j constant, consider the effect of an increase in price at date T on date-t industry supply, where $T \geq t$. All deposits developed at or before t contribute to S_t. Consequently the effect is the sum of the changes in the supply from all deposits such that $\tau^j \leq t \leq T$: if $t < T$, an increase in p_T reduces extraction from all active deposits at t reducing total supply at t. If $t = T$, a rise in p_T increases the contribution from all deposits, thus increasing total supply at t.

Now allow optimal development dates to adjust to the price change. As in Section 2., a "*" next to a variable or function refers to the optimal unrestricted level of the variable or to the unrestricted function. The optimal development date $\tau^{j*}(p)$ of deposit j may be a corner solution $\tau^{j*}(p) = 0$ or, if it is an interior solution, it is an integer value $\tau^{j*}(p) > 0$ within the set of possible dates. For an interior solution, the Lagrangian $\mathcal{L}(x^{j*}, X^{j*}, \tau, \lambda^{j*})$ for Problem (42) must be bigger at $\tau^{j*}(p)$ than at $\tau^{j*}(p) - 1$ and at $\tau^{j*}(p) + 1$. The implied inequalities $\mathcal{L}(x^{j*}, X^{j*}, \tau, \lambda^{j*}) - \mathcal{L}(x^{j*}, X^{j*}, \tau - 1, \lambda^{j*}) > 0$ and $\mathcal{L}(x^{j*}, X^{j*}, \tau, \lambda^{j*}) - \mathcal{L}(x^{j*}, X^{j*}, \tau + 1, \lambda^{j*}) > 0$ when $\tau = \tau^{j*}$ can be written as

$$rE_{\tau-1}^j(X^{j*}) + E_{\tau-1}^j(X^{j*}) - E_\tau^j(X^{j*}) \quad > \quad \pi_{\tau-1}^j(x_{\tau-1}^{j*}, p_t)(1+r) - (1+r)^\tau \lambda^{j*} x_{\tau-1}^{j*}, \tag{45}$$

$$\frac{r}{1+r}E_{\tau+1}^j(X^{j*}) + E_\tau^j(X^{j*}) - E_{\tau+1}^j(X^{j*}) \quad < \quad \pi_\tau^j(x_\tau^{j*}, p_t) - (1+r)^\tau \lambda^{j*} x_\tau^{j*}. \tag{46}$$

The assumption that the optimum is global further implies that the Lagrangian is increasing in τ before $\tau^{j*} - 1$ and decreasing after $\tau^{j*} + 1$.

Consider a change in the price schedule from p to p' where p' departs from p because of an increase in p_T at some date $T > \tau = \tau^{j*}(p)$. We know that $\lambda^{j*}(p') > \lambda^{j*}(p)$. Then since p_t is unchanged at any $t < T$, the concavity of π_t^j in x_t^j implies that $x_t^{j\prime*} < x_t^{j*}$, $\forall t \in \{\tau^{j*}(p'), ..., T-1\}$ by (43). However, the fact that production from any existing deposit j is diminished at dates

$t \in \{\tau^{j*}(p'), ..., T-1\}$ following the increase in p_T is not sufficient to conclude to a drop in production at these dates: it is also necessary that no deposit comes into stream at the new price if it was not in production at the old price vector at any of these dates. That is to say, if a deposit is developed at $\tau^{j*}(p') = t$, $t \in \{0, ..., T-1\}$, it must be the case that it was under operation at, or before, t at the old price vector: we must show that $\forall j$, $\tau^{j*}(p') \geq \tau^{j*}(p)$.

Consider (45) and (46) at $\tau = \tau^{j*}(p) < T$ for any j when $\lambda^{j*}(p)$ increases to $\lambda^{j*}(p')$. The right-hand sides of both expressions are diminished. Since X^{j*} increases by (44) as a result of the increase in λ^{j*}, the monotonicity of E_t^j and Assumption (41) imply that the left-hand sides of both expressions increase. This means that (45) remains satisfied if τ^j is not adjusted, while (46) may become violated; in such case, only an increase in τ^j can reestablish the optimality conditions. Consequently, $\tau^{j*}(p') \geq \tau^{j*}(p)$, completing the proof that, if $t < T$, an increase in p_T not only reduces extraction from active deposits but may also cause the development of some deposits at or before t to be postponed.

Similar arguments show that, if $t = T$, an increase in p_T increases supply at t.