2021

AP® CollegeBoard

AP® Physics 1: Algebra-Based

Sample Student Responses and Scoring Commentary

Inside:

Free Response Question 3

☑ Scoring Guideline

☑ Student Samples

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Question 3: Qualitative/Quantitative Translation — 12 points

(a) i.

For a correct answer $v_D = \frac{F_H t_f}{M_D}$ — **1 point**

ii.

For indicating the total momentum of the system is the same before and after the collision — **1 point**

Scoring Note: If the response only includes a correct final answer of $\frac{M_S}{M_D}$, the response earns this point but not the next point.

For correctly substituting the appropriate variables into a conservation of momentum equation — **1 point**

AND

an answer in the form $\frac{v_D}{v_S} = \ldots$

Scoring Notes:

This point can be earned only if the first point is earned.

The answer need not be correct to earn this point.

Example response for part (a)(ii)

$p_i = p_f$

$0 = M_S v_S - M_D v_D$

$\frac{v_D}{v_S} = \frac{M_S}{M_D}$

Total for part (a) — 3 points

(b)

For two functions that are straight segments for $t < t_f$, — **1 point**

AND

begin at the origin,

AND

have two different positive slopes

For two functions that are horizontal functions for $t > t_f$ — **1 point**

AND

are continuous over the entire time range $0 < t < 2t_f$

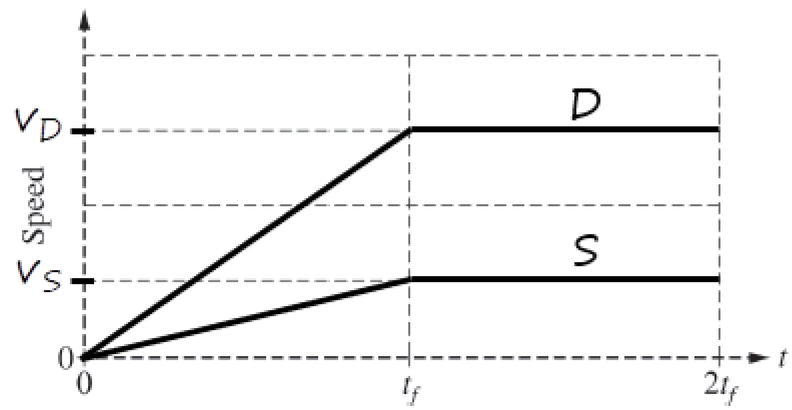
For labeling values on the vertical axis with $v_D > v_S$ — **1 point**

OR

The curve labeled D is greater than the curve labeled S for all t > 0

Scoring note: This point can still be earned if the labels are not on the vertical axis but clearly indicate that $v_D > v_S$.

Example response for part (b)



		Total for part (b)	**3 points**
(c)	**i.**	For stating or mathematically representing that if the disk is much more massive, then the block will have little effect on the motion of disk 1 **OR** For stating or mathematically representing that when $M_D >> M_B$, $v_{cm} = v_1$	**1 point**
	ii.	For correct reasoning. Correct answer: When $M_D << M_B$, $v_{cm} = 0$	**1 point**
		Example response for part (c) (ii) *If the block is much more massive, then it will barely move when the disk collides and sticks to it.*	
	iii.	For using conservation of momentum	**1 point**
		For a correct answer $v_{cm} = \dfrac{m_D v_1}{m_D + m_B}$	**1 point**
	iv.	For an attempt to use limiting-case reasoning or functional dependence with the equation in part (c)(iii)	**1 point**
		For recognizing the equation from (c)(iii) reduces to a simpler form and the simplified form is correctly compared to their answer in (c)(i)	**1 point**
		Example 1 response for part (c) (iv) *Yes. If M_B is very small, then the denominator of the equation simplifies to M_D, which then can cancel out of the equation leaving $v_{cm} = v_1$.*	
		Total for part (c)	**6 points**
		Total for question 3	**12 points**

Begin your response to QUESTION 3 on this page.

3. (12 points, suggested time 25 minutes)

(a) A student of mass M_S, standing on a smooth surface, uses a stick to push a disk of mass M_D. The student exerts a constant horizontal force of magnitude F_H over the time interval from time $t = 0$ to $t = t_f$ while pushing the disk. Assume there is negligible friction between the disk and the surface.

i. Assuming the disk begins at rest, determine an expression for the final speed v_D of the disk relative to the surface. Express your answer in terms of F_H, t_f, M_S, M_D, and physical constants, as appropriate.

$a = \frac{V_D}{T_f}$

$F_H = M_S a$

$F_H = (\quad + M_D)\left(\frac{V_D}{T_F}\right)$

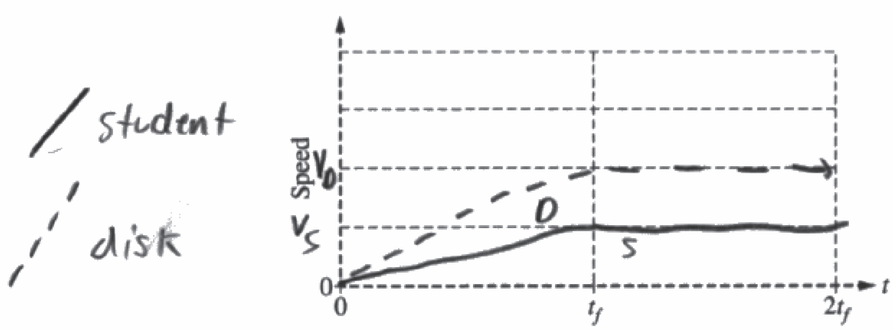
$F_H T_F = (\quad + M_D) V_D$

$V_D = \frac{F_H T_F}{M_D}$

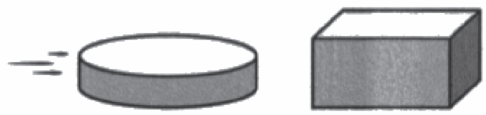
ii. Assume there is negligible friction between the student's shoes and the surface. After time t_f, the student slides with speed v_S. Derive an equation for the ratio v_D / v_S. Express your answer in terms of M_S, M_D, and physical constants, as appropriate.

$\frac{V_D M_D}{V_S M_S}$

(b) Assume that the student's mass is greater than that of the disk ($M_S > M_D$). On the grid below, sketch graphs of the speeds of both the student and the disk as functions of time t between $t = 0$ and $t = 2t_f$. Assume that neither the disk nor the student collides with anything after $t = t_f$. On the vertical axis, label v_D and v_S. Label the graphs "S" and "D" for the student and the disk, respectively.



Continue your response to **QUESTION 3** on this page.



Disk Block

(c) The disk is now moving at a constant speed v_1 on the surface toward a block of mass M_B, which is at rest on the surface, as shown above. The disk and block collide head-on and stick together, and the center of mass of the disk-block system moves with speed v_{cm}.

i. Suppose the mass of the disk is much <u>greater</u> than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

The velocity of the system will be slightly less than V_1. COP - $mv = (m+M_B)v$. Because the mass of the disk-block system is slightly greater than the disk.

ii. Suppose the mass of the disk is much <u>less</u> than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

The velocity of the disk-block system will be much less than V_1. Due to the law of conservation of momentum a larger mass collision decreases velocity

iii. Now suppose that neither object's mass is much greater than the other but that they are not necessarily equal. Derive an equation for v_{cm}. Express your answer in terms of v_1, M_D, M_B, and physical constants, as appropriate.

$$V_1 M_D = V_{cm}(M_D + M_B)$$

$$V_{cm} = \frac{V_1 M_D}{(M_D + M_B)}$$

iv. Consider the scenario from part (c)(i), where the mass of the disk was much <u>greater</u> than the mass of the block. Does your equation for v_{cm} from part (c)(iii) agree with your reasoning from part (c)(i) ?

X Yes ____ No

Explain your reasoning by addressing why, according to your equation, v_{cm} becomes (or approaches) a certain value when M_D is much <u>greater</u> than M_B.

Because V_{cm} is equal to the ratio of M_D to $M_D + M_B$ multiplied by V_1. In this scenario the ratio of $M_D/(M_D + M_B)$ is slightly less than 1 which would make V_1 slightly less than V_{cm}

Begin your response to **QUESTION 3** on this page.

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(a) A student of mass M_S, standing on a smooth surface, uses a stick to push a disk of mass M_D. The student exerts a constant horizontal force of magnitude F_H over the time interval from time $t = 0$ to $t = t_f$ while pushing the disk. Assume there is negligible friction between the disk and the surface.

i. Assuming the disk begins at rest, determine an expression for the final speed v_D of the disk relative to the surface. Express your answer in terms of F_H, t_f, M_S, M_D, and physical constants, as appropriate.

$\Sigma F_x = ma$

$F_H = M_D a$

$\frac{F_H}{M_D} = a$

$v_f = v_i + at$

$V_D = 0 + \frac{F_H}{M_D} t_f$

$V_D = \frac{F_H}{M_D} t_f$

ii. Assume there is negligible friction between the student's shoes and the surface. After time t_f, the student slides with speed v_S. Derive an equation for the ratio v_D / v_S. Express your answer in terms of M_S, M_D, and physical constants, as appropriate.

$KE = \frac{1}{2}mv^2$

$KE_D = \frac{1}{2}M_D v_D^2$

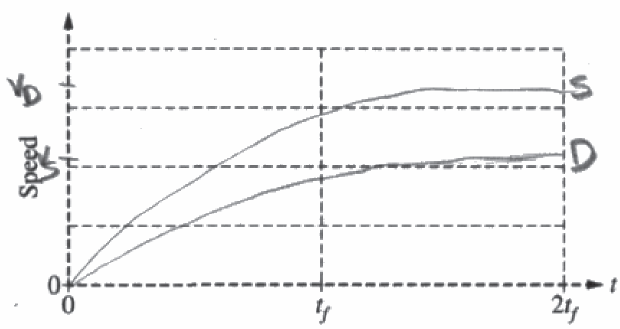
$KE_S = \frac{1}{2}M_S v_S^2$

$V_D = \sqrt{\frac{1}{2}M_D}$

$V_S = \sqrt{\frac{1}{2}M_S}$

$\frac{V_D}{V_S} = \frac{\sqrt{\frac{1}{2}M_D}}{\sqrt{\frac{1}{2}M_S}}$

(b) Assume that the student's mass is greater than that of the disk ($M_S > M_D$). On the grid below, sketch graphs of the speeds of both the student and the disk as functions of time t between $t = 0$ and $t = 2t_f$. Assume that neither the disk nor the student collides with anything after $t = t_f$. On the vertical axis, label v_D and v_S. Label the graphs "S" and "D" for the student and the disk, respectively.



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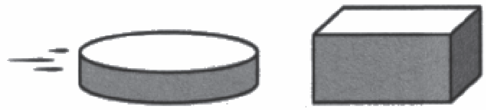

(c) The disk is now moving at a constant speed v_1 on the surface toward a block of mass M_B, which is at rest on the surface, as shown above. The disk and block collide head-on and stick together, and the center of mass of the disk-block system moves with speed v_{cm}.

i. Suppose the mass of the disk is much greater than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically. The velocity of the disk-block system should be very close to that of v_1. This is because if the mass of the block is very small compared to that of the disk when the masses combine the total mass will only slightly increase. Following the conservation of momentum, if the final mass barely increases the final velocity will barely decrease.

ii. Suppose the mass of the disk is much less than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically. v_{cm} would be very close to 0. This is because if M_B is much greater than M_D, when the masses combine the total mass will greatly increase. In order for the conservation of momentum to be true, v_{cm} must greatly decrease in response to a large increase in the system's mass since initial momentum must equal final momentum.

iii. Now suppose that neither object's mass is much greater than the other but that they are not necessarily equal. Derive an equation for v_{cm}. Express your answer in terms of v_1, M_D, M_B, and physical constants, as appropriate.

$\Sigma p_i = \Sigma p_f$

$m_D v_1 = (M_D + M_B) v_{cm}$

$v_{cm} = \frac{M_D v_1}{M_D + M_B}$

iv. Consider the scenario from part (c)(i), where the mass of the disk was much greater than the mass of the block. Does your equation for v_{cm} from part (c)(iii) agree with your reasoning from part (c)(i) ?

✓ Yes ____ No

Explain your reasoning by addressing why, according to your equation, v_{cm} becomes (or approaches) a certain value when M_D is much greater than M_B.

According to the equation, $M_D v_1$ is divided by $M_D + M_B$ to solve for vcm. If M_D is much larger than M_B, the addition of M_B to M_D will minimally affect the total velocity of the system because the denominator is only changing slightly. As a result vcm would almost equal v_1 because if M_B was 0 M_D would cancel from the equation and vcm would equal v_1. Since it is known that M_B is very small but greater than 0, it can be deduced that the change to vcm will be very small, therefore vcm will approach but not equal v_1.

GO ON TO THE NEXT PAGE.

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3. (12 points, suggested time 25 minutes)

(a) A student of mass M_S, standing on a smooth surface, uses a stick to push a disk of mass M_D. The student exerts a constant horizontal force of magnitude F_H over the time interval from time $t = 0$ to $t = t_f$ while pushing the disk. Assume there is negligible friction between the disk and the surface.

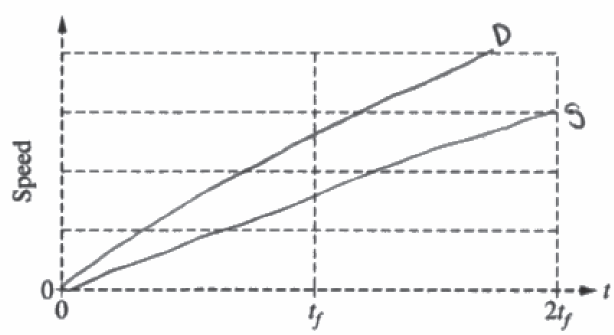
i. Assuming the disk begins at rest, determine an expression for the final speed v_D of the disk relative to the surface. Express your answer in terms of F_H, t_f, M_S, M_D, and physical constants, as appropriate.

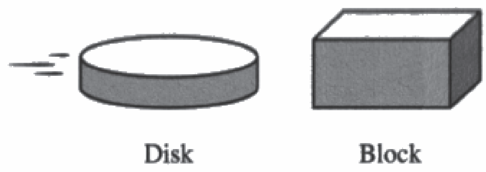
$V_D = \sqrt{V_0 + 2a(x - x_0)}$

ii. Assume there is negligible friction between the student's shoes and the surface. After time t_f, the student slides with speed v_S. Derive an equation for the ratio v_D / v_S. Express your answer in terms of M_S, M_D, and physical constants, as appropriate.

$\frac{V_D}{V_S} = \frac{m_S - m_D}{r}$

(b) Assume that the student's mass is greater than that of the disk ($M_S > M_D$). On the grid below, sketch graphs of the speeds of both the student and the disk as functions of time t between $t = 0$ and $t = 2t_f$. Assume that neither the disk nor the student collides with anything after $t = t_f$. On the vertical axis, label v_D and v_S. Label the graphs "S" and "D" for the student and the disk, respectively.





(c) The disk is now moving at a constant speed v_1 on the surface toward a block of mass M_B, which is at rest on the surface, as shown above. The disk and block collide head-on and stick together, and the center of mass of the disk-block system moves with speed v_{cm}.

i. Suppose the mass of the disk is much greater than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

the velocity would be the same if the masses were the same bc if the velocity you pushed the disk at is the same then the velocity of the com would be the same.

ii. Suppose the mass of the disk is much less than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

The velocity would be the same bc the velocity doesn't change unless initial velocity changes

iii. Now suppose that neither object's mass is much greater than the other but that they are not necessarily equal. Derive an equation for v_{cm}. Express your answer in terms of v_1, M_D, M_B, and physical constants, as appropriate.

$V_{cm} = V_0 + 2a(x-x_0)$

iv. Consider the scenario from part (c)(i), where the mass of the disk was much greater than the mass of the block. Does your equation for v_{cm} from part (c)(iii) agree with your reasoning from part (c)(i) ?

____ Yes ✓ No

Explain your reasoning by addressing why, according to your equation, v_{cm} becomes (or approaches) a certain value when M_D is much greater than M_B.

my equation is finding the final velocity

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

Responses to this question were expected to demonstrate an understanding of the center of mass of a system of two objects involved in a collision. To successfully complete the problem, students must:

- Predict the velocity of the center of mass of a system.
- Derive an equation for the velocity of the center of mass.
- Graph the speed of two objects involved in a conservation of momentum scenario.
- Determine an expression for final speed when an impulse is applied.
- Apply limiting reasoning and functional dependence to support a claim.

Sample: P1 Q3 A
Score: 10

Part (a)(i) earned 1 point for the correct expression. Part (a)(ii) earned no points. The response does not show a valid starting point for conservation of momentum. The second point cannot be earned without the first point. Part (b) earned 3 points. One point was earned for a graph that contains two straight functions that both start at the origin and each have a different positive slope before time t_f. One point was earned for a graph that contains two horizontal line segments after time t_f and is continuous for all times. One point was earned for a graph for D that is greater than the graph for S for all times greater than zero. Part (c)(i) earned 1 point for a response that correctly estimates the velocity of the center of mass with an explanation. Part (c)(ii) earned 1 point for a response that correctly estimates the velocity of the center of mass with an explanation. Part (c)(iii) earned 2 points. One point was earned for a response that starts with the law of conservation of momentum and uses the law in a derivation. One point was earned for reaching the correctly derived equation. Part (c)(iv) earned 2 points. One point was earned for an attempt to apply a limiting case or functional dependence to the equation for part (c)(iii). One point was earned for explaining how the equation in part (c)(iii) supports the estimate in part (c)(i). Note: While the response does not explicitly state that M_B is much smaller than M_D, this is implied because the prompt in part (c)(iv) refers to the scenario in part (c)(i) where this is the case.

Sample: P1 Q3 B
Score: 6

Part (a)(i) did not earn any points for the correct expression. Part (a)(ii) earned no points. The response does not show a valid starting point for conservation of momentum. The second point cannot be earned without the first point. Part (b) earned no points for this graph. The response shows curved lines for the entire time period and graph S is greater than graph D at all times greater than zero. Part (c)(i) earned 1 point for a response that correctly estimates the velocity of the center of mass with an explanation. Part (c)(ii) earned 1 point for a response that correctly estimates the velocity of the center of mass with an explanation. Part (c)(iii) earned 2 points. One point was earned for a response that starts with the law of conservation of momentum and uses the law in the derivation. One point was earned for reaching the correctly derived equation. Part (c)(iv) earned 2 points. One point was earned for an attempt to apply a limiting case or functional dependence to the equation for part (c)(iii). One point was earned for explaining how the equation in part (c)(iii) supports the estimate in part (c)(i). Note: While the response states "M_D+M_D" in the first line, the response later clarifies in the third line that it is "the addition of M_B to M_D."

Question 3 (continued)

Sample: P1 Q3 C
Score: 2

Part (a)(i) did not earn the point for the correct expression. Part (a)(ii) earned no points. The response does not show a valid starting point for conservation of momentum. The second point cannot be earned without the first point. Part (b) earned 2 points. One point was earned for a graph that contains two straight functions that both start at the origin and each have a different positive slope before time t_f. One point was earned for a graph for D that is greater than the graph for S for all times greater than zero. Note: The response shows that the D line ends before $2t_f$, the implication is that the line continues. The response does not show two horizontal functions after t_f. Parts (c)(i) and (c)(ii) earned no points. Neither response estimates the velocity of the center of mass of the system in relation to the original velocity of the disk. It does not appear in the response that the premise of the unequal masses is being addressed. Part (c)(iii) earned no points because the response does not start with the law of conservation of momentum and does not show the correct equation. Part (c)(iv) earned no points because the response shows no attempt to apply a limiting case or functional dependence to the equation in part (c)(iii) and does not link the equation in part (c)(iii) to the estimate in part (c)(i).