Question 4

10 points total | **Distribution of points**

(a)

i. 1 point

For indicating that $I = \mathcal{E}/R$ and $V_C = 0$ — 1 point

Because there is no charge on the capacitor, there is no potential difference across it. Therefore, entire battery potential is across the resistor, so the current is that potential divided by the resistance.

ii. 1 point

For indicating that $I = 0$ and $V_C = \mathcal{E}$ — 1 point

Once the capacitor is fully charged, it allows no current to pass. Because all the components are in series, there is no current at all in the circuit. With no current, there is no potential difference across the resistor, so the entire battery potential is across the capacitor.

(b)

i. 2 points

For a calculation that indicates one of the following: — 1 point

- The potential difference across each capacitor in the new circuit is half that across the single capacitor in the original circuit
- The equivalent capacitance of the new circuit is one-half the capacitance of the original circuit

$U_1 = (1/2)C\mathcal{E}^2$

$U_2 = 2\left[(1/2)C(\mathcal{E}/2)^2\right]$ or $(1/2)(C/2)\mathcal{E}^2$, which both equal $C\mathcal{E}^2/4$

For correctly calculating the ratio — 1 point

$U_1/U_2 = \left(C\mathcal{E}^2/2\right)/\left(C\mathcal{E}^2/4\right) = 2$

ii. 1 point

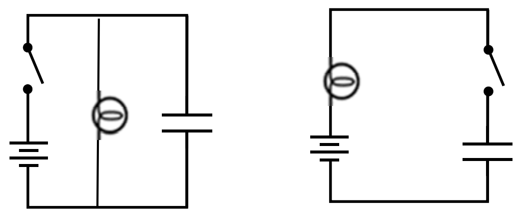
For any combination of area and spacing that is consistent with the student's answer for the ratio in part (b)(i), with a proper principle or model as support — 1 point

Example: $U = (1/2)CV^2$. The potential difference across each of the single capacitors is the same. For the energy stored in the single new capacitor to be half that of the original single capacitor, the new capacitor must have half the capacitance. $C = \varepsilon_0 A/d$, so half the plate area with the same distance between the plates will accomplish this.

Question 4 (continued)

Distribution of points

(c) 5 points



Arrangement 1 Arrangement 2

For two correct circuit diagrams, each matched with the correct situation — arrangement 1 has lightbulb and capacitor in parallel, and arrangement 2 has them in series — 1 point

For indicating that the lightbulb is brightest when the current through it is maximum and that the capacitor eventually stops current from flowing in its branch when the potential difference across its plates is equal in magnitude to the emf of the battery (or something similar) — 1 point

Response using current	Response using potential difference	
For indicating that in the series circuit (where the same current flows through both components) the most current flows right after the switch is closed and decreases as the capacitor charges	For indicating that in the series circuit (where the potential is shared) the resistor has its maximum potential difference right after the switch is closed, because the capacitor starts out uncharged (no potential difference) and then charges until it has the same potential as the battery	1 point
For indicating that in the parallel circuit (where the current is shared between the components) the most current flows through the lightbulb a long time after the switch is closed, because the full current initially goes through the capacitor branch because it acts like a wire (very low potential difference), then ends up all through the lightbulb once the fully charged capacitor acts like an open circuit (same potential difference as battery)	For indicating that in a parallel circuit (where both components have the same potential difference) the bulb starts out with the same zero potential difference as the capacitor and ends up with the total battery potential	1 point

For a response that has sufficient paragraph structure, as described in the published requirements for the paragraph-length response — 1 point

P2 Q4 A1

4. (10 points, suggested time 20 minutes)

Some students are investigating the behavior of a circuit with four components in series: a resistor of resistance R, a capacitor of capacitance C, a battery with potential difference $\mathcal{E}$, and a switch. Initially, the capacitor is uncharged and the switch is open.

(a)

i. Determine the current in the resistor and the potential difference across the capacitor immediately after the switch is closed.

Across capacitor, potential difference = 0 Volts

Current in resistor = $\frac{\mathcal{E}}{R}$ amps

ii. Determine the current in the resistor and the potential difference across the capacitor a long time after the switch is closed.

Current in resistor = 0 amps

Potential difference across capacitor = $\mathcal{E}$ Volts

(b) The switch is opened, the capacitor is discharged, and a second, identical capacitor is added to the circuit in series with the other components. The switch is then closed again.

i. A long time after the switch is closed, the energy stored in the single capacitor in the original circuit is U_1, and the total energy stored in the two capacitors in the new circuit is U_2. Calculate the ratio U_1/U_2.

$U_1: \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\mathcal{E}^2 \rightarrow \frac{U_1}{U_2} = \frac{\frac{1}{2}\mathcal{E}^2}{\frac{1}{4}\mathcal{E}^2} = 2\mathcal{E}^2$

$U_2 = \frac{1}{2}C\left(\frac{\mathcal{E}}{2}\right)^2 + \frac{1}{2}C\left(\frac{\mathcal{E}}{2}\right)^2 = \frac{1}{8}C\mathcal{E}^2 + \frac{1}{8}C\mathcal{E}^2 = \frac{1}{4}C\mathcal{E}^2$

ii. The two capacitors in series are to be replaced with a single capacitor that will have the same energy U_2. Indicate a plate area and a distance between the plates for the new capacitor, compared with one of the original capacitors, that will accomplish this. Support your reasoning using appropriate physics principles and/or mathematical models.

$U_{2\,initial} = \frac{1}{4}C\mathcal{E}^2$

$U = \frac{1}{2}C\Delta V^2$

new $\Delta V = \mathcal{E}$, so new capacitance must be $\frac{C}{2}$ to have $U_{2new} = \frac{1}{4}C\mathcal{E}^2$

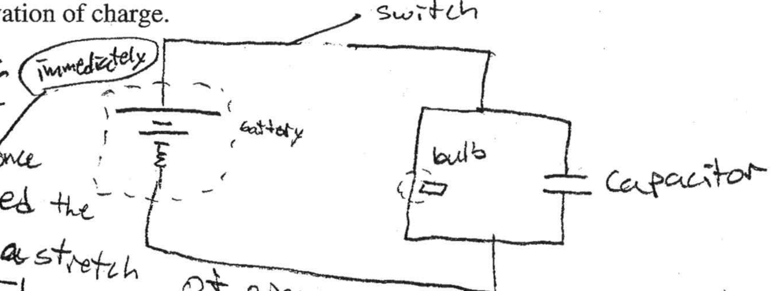
To half capacitance, one can either halve the area per plate or double the distance between them, b/c $C = \frac{\varepsilon_0 A}{d}$.

GO ON TO THE NEXT PAGE.

The students are then asked to design two circuits each containing a switch, a battery with a small internal resistance, a lightbulb, and a capacitor. In arrangement 1, the bulb should gradually light up after the switch is closed, becoming brightest after the switch has been closed a long time. In arrangement 2, the bulb should be brightest when the switch is first closed, getting dimmer with time, and going out completely when the switch has been closed for a long time.

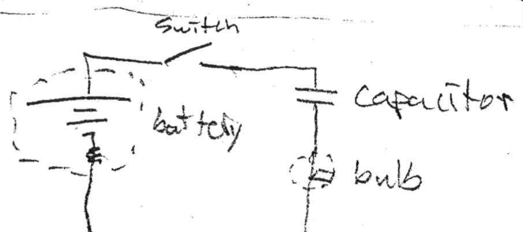
(c) Using standard symbols, draw two circuit diagrams, one showing a possible circuit for arrangement 1 and the other showing a possible circuit for arrangement 2. Justify your circuit diagrams with a paragraph-length explanation referring to the properties of lightbulbs and capacitors in circuits and the conservation of energy and/or the conservation of charge.

Arrangement 1: (immediately)



In this arrangement, once the switch is closed the capacitor acts like a stretch of open wire. Thus, by the junction rule,
when current flows into the parallel system in the circuit, the vast
majority of it will flow into the branch w/ less resistance (the capacitor)
so the bulb will be very dim, b/c brightness is proportional to power, which
= IV. After a while, the capacitor will develop a higher and higher
resistance as it fills w/ charge & its stopping potential increases.
As the resistance in the capacitor increases, more current goes through
the bulb in order to conserve charge, so b/c P=IV, its brightness increases.
Eventually, when the capacitor is completely charged, the bulb will
get all of the circuits current & thus be @ its brightest.

Arrangement 2:



Initially, the capacitor acts like an open wire. Thus, b/c power $= I^2R$ and power is proportional to brightness, the bulb will be brightest @ this time, b/c $I = \frac{V}{R_{total}}$ & R_{total} will be at a minimum.
As the capacitor fills w/ charge, its resistance increases, so the current going through the bulb decreases and the bulbs brightness decreases. Eventually, the capacitor fills w/ charge, and develops a stopping potential equal to the battery's voltage, so no current flows through the bulb, and thus the bulb goes out.

GO ON TO THE NEXT PAGE.

4. (10 points, suggested time 20 minutes)

Some students are investigating the behavior of a circuit with four components in series: a resistor of resistance R, a capacitor of capacitance C, a battery with potential difference $\mathcal{E}$, and a switch. Initially, the capacitor is uncharged and the switch is open.

(a)

i. Determine the current in the resistor and the potential difference across the capacitor immediately after the switch is closed.

$I = \frac{\mathcal{E}}{R}$

ii. Determine the current in the resistor and the potential difference across the capacitor a long time after the switch is closed.

$I = 0$ $V = \mathcal{E}$

(b) The switch is opened, the capacitor is discharged, and a second, identical capacitor is added to the circuit in series with the other components. The switch is then closed again.

i. A long time after the switch is closed, the energy stored in the single capacitor in the original circuit is U_1, and the total energy stored in the two capacitors in the new circuit is U_2. Calculate the ratio U_1/U_2.

Original
capacitance $= C$
$U_1 = \frac{1}{2}C(\Delta V)^2$

New
capacitance: $\frac{1}{C_1} = \frac{1}{C} + \frac{1}{C}$
$C_1 = \frac{C}{2}$
$U_2 = \frac{1}{2}\left(\frac{C}{2}\right)\Delta V^2$

$\frac{U_1}{U_2} = \frac{\frac{1}{2}C(\Delta V)^2}{\frac{1}{2}\left(\frac{C}{2}\right)(\Delta V)^2}$
$= 2$

ii. The two capacitors in series are to be replaced with a single capacitor that will have the same energy U_2. Indicate a plate area and a distance between the plates for the new capacitor, compared with one of the original capacitors, that will accomplish this. Support your reasoning using appropriate physics principles and/or mathematical models.

$C = K\varepsilon_0\frac{A}{d}$

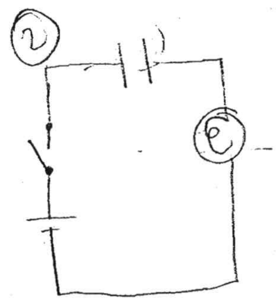
The 2 capacitors in series have a total capacitance of $\frac{C}{2}$. Capacitance is directly related to plate area of distance between plates. So, either the area of the plates can be halved or the distance between the plates can be doubled.

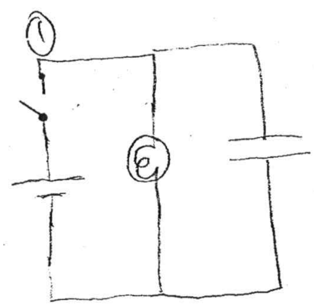
6

$\frac{4}{6} = \frac{A}{d}$

The students are then asked to design two circuits each containing a switch, a battery with a small internal resistance, a lightbulb, and a capacitor. In arrangement 1, the bulb should gradually light up after the switch is closed, becoming brightest after the switch has been closed a long time. In arrangement 2, the bulb should be brightest when the switch is first closed, getting dimmer with time, and going out completely when the switch has been closed for a long time.

(c) Using standard symbols, draw two circuit diagrams, one showing a possible circuit for arrangement 1 and the other showing a possible circuit for arrangement 2. Justify your circuit diagrams with a paragraph-length explanation referring to the properties of lightbulbs and capacitors in circuits and the conservation of energy and/or the conservation of charge.





4. (10 points, suggested time 20 minutes)

Some students are investigating the behavior of a circuit with four components in series: a resistor of resistance R, a capacitor of capacitance C, a battery with potential difference $\mathcal{E}$, and a switch. Initially, the capacitor is uncharged and the switch is open.

(a)

i. Determine the current in the resistor and the potential difference across the capacitor immediately after the switch is closed.

Immediately after the switch is closed, the current in the resistor is $I = 0$. The V across the capacitor is $\frac{Q}{C}$.

ii. Determine the current in the resistor and the potential difference across the capacitor a long time after the switch is closed.

After a long time: $V = 0$

$I = \frac{\mathcal{E}}{R}$

(b) The switch is opened, the capacitor is discharged, and a second, identical capacitor is added to the circuit in series with the other components. The switch is then closed again.

i. A long time after the switch is closed, the energy stored in the single capacitor in the original circuit is U_1, and the total energy stored in the two capacitors in the new circuit is U_2. Calculate the ratio U_1/U_2.

$U_1 = \frac{1}{2} C(\Delta V)^2$

$U_2 = C(\Delta V)^2$

$\frac{U_1}{U_2} = \frac{1}{2}$

ii. The two capacitors in series are to be replaced with a single capacitor that will have the same energy U_2. Indicate a plate area and a distance between the plates for the new capacitor, compared with one of the original capacitors, that will accomplish this. Support your reasoning using appropriate physics principles and/or mathematical models.

The plate are would have to be double that of the original capacitors. The distance can remain the same.

$C \propto \frac{A}{d}$

If d is constant

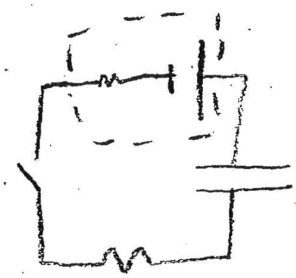
$C \propto A$

GO ON TO THE NEXT PAGE.

The students are then asked to design two circuits each containing a switch, a battery with a small internal resistance, a lightbulb, and a capacitor. In arrangement 1, the bulb should gradually light up after the switch is closed, becoming brightest after the switch has been closed a long time. In arrangement 2, the bulb should be brightest when the switch is first closed, getting dimmer with time, and going out completely when the switch has been closed for a long time.

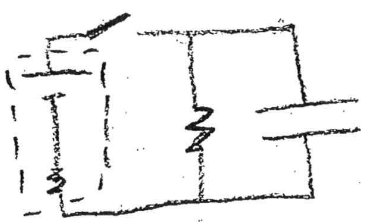
(c) Using standard symbols, draw two circuit diagrams, one showing a possible circuit for arrangement 1 and the other showing a possible circuit for arrangement 2. Justify your circuit diagrams with a paragraph-length explanation referring to the properties of lightbulbs and capacitors in circuits and the conservation of energy and/or the conservation of charge.

①



As time passes, the capacitor absorbs less charge as it fills up and more current can reach the bulb, increasing its brightness.

②



Immediately after the switch closes the entirety of the current goes through the bulb. As time passes, the amount of current going through the capacitor increases taking away from the bulb.

AP® PHYSICS 2
2016 SCORING COMMENTARY

Question 4

Overview

This question assessed learning objectives 4.E.4.1, 4.E.5.1, 4.E.5.2, and 5.B.9.5. The intent of this question was to see if students could analyze the behavior of DC circuits with capacitors and resistors and determine how changes in the circuit would change that behavior. Students were asked to present their analyses in both mathematical and coherent paragraph form.

Sample: P2 Q4 A
Score: 9

Part (a) earned 2 points for correct current and potential difference values for both cases. Part (b)(i) earned 1 point for the calculation of U_2 that indicates that the potential difference across each capacitor is now half that in the original circuit. An algebra mistake leads to an incorrect final answer. The response in part (b)(ii) explains that the new capacitance must be half the previous value based on the energy stored in the capacitors, and earned 1 point for stating both a correct area and distance change, either of which would halve the capacitance. Part (c) earned all 5 points. The response has correct circuit diagrams matched to correct situations, correctly relates the brightness of the bulb to the current through it, and correctly describes the behavior of the capacitor as it charges. It also correctly describes the changes in current for each circuit and is written in coherent, logical paragraphs.

Sample: P2 Q4 B
Score: 5

Part (a)(i) earned no credit because only the correct current is given. In part (a)(ii) both the current and potential are correct, and 1 point was earned. In part (b)(i) the equivalent capacitance is correctly calculated in terms of the original capacitance, and these are used to correctly determine the ratio of energies, so 2 points were earned. Part (b)(ii) earned 1 point for correctly using a comparison of original and new capacitance and stating both a correct area and distance change, either of which would halve the capacitance. Part (c) earned 1 point for correct circuit diagrams.

Sample: P2 Q4 C
Score: 2

Both (a)(i) and (ii) are incorrect. The responses show a possibility of having the two situations backward because the correct answer to (a)(i) is given in (a)(ii). Part (b)(i) earned no credit. The expression for U_2 does not correctly account for either the change in capacitance or the new distribution of potential difference, so the ratio is incorrect. Part (b)(ii) also earned no credit, because there is no supporting reasoning to indicate how the indicated capacitance is consistent with the result of (b)(i). Part (c) earned 2 points. The response correctly relates the brightness of the bulb to the current through it and correctly describes the behavior of the capacitor as it charges. It also has sufficient paragraph structure. The explanations are correctly associated to the two situations, but the circuit diagrams are not.