AP® PHYSICS C: ELECTRICITY AND MAGNETISM
2007 SCORING GUIDELINES

General Notes About 2007 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

3. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution contains the application of that equation to the problem but the student does not write the basic equation, the point is still awarded. However, when students are asked to derive and expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the AP Physics exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics Course Description* .

4. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

5. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 2

15 points total | **Distribution of points**

(a)

(i) 2 points

$\oint \mathbf{E} \cdot d\mathbf{A} = \dfrac{Q_{enc}}{\epsilon_0}$, where Q_{enc} is the charge enclosed by the Gaussian surface

Use a concentric sphere of radius $r < a$ as the Gaussian surface.

For correctly calculating Q_{enc} — 1 point

$$Q_{enc} = \rho V = \left(\frac{Q}{\frac{4}{3}\pi a^3}\right)\left(\frac{4}{3}\pi r^3\right) = \frac{Qr^3}{a^3}$$

E is normal to the surface everywhere, so applying Gauss's law,

$$E\left(4\pi r^2\right) = \frac{1}{\epsilon_0}\left(\frac{Qr^3}{a^3}\right)$$

For the correct answer (This point was not awarded if no supporting work was shown.) — 1 point

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{a^3} \quad \text{or} \quad E = \frac{kQr}{a^3}$$

(ii) 2 points

Use a concentric sphere of radius $a < r < 2a$ as the Gaussian surface.

For correctly identifying Q_{enc} — 1 point

$$Q_{enc} = Q$$

E is normal to the surface everywhere, so applying Gauss's law,

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}$$

For the correct answer (This point was not awarded if no supporting work was shown.) — 1 point

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \quad \text{or} \quad E = \frac{kQ}{r^2}$$

Question 2 (continued)

Distribution of points

(iii) 4 points

Use a concentric sphere of radius $2a < r < 3a$ as the Gaussian surface.

For recognizing that Q_{enc} is the sum of two charges (or that E_{total} is the sum of two components) — 1 point

$Q_{enc} = Q - \rho_0 V_0$, where ρ_0 is the charge density of the outer sphere and V_0 is the volume of the outer sphere enclosed by the Gaussian surface

$$\rho_0 = \frac{Q}{\frac{4}{3}\pi(3a)^3 - \frac{4}{3}\pi(2a)^3} = \frac{Q}{\frac{4}{3}\pi a^3(19)}$$

$$V_0 = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi(2a)^3 = \frac{4}{3}\pi\left(r^3 - 8a^3\right)$$

For correctly calculating Q_{enc} or just the charge enclosed by the shell — 1 point

$$Q_{enc} = Q - \left(\frac{Q}{\frac{4}{3}\pi a^3(19)}\right)\left(\frac{4}{3}\pi\left(r^3 - 8a^3\right)\right) = Q - \frac{Qr^3}{19a^3} + \frac{8Q}{19} = \frac{Q}{19}\left(27 - \frac{r^3}{a^3}\right)$$

Gauss's law, $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{enc}}{\epsilon_0}$, is applied with **E** normal to the surface everywhere.

For recognizing that $\oint \mathbf{E} \cdot d\mathbf{A} = E\left(4\pi r^2\right)$ — 1 point

For correctly substituting Q_{enc} into Gauss's law to find E — 1 point

$$E\left(4\pi r^2\right) = \frac{1}{\epsilon_0}\left(\frac{Q}{19}\left(27 - \frac{r^3}{a^3}\right)\right)$$

$$E = \frac{Q}{76\pi\epsilon_0 r^2}\left(27 - \frac{r^3}{a^3}\right) \text{ or equivalent}$$

(iv) 2 points

Use a concentric sphere of radius $3a < r$ as the Gaussian surface.

For correctly calculating Q_{enc} — 1 point

$$Q_{enc} = +Q - Q = 0$$

Applying Gauss's law

$$E\left(4\pi r^2\right) = \frac{Q_{enc}}{\epsilon_0} = 0$$

For the correct answer (This point was not awarded if no supporting work was shown.) — 1 point

$E = 0$

Question 2 (continued)

Distribution of points

(b) 3 points

For correctly identifying that $V = 0$ — 1 point

For a correct, complete explanation — 2 points

Examples:

$V = -\int_{\infty}^{3a} \mathbf{E} \cdot d\mathbf{r}$, and since $E = 0$ and $V_{\infty} = 0$, it follows that $V = 0$

<u>OR</u> Work to bring a charge from ∞ to the outer surface is 0 since $E = 0$, so
$W = q\left(V_{3a} - V_{\infty}\right) = q\left(V_{3a} - 0\right) = 0$. Thus $V_{3a} = 0$.

<u>Note</u>: 1 point only was awarded for a correct but incomplete explanation such as $\sum q = 0$ *or* $E = 0$.

(c) 2 points

Let V_0 be the potential inside the outer sphere due to charge on the outer sphere. Use superposition of the potentials resulting from the charges on inner and outer spheres.

For correctly identifying V_X — 1 point

$$V_X = V_0 + \frac{Q}{4\pi\epsilon_0 a}$$

For correctly identifying V_Y — 1 point

$$V_Y = V_0 + \frac{Q}{4\pi\epsilon_0 (2a)}$$

$$V_X - V_Y = \left(V_0 + \frac{Q}{4\pi\epsilon_0 a}\right) - \left(V_0 + \frac{Q}{4\pi\epsilon_0 (2a)}\right) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{2a}\right)$$

$$V_X - V_Y = \frac{Q}{8\pi\epsilon_0 a}$$

Alternate solution — *Alternate points*

$$E = -\frac{dV}{dr}$$

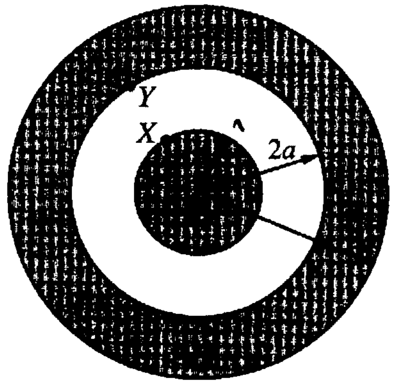
$$\Delta V = \int E\, dr$$

For setting up a correct integral with proper limits and sign (limits could be switched from those shown below if the integral had a positive sign) — *1 point*

For correct substitution of E in to the integral — *1 point*

$$V_X - V_Y = -\int_{2a}^{a} \frac{Q}{4\pi\epsilon_0 r^2}\, dr = -\frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{r}\Big|_{2a}^{a}\right) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{2a}\right)$$

$$V_X - V_Y = \frac{Q}{8\pi\epsilon_0 a}$$



E&M 2.

In the figure above, a nonconducting solid sphere of radius a with charge $+Q$ uniformly distributed throughout its volume is concentric with a nonconducting spherical shell of inner radius $2a$ and outer radius $3a$ that has a charge $-Q$ uniformly distributed throughout its volume. Express all answers in terms of the given quantities and fundamental constants.

(a) Using Gauss's law, derive expressions for the magnitude of the electric field as a function of radius r in the following regions.

i. Within the solid sphere ($r < a$)

$\int E\,dA = \frac{Q_{in}}{\varepsilon_0}$ $\quad \rho = \frac{Q}{\frac{4}{3}\pi a^3} \cdot \frac{4}{3}\pi r^3 \quad \frac{Qr^3}{a^3}$

$E(4\pi r^2) = \frac{Qr^3}{\varepsilon_0 a^3}$

$\boxed{E = \frac{Qr}{4\pi\varepsilon_0 a^3}}$

ii. Between the solid sphere and the spherical shell ($a < r < 2a$)

Spher acts as point charge.

$\boxed{\frac{KQ}{r^2}}$

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iii. Within the spherical shell ($2a < r < 3a$)

$$\frac{\cdot Q}{\frac{4}{3}\pi(3a)^3 - 4/3\pi(2a)^3}$$

$$\rho = \frac{Q}{76\pi r^3 / 3} \quad 4/3\pi r^3 - 4/3\pi(2a)^3$$

$$= \left(\frac{r^3 - 8a^3}{19a^3}\right) \cdot Q$$

$$Q'_{in} = \underline{Q - Q\left(\frac{r^3 - 8a^3}{19a^3}\right)}$$

$$\oint E\,dA = E(4\pi r^2) = \frac{Q - Q\left(\frac{r^3-8a^3}{19a^3}\right)}{\varepsilon_0}$$

$$\boxed{E = \frac{Q - Q\left(\frac{r^3 - 8a^3}{19a^3}\right)}{4\pi r^2 \varepsilon_0}}$$

iv. Outside the spherical shell ($r > 3a$)

$$Q'_{in} = Q_{net} = 0$$

so $\boxed{E = 0}$

(b) What is the electric potential at the outer surface of the spherical shell ($r = 3a$)? Explain your reasoning.

$$-\frac{kQ}{3a} + \frac{kQ}{3a} = 0$$

(c) Derive an expression for the electric potential difference $V_X - V_Y$ between points X and Y shown in the figure.

$V = Ed$

$$\frac{kQ}{a} - \frac{kQ}{2a} = V_X - V_Y$$

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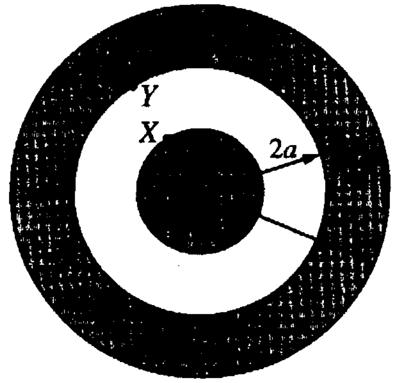

E&M 2.

In the figure above, a nonconducting solid sphere of radius a with charge $+Q$ uniformly distributed throughout its volume is concentric with a nonconducting spherical shell of inner radius $2a$ and outer radius $3a$ that has a charge $-Q$ uniformly distributed throughout its volume. Express all answers in terms of the given quantities and fundamental constants.

(a) Using Gauss's law, derive expressions for the magnitude of the electric field as a function of radius r in the following regions.

i. Within the solid sphere ($r < a$)

Gauss's law:

$\int E\, dA = \frac{Q_{enc}}{\varepsilon_0}$

$S = 4\pi r^2$

$\frac{Q_{ENC}}{\cancel{\frac{4}{3}}\pi r^3} = \frac{Q}{\frac{4}{3}\pi a^3}$

$Q_{ENC} = \frac{r^3 Q}{a^3}$

$E = \frac{Q r^{\cancel{3}}}{4\pi \cancel{r^2} \varepsilon_0 a^3} = \boxed{\frac{Qr}{4\pi\varepsilon_0 a^3}\ \frac{N}{C}}$

ii. Between the solid sphere and the spherical shell ($a < r < 2a$)

$Q_{ENC} = Q$

$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}\ N/C$

iii. Within the spherical shell ($2a < r < 3a$)

iv. Outside the spherical shell ($r > 3a$)

$\boxed{E = 0}$ b/c $Q_{ENC} = Q - Q = 0$

(b) What is the electric potential at the outer surface of the spherical shell ($r = 3a$)? Explain your reasoning.

$V = \frac{kQ}{r}$ $\boxed{V = 0}$ b/c $Q_{ENC} = Q - Q = 0$

(c) Derive an expression for the electric potential difference $V_X - V_Y$ between points X and Y shown in the figure.

$$\Delta V = -\int_a^{2a} E\, da = -\int_a^{2a} \frac{Q}{4\pi \varepsilon_0 r^2}\, da = -\left[\frac{Q}{4\pi\varepsilon_0}\right]\int_a^{2a} \frac{da}{r^2} = \left[\frac{Q}{4\pi\varepsilon_0}\right]\frac{1}{r}\Big|_a^{2a}$$

$$\boxed{\Delta V = \left[\frac{Q}{4\pi\varepsilon_0}\right]\left[\frac{1}{2a} - \frac{1}{a}\right] V}$$

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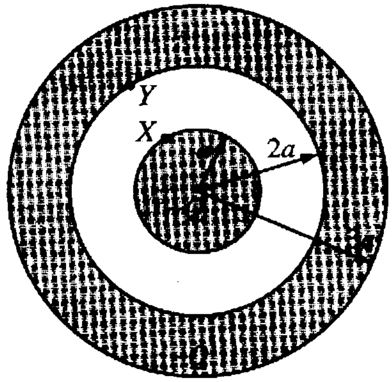

E&M 2.

In the figure above, a nonconducting solid sphere of radius a with charge $+Q$ uniformly distributed throughout its volume is concentric with a nonconducting spherical shell of inner radius $2a$ and outer radius $3a$ that has a charge $-Q$ uniformly distributed throughout its volume. Express all answers in terms of the given quantities and fundamental constants.

(a) Using Gauss's law, derive expressions for the magnitude of the electric field as a function of radius r in the following regions.

i. Within the solid sphere ($r < a$)

$\oint E\,da = 4\pi K Q_+$, $Q_+ = \rho V = \rho(\frac{4}{3}\pi r^3)$

$E(4\pi r^2) = 4\pi K(\rho \frac{4}{3}\pi r^3)$

$E = \frac{4}{3}K\rho\pi r$ N/C

ii. Between the solid sphere and the spherical shell ($a < r < 2a$)

$\oint E\,da = 4\pi K Q_+$ $Q_+ = Q$ (all the charge)

$E(4\pi r^2) = 4\pi K(Q)$

$E = \frac{KQ}{r^2}$ N/C

GO ON TO THE NEXT PAGE.

iii. Within the spherical shell ($2a < r < 3a$)

$\int E da = 4\pi K Q_+$

iv. Outside the spherical shell ($r > 3a$)

$\int E da = 4\pi K Q_+$, $Q_+ = 0$ (inside and outside charges cancel out)

$E = 0 \; N/C$

(b) What is the electric potential at the outer surface of the spherical shell ($r = 3a$)? Explain your reasoning.

The electric potential is zero because there is no charge or E field on the surface

(c) Derive an expression for the electric potential difference $V_X - V_Y$ between points X and Y shown in the figure.

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Question 2

Overview

The purpose of this question was to evaluate students' ability to use Gauss's law to find the electric field that resulted from a specified spherically symmetric charge distribution, as well as their understanding of the relationship between an electric field and electric potential.

Sample: E2A
Score: 14

This very succinctly written response lost only 1 point for an incomplete explanation in part (b). In part (a)(ii) the explicit statement of the sphere acting as a point charge was sufficient for full credit. In part (a)(iv) the indication that the enclosed charge is zero implies applying Gauss's law. Part (c) illustrates the bare minimum to get full credit for the superposition method.

Sample: E2B
Score: 9

Parts (a)(i), (a)(ii), and (a)(iv) received full credit, but part (a)(iii) shows no work and earned no credit. Part (b) received 2 points, but the explanation is incomplete, referring only to sum of the charges. Part (c) uses the alternate method of integrating over the field and correctly substitutes for E, but the sign is incorrect for the limits of integration shown, so only 1 point was awarded.

Sample: E2C
Score: 6

Part (a)(i) received no credit. Parts (a)(ii) and (a)(iv) each received 2 points full credit. No credit was given for part (a)(iii) for showing no work beyond writing down Gauss's law. Part (b) was given only 2 points because the explanation is incomplete. Part (c) was left blank and hence earned no credit.