AP® PHYSICS C: ELECTRICITY AND MAGNETISM
2010 SCORING GUIDELINES

General Notes

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for the solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong — for example, a speed faster than the speed of light in vacuum.

3. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point and a student's solution contains the application of that equation to the problem but the student does not write the basic equation, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the AP Physics Exams equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics Course Description*.

4. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

5. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

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Question 3

15 points total | **Distribution of points**

(a) 4 points

For a correct indication that the current in the loop is in the counterclockwise direction	1 point
For indicating that the magnetic field through the loop is directed out of the page (which can be done on the diagram)	1 point
For indicating that the current in the wire is decreasing, either explicitly or by indicating a decrease in field or flux through the loop	1 point
For indicating that the direction of the induced current is such as to oppose the change in flux	1 point

Example: The flux due to current I is out of the plane of the page (by the right-hand rule) and is decreasing with time. The induced current will be in the direction that will produce a compensating flux (by Lenz's law). Again using a right-hand rule, the current must be counterclockwise.

(b) 2 points

For a correct indication that the brightness of the lightbulb remains the same	1 point
For a correct justification	1 point

Example: The field and the flux both vary linearly with time. The emf, which is the time derivative of the flux, must then be constant. Since the power output of the lightbulb depends only on the emf and resistance (which are both constant), the power must be constant.

(c) 2 points

$$\oint \mathbf{B} \bullet d\ell = \mu_0 I$$

$$B 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B = \left.\frac{\mu_0 I}{2\pi r}\right|_{t=0}$$

For the correct answer — 2 points

$$B = \frac{\mu_0 I_0}{2\pi r}$$

One point partial credit could be earned for either correctly applying Ampere's law and leaving the result in terms of I, or obtaining an incorrect expression for B in terms of I_0.

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Question 3 (continued)

Distribution of points

(d) 4 points

$\phi = \int \mathbf{B} \cdot d\mathbf{A}$

For correctly substituting the expression for B as a function of r into the flux equation — 1 point

$\phi = \int \left(\frac{\mu_0 I}{2\pi r} \right) dA$

For correctly recognizing that $dA = b\ dr$ — 1 point

$\phi = \int \left(\frac{\mu_0 I}{2\pi r} \right) b\ dr$

For correctly integrating with respect to r — 1 point

$\phi = \frac{\mu_0 I b}{2\pi} \int_d^{d+a} \frac{dr}{r}$

$\phi = \frac{\mu_0 I b}{2\pi} \ln(r) \Big|_d^{d+a}$

$\phi = \frac{\mu_0 I b}{2\pi} \ln\left(\frac{d+a}{d} \right)$

For correctly substituting the current as a function of time — 1 point

$\phi = \frac{\mu_0 (I_0 - Kt) b}{2\pi} \ln\left(\frac{d+a}{d} \right)$

(e) 3 points

$P = V^2 / R$ (which can be derived from $P = I^2 R$ and $V = IR$)

For recognizing that the voltage across the bulb is the induced emf in the loop, and using that emf in the above expression for power — 1 point

$P = \mathcal{E}^2 / R$, where $\mathcal{E} = -d\phi/dt$

For correctly substituting the flux from part (d) into the above equation for emf — 1 point

$\mathcal{E} = -\frac{d}{dt}\left[\frac{\mu_0 b (I_0 - Kt)}{2\pi} \ln\left(\frac{d+a}{d} \right) \right]$

For correctly taking the derivative (with respect to time) of the flux — 1 point

$\mathcal{E} = -\frac{\mu_0 b}{2\pi} \ln\left(\frac{d+a}{d} \right) \left[\frac{d}{dt}(I_0 - Kt) \right]$

$\mathcal{E} = -\frac{\mu_0 b}{2\pi} \ln\left(\frac{d+a}{d} \right)(-K)$

$P = \frac{\mathcal{E}^2}{R} = \frac{1}{R}\left[\frac{\mu_0 b K}{2\pi} \ln\left(\frac{d+a}{d} \right) \right]^2$

Note: If $P = I\mathcal{E}$ is used with the expression for the current in the long wire (rather than the loop) being substituted for I, the last 2 points for correctly determining the emf could still be earned.

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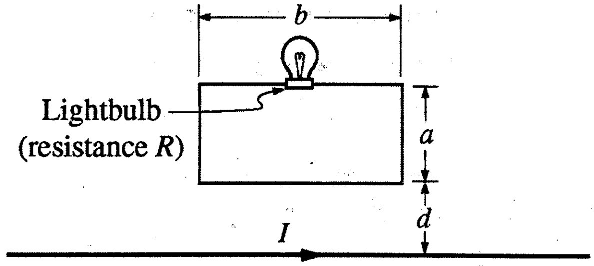

E&M. 3.

The long straight wire illustrated above carries a current I to the right. The current varies with time t according to the equation $I = I_0 - Kt$, where I_0 and K are positive constants and I remains positive throughout the time period of interest. The bottom of a rectangular loop of wire of width b and height a is located a distance d above the long wire, with the long wire in the plane of the loop as shown. A lightbulb with resistance R is connected in the loop. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Indicate the direction of the current in the loop.

____Clockwise __X__Counterclockwise

Justify your answer.

the magnetic field, by right hand rule, is directed out of the page as it goes through the loop, and it is decreasing, because the current in the wire is decreasing the induced current must act to counteract the decreasing magnetic flux, and therefore travels counterclockwise

(b) Indicate whether the lightbulb gets brighter, gets dimmer, or stays the same brightness over the time period of interest.

____Gets brighter ____Gets dimmer __X__Remains the same

Justify your answer.

~~Eventually, there will be no current left in the~~

the induced current will be constant, because the magnetic flux is dependent on only the first degree of t, and will therefore have a derivative that is constant

(c) Determine the magnetic field at $t = 0$ due to the current in the long wire at distance r from the long wire.

$\oint B \cdot d\ell = \mu_0 I$

$B\ell = \mu_0 I_0$

$B(2\pi r) = \mu_0 I_0$

$B = \frac{\mu_0 I_0}{2\pi r}$

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(d) Derive an expression for the magnetic flux through the loop as a function of time.

$$\Phi_B = \int B \cdot dA$$
$$\Phi_B = \int_d^{d+a} \frac{\mu_0 I}{2\pi r}(b\,dr)$$
$$\Phi_B = \frac{\mu_0 I b}{2\pi}\int_d^{d+a} \frac{dr}{r}$$
$$\Phi_B = \frac{\mu_0 I b}{2\pi} \ln r \Big|_d^{d+a}$$
$$\Phi_B = \frac{\mu_0 I b}{2\pi} \ln\left(1+\frac{a}{d}\right)$$
$$\Phi_B = \frac{\mu_0 b}{2\pi} \ln\left(1+\frac{a}{d}\right)(I_0 - Kt)$$

(e) Derive an expression for the power dissipated by the lightbulb.

$$\mathcal{E} = \frac{d\Phi_B}{dt}$$
$$\mathcal{E} = \frac{\mu_0 b}{2\pi} \ln\left(1+\frac{a}{d}\right) K$$

$$V = IR$$
$$I = \frac{V}{R}$$
$$I = \frac{\mu_0 b K}{2\pi R} \ln\left(1+\frac{a}{d}\right)$$

$$P = I^2 R$$
$$P = \left(\frac{\mu_0 b K}{2\pi R} \ln\left(1+\frac{a}{d}\right)\right)^2 R$$
$$P = \frac{\mu_0^2 b^2 K^2}{4\pi^2 R} \ln\left(1+\frac{a}{d}\right)^2$$

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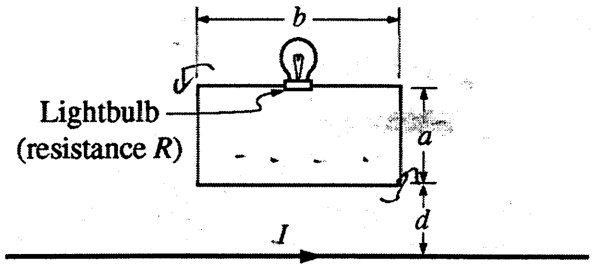

E&M. 3.

X X X X

The long straight wire illustrated above carries a current I to the right. The current varies with time t according to the equation $I = I_0 - Kt$, where I_0 and K are positive constants and I remains positive throughout the time period of interest. The bottom of a rectangular loop of wire of width b and height a is located a distance d above the long wire, with the long wire in the plane of the loop as shown. A lightbulb with resistance R is connected in the loop. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Indicate the direction of the current in the loop.

____Clockwise ✓ Counterclockwise

Justify your answer.

I decreases, Φ_m decreases as well
Φ_m was in ⊙ direction, thus the current is induced counterclockwise to resist the change of Φ_m and induce ⊙ direction Φ_m.

(b) Indicate whether the lightbulb gets brighter, gets dimmer, or stays the same brightness over the time period of interest.

____Gets brighter ____Gets dimmer ✓ Remains the same

Justify your answer.

I is decreasing at a constant rate with time, which means that $\frac{d\phi_m}{dt}$ is also constant with time.
Therefore, $\varepsilon = -\frac{d\phi_m}{dt}$ and ε stays constant, giving same power and brightness $(P = \frac{\varepsilon^2}{R})$

(c) Determine the magnetic field at $t = 0$ due to the current in the long wire at distance r from the long wire.

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{en}$

at $t=0$, $I = I_0$.

$B \cdot 2\pi r = \mu_0 I_0$

$B = \frac{\mu_0 I_0}{2\pi r}$

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(d) Derive an expression for the magnetic flux through the loop as a function of time.

$\phi_m = \int B \cdot dA$

$A = ab$

$dA = b\,da$

$\phi_m = b \int_d B\, da$

(e) Derive an expression for the power dissipated by the lightbulb.

$P = \frac{\mathcal{E}^2}{R}$

$\mathcal{E} = -\frac{d\phi_m}{dt}$

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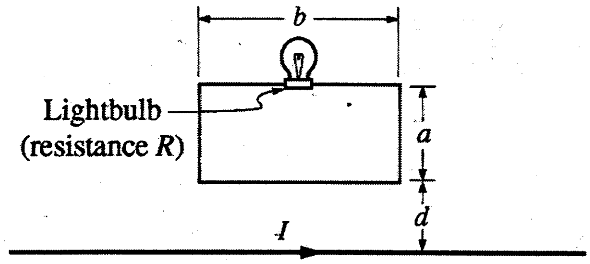

E&M. 3.

The long straight wire illustrated above carries a current I to the right. The current varies with time t according to the equation $I = I_0 - Kt$, where I_0 and K are positive constants and I remains positive throughout the time period of interest. The bottom of a rectangular loop of wire of width b and height a is located a distance d above the long wire, with the long wire in the plane of the loop as shown. A lightbulb with resistance R is connected in the loop. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Indicate the direction of the current in the loop.

____Clockwise　　　X Counterclockwise

Justify your answer.

The magnetic field created by the moving current induces current in the wire above in the same direction as it is moving. It does this because the ~~mg~~ magnetic field ~~change~~ around the lightbulb wire is coming out of the page. Using RHR, the direction is determined

(b) Indicate whether the lightbulb gets brighter, gets dimmer, or stays the same brightness over the time period of interest.

____Gets brighter　　　X Gets dimmer　　　____Remains the same

Justify your answer.

The brightness depends on the voltage. Since $V=IR$, and R is a constant, when I decreases, V decreases. I will decrease because, ~~the~~ as the equation states, I is decreasing over time.

(c) Determine the magnetic field at $t = 0$ due to the current in the long wire at distance r from the long wire.

$$\cancel{B_s = \mu_0 n I = \mu_0 \cdot I \cdot 1 = \boxed{\mu_0 I}}$$

$$\frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} \qquad \boxed{\frac{I\mu_0}{4\pi r^2}}$$

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(d) Derive an expression for the magnetic flux through the loop as a function of time.

$$\Phi_m = \int B \cdot dA = \frac{I \mu_0}{4\pi r^2} \cdot 2b = \boxed{\frac{b\mu_0 (I_0 - kt)}{2\pi d^2}}$$

only sides parallel to the wire matter

(e) Derive an expression for the power dissipated by the lightbulb.

$$P = \frac{V^2}{R} \qquad \mathcal{E} = \frac{d\Phi_m}{dt}$$

$$\mathcal{E} = \cancel{b\mu_0 (I - kt)} \; \frac{-kb\mu_0}{2\pi d^2}$$

$$P = \frac{\mathcal{E}^2}{R} \qquad \boxed{P = \frac{k^2 b^2 \mu_0^2}{4\pi^2 d^4 R}}$$

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Question 3

Overview

This question was intended to assess students' understanding of electromagnetism. Part (a) asked students to determine the direction of the induced current in a loop of wire near a long wire. Students had to give a justification for their choice based on Lenz's law. Part (b) asked students whether the bulb brightness would increase, decrease or remain the same, and students were to justify their choice by discussing the rate of change of the flux. Part (c) asked students for the strength of the magnetic field as a function of position at time $t = 0$. Students were asked in part (d) to derive an expression for the flux through the loop as a function of time, for which they had to integrate the magnetic field over the area of the loop. In part (e) they were asked to derive an expression for the power dissipated by the lightbulb as a function of time.

Sample: E3-A
Score: 15

This full-credit response does a nice job of justifying the choices in parts (a) and (b), clearly stating each step in the thought process. The derivation of the flux in part (d) is nicely done. In part (e) the current induced in the loop is correctly calculated and then used in the expression $P = I^2R$ to determine the power.

Sample: E3-B
Score: 9

This response earned full credit for parts (a) through (c). The explanations in (a) and (b) are nicely done, and the derivation using Ampere's law in part (c) shows all the relevant steps. No credit was earned for part (d). The response starts with the correct flux equation but does not have any correct substitutions. Part (e) did earn 1 point, since the two equations shown indicate that the emf used to calculate the power is the rate of change of flux through the loop.

Sample: E3-C
Score: 7

Part (a) earned 2 points. The justification is incomplete since it mentions only that the magnetic field is out of the page through the loop. The incorrect answer to part (b) is a common misconception and received no credit. In response to this part, students often stated that since the current is decreasing, the brightness is decreasing, rather than relating the brightness to the induced current, which is constant. No credit was earned for an incorrect answer to part (c). However, the student correctly substitutes this answer into the expression for flux in part (d) and thus earned 1 point. The second point earned in part (d) was for substituting the expression for the current as a function of time. Full credit was earned for part (e) since the expression for the flux derived in part (d) is correctly used.

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