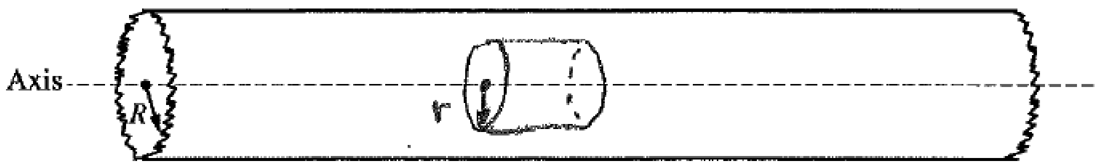
Question 1

15 points total | **Distribution of points**

(a) 5 points



For drawing a Gaussian surface that is a cylinder, contained within and sharing the axis of the given cylinder — 1 point

For using a correct statement of Gauss's law — 1 point

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \bullet dA$$

For a correct expression for q_{enc} including the expression for the volume of a cylinder — 1 point

$$q_{enc} = \rho V_{enc} = \rho\left(\pi r^2 L\right)$$

For a correct expression for the surface area of the sides of a cylinder — 1 point

$$A = 2\pi r L$$

Substitute expressions into Gauss's law:

$$\frac{\rho\left(\pi r^2 L\right)}{\varepsilon_0} = E(2\pi r L)$$

For a correct answer — 1 point

$$E = \frac{\rho r}{2\varepsilon_0}$$

(b) 1 point

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \bullet dA$$

$$q_{enc} = \rho V_{tot} = \rho\left(\pi R^2 L\right)$$

$$\frac{\rho\left(\pi R^2 L\right)}{\varepsilon_0} = E(2\pi r L)$$

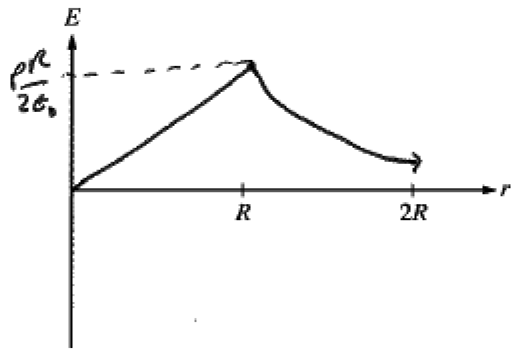
For a correct answer — 1 point

$$E = \frac{\rho R^2}{2\varepsilon_0 r}$$

Question 1 (continued)

Distribution of points

(c) 3 points



For a graph in the region $0 < r < R$ that is a straight line starting at the origin, and with a positive slope — 1 point

For a graph in the region $R < r < 2R$ that is continuous, decreasing, concave upward, and not zero at $r = 2R$ — 1 point

For labeling the maximum value of E on the vertical axis, consistent with part (a) — 1 point

(d)

i. 3 points

For stating the correct relation between the magnitude of the potential difference and the integral of the electric field — 1 point

$\Delta V = -\int_0^R E \bullet dr$ (negative sign not required)

For substituting the expression for electric field from part (a) — 1 point

$$\Delta V = -\int_0^R \frac{\rho r}{2\varepsilon_0} dr$$

For integrating with the proper limits or correctly evaluating the constant of integration — 1 point

$$V(R) - V(0) = -\frac{\rho}{2\varepsilon_0}\left[\frac{r^2}{2}\right]_0^R = -\frac{\rho}{2\varepsilon_0}\left(\frac{R^2}{2} - 0\right) = -\frac{\rho R^2}{4\varepsilon_0}$$

$$|\Delta V| = \frac{\rho R^2}{4\varepsilon_0} \left(\text{leaving the answer } -\frac{\rho R^2}{4\varepsilon_0} \text{ is also acceptable}\right)$$

Note: Graphical integration could also be performed. For students taking this approach, 1 point was earned for evaluating the area for $0 < r < R$, 1 point for calculating the area of a triangle, and 1 point for a magnitude consistent with previous results.

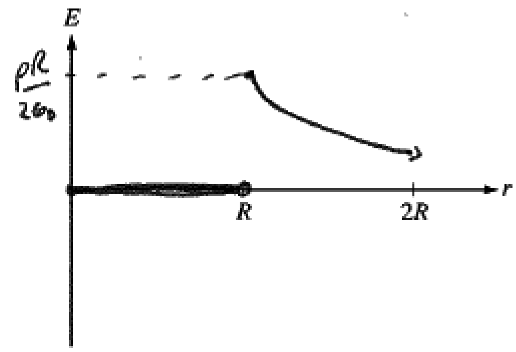
Question 1 (continued)

Distribution of points

ii. 1 point

For selecting "$r = 0$" — 1 point

(e) 2 points



For $0 < r < R$, drawing a horizontal graph with $E = 0$ — 1 point

For $R < r < 2R$, drawing a graph consistent with the graph in part (c) in that region — 1 point

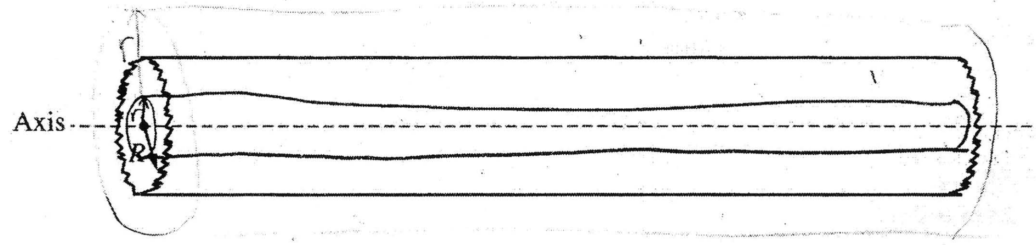
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M 1.

A very long, solid, nonconducting cylinder of radius R has a positive charge of uniform volume density ρ. A section of the cylinder far from its ends is shown in the diagram above. Let r represent the radial distance from the axis of the cylinder. Express all answers in terms of r, R, ρ, and fundamental constants, as appropriate.

(a) Using Gauss's law, derive an expression for the magnitude of the electric field at a radius $r < R$. Draw an appropriate Gaussian surface on the diagram.

$Q_{encl} = \rho \cdot \text{volume} = \rho \pi (r-R)^2 \lambda$

$$\oint E \cdot dA = \frac{Q_{encl}}{\varepsilon_0}$$

$$E(2\pi r \ell) = \frac{\rho \pi r^2 \ell}{\varepsilon_0}$$

$$\boxed{E = \frac{\rho r}{2\varepsilon_0}}$$

(b) Using Gauss's law, derive an expression for the magnitude of the electric field at a radius $r > R$.

$$\oint E \cdot dA = \frac{Q_{encl}}{\varepsilon_0}$$

$Q_{encl} = \rho \text{ volume} = \rho \pi R^2 \ell$

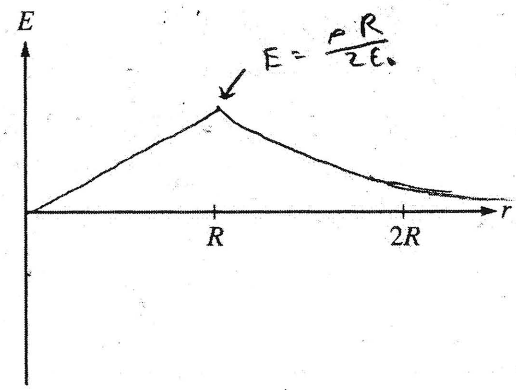
$$E(2\pi r \ell) = \frac{\rho(\pi R^2 \ell)}{\varepsilon_0}$$

$$\boxed{E = \frac{\rho R^2}{2\varepsilon_0 r}}$$

GO ON TO THE NEXT PAGE.

(c) On the axes below, sketch the graph of electric field E as a function of radial distance r for $r = 0$ to $r = 2R$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

E1 A2



(d)

i. Derive an expression for the magnitude of the potential difference between $r = 0$ and $r = R$.

$$E = -\frac{dV}{dr}$$

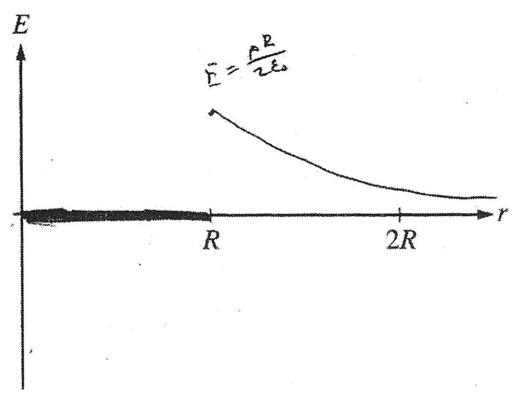
$$\int dV = \int -E\, dr$$

$$V = -\int E \cdot dr = -\int_0^R \frac{\rho r}{2\varepsilon_0}\, dr = -\frac{\rho r^2}{4\varepsilon_0}\Big|_0^R = \boxed{-\frac{\rho R^2}{4\varepsilon_0}}$$

ii. Is the potential higher at $r = 0$ or $r = R$?

__✓__ $r = 0$ ______ $r = R$

(e) The nonconducting cylinder is replaced with a conducting cylinder of the same shape and same <u>linear</u> charge density. On the axes below, sketch the electric field E as a function of r for $r = 0$ to $r = 2R$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes
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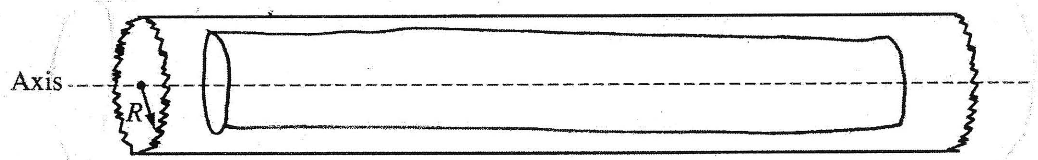


E&M 1.

A very long, solid, nonconducting cylinder of radius R has a positive charge of uniform volume density ρ. A section of the cylinder far from its ends is shown in the diagram above. Let r represent the radial distance from the axis of the cylinder. Express all answers in terms of r, R, ρ, and fundamental constants, as appropriate.

(a) Using Gauss's law, derive an expression for the magnitude of the electric field at a radius $r < R$. Draw an appropriate Gaussian surface on the diagram.

$$\int E\,dA = \frac{Q_{in}}{\varepsilon_0}$$

$$E(\pi r^2 \ell) = \frac{Q_{in}}{\varepsilon_0}$$

$$E = \frac{\rho}{\pi r^2 \varepsilon_0}$$

(b) Using Gauss's law, derive an expression for the magnitude of the electric field at a radius $r > R$.

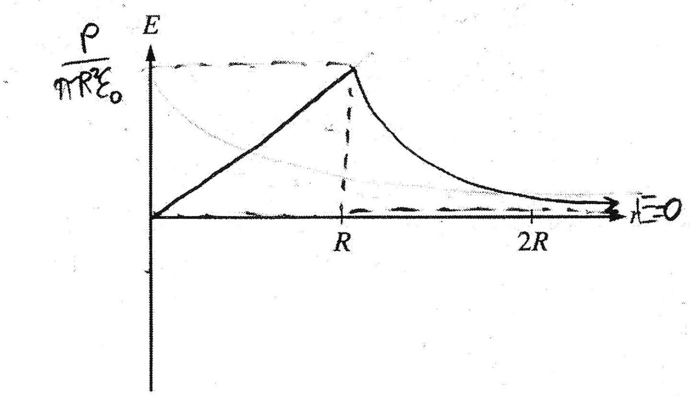
$$\int E\,dA = \frac{Q_{in}}{\varepsilon_0}$$

$$E(\pi R^2 \ell) = \frac{Q_{in}}{\varepsilon_0}$$

$$E = \frac{\rho}{\pi R^2 \varepsilon_0}$$

GO ON TO THE NEXT PAGE.

(c) On the axes below, sketch the graph of electric field E as a function of radial distance r for $r = 0$ to $r = 2R$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(d)

i. Derive an expression for the magnitude of the potential difference between $r = 0$ and $r = R$.

$$E = \frac{-dV}{dr}$$

$$-E\,dr = dV$$

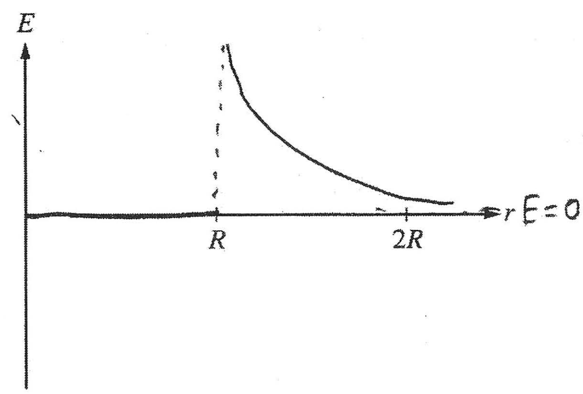
$$dV = \frac{-\rho\,dr}{\pi R^2 \varepsilon_0} \Rightarrow \int dV = \int_0^R \frac{-\rho}{\pi R^2 \varepsilon_0}\,dr$$

$$V = \frac{-\rho}{\pi R \varepsilon_0}$$

ii. Is the potential higher at $r = 0$ or $r = R$?

✓ $r = 0$ ____ $r = R$

(e) The nonconducting cylinder is replaced with a conducting cylinder of the same shape and same linear charge density. On the axes below, sketch the electric field E as a function of r for $r = 0$ to $r = 2R$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



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E1 C1

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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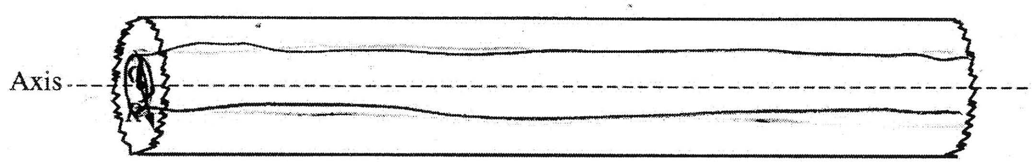


E&M 1.

A very long, solid, nonconducting cylinder of radius R has a positive charge of uniform volume density ρ. A section of the cylinder far from its ends is shown in the diagram above. Let r represent the radial distance from the axis of the cylinder. Express all answers in terms of r, R, ρ, and fundamental constants, as appropriate.

(a) Using Gauss's law, derive an expression for the magnitude of the electric field at a radius $r < R$. Draw an appropriate Gaussian surface on the diagram.

$\oint E \cdot dA = \frac{q_{enclosed}}{\varepsilon_0}$ Uniform charge density

$E(\pi r^2 h) = \frac{\rho \times \pi r^2 h}{\varepsilon_0}$ $E = \frac{\rho \cdot r^2}{\pi r^2}$

(b) Using Gauss's law, derive an expression for the magnitude of the electric field at a radius $r > R$.

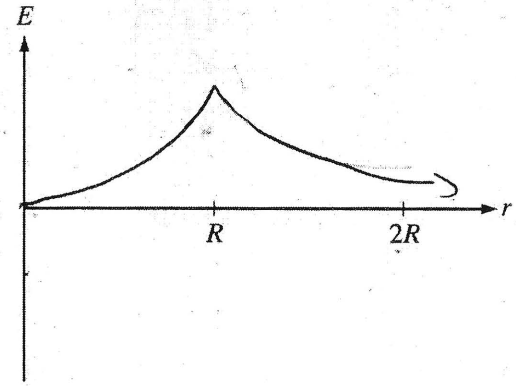
$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$ $E(\pi r^2 h) =$

$E = \frac{kQ}{r^2}$ $E =$

GO ON TO THE NEXT PAGE.

E1 C2

(c) On the axes below, sketch the graph of electric field E as a function of radial distance r for $r = 0$ to $r = 2R$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(d)

i. Derive an expression for the magnitude of the potential difference between $r = 0$ and $r = R$.

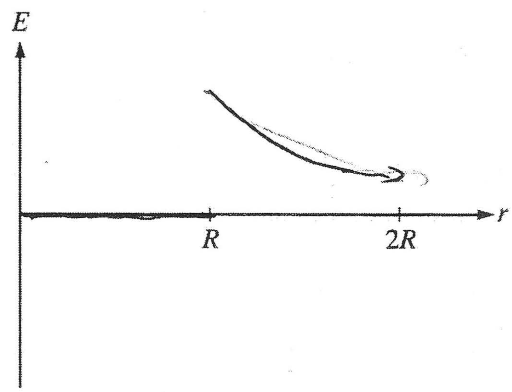
$V = \frac{kQ}{r}$ $V_{@\, r=0} = 0$

$\frac{kQ}{R} - 0 = \boxed{\frac{kQ}{R}}$

ii. Is the potential higher at $r = 0$ or $r = R$?

____ $r = 0$ __✓__ $r = R$

(e) The nonconducting cylinder is replaced with a conducting cylinder of the same shape and same <u>linear</u> charge density. On the axes below, sketch the electric field E as a function of r for $r = 0$ to $r = 2R$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



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Question 1

Overview

This question evaluated students' understanding of Gauss's law in a classic problem. In the first two parts of the problem, students were asked to derive an expression for the electric field E as a function of the distance r from the axis of a cylindrical non-conductor with a uniform charge distribution ρ and radius R. In part (a), students performed the derivation for the region inside the cylinder, and in part (b) students repeated the process for the region outside of the cylinder. In part (c), students sketched a qualitative graph of E as a function of r for $0 < r < 2R$. Part (d) required students to use their expression for E inside the cylinder to derive an expression for the potential difference ΔV between the cylinder axis and its surface, and to identify the point of highest potential as either $r = 0$ or $r = R$. In part (e), students sketched another qualitative graph of E as a function of r for $0 < r < 2R$, this time for a cylinder of identical radius and linear charge distribution but made of a conducting material.

Sample: E1-A
Score: 15

This response is a well-illustrated and completely correct solution. Parts (a) and (b) are both classic representations of Gauss's law derivations with an appropriate Gaussian surface drawn in the diagram. Part (c) is a clear and properly labeled graph sketched with the proper shape which clearly does not touch the horizontal axis at $r = 2R$. In part (d)(i), there is no deduction for the negative sign even though a magnitude calculation is requested. The student correctly identifies the higher potential at $r = 0$ in part (d)(ii). Part (e) shows a correct graph from $r = 0$ to $r = R$, earning the first point. The second point is earned for the similarity between this graph and the graph in part (c) for $r > R$.

Sample: E1-B
Score: 10

Part (a) received two points for the correct calculation of the surface area, and for drawing an appropriate Gaussian surface. The point for Gauss's law is not awarded because there is no indication that the integral is done over a closed surface. The student does not correctly calculate enclosed charge using the volume charge density and does not get a correct answer; therefore, 3 points were not earned. Part (b) earned no credit for the incorrect answer. In part (c), the student correctly sketches the graph of the expected function, with a maximum consistent with the answer to part (b), and earned full credit. In part (d)(i), the student starts with a correct integral expression and uses appropriate limits and earned 2 points. However, the substitution of the electric field function does not come from part (a), so the point was not earned. The student correctly identifies the higher potential at $r = 0$ and earned the point in part (d)(ii). The graph in part (e) earned full credit.

Sample: E1-C
Score: 5

Part (a) earned 2 points for a correct representation of Gauss's law and for an appropriate sketch of a Gaussian surface. The student earned no additional credit in this section, or in part (b). There is an attempt to calculate the area of the Gaussian surface, but the formula for volume is used. The sketch in part (c) has the shape of the curve where $r > R$, but is incorrect for $r < R$ and does not label the maximum so only 1 point is earned. No credit was earned in part (d) for the use of the expression for absolute potential about a spherical charge distribution and an incorrect selection of $r = R$. Full credit was earned in part (e) for the zero electric field inside a conducting charged cylinder and for $r > R$ consistent with part (c).