## Question 2

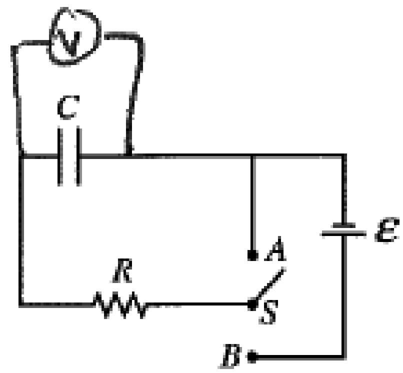
**15 points total** | **Distribution of points**

(a)

i. 1 point

For selecting $B$ — 1 point

ii. 1 point




For correctly drawing the voltmeter on the diagram in parallel with the capacitor — 1 point

(b)

i. 2 points

For stating two correct variables that would yield a linear graph — 2 points

Examples: $t$ and $\ln V$

$t$ and $-\ln V$

$t$ and $\ln(V/V_0)$, where $V_0 = 252$ V

*Note:* Students who do not come up with a correct pair of variables but who use Kirchhoff's loop rule to analyze the circuit, or write an appropriate differential equation or exponential relationship may earn one point.

ii. 1 point

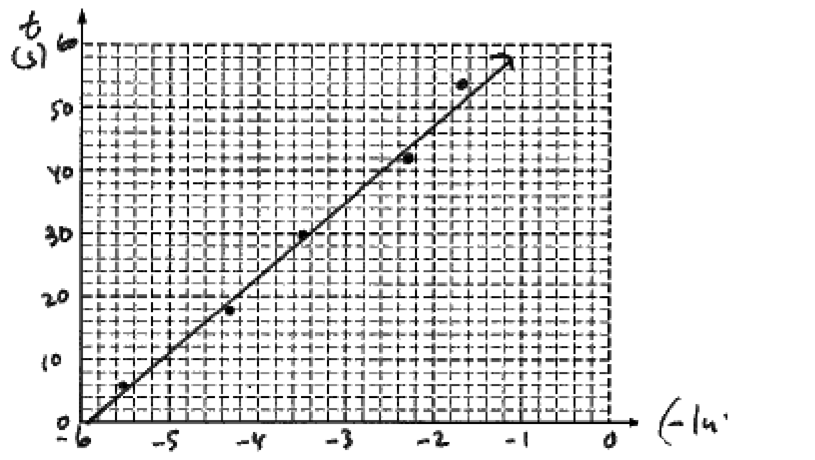
| t (s) | 6 | 18 | 30 | 42 | 54 |
|---|---|---|---|---|---|
| $V$(V) | 252 | 74 | 33 | 10 | 6 |
| $-\ln V$ | −5.53 | −4.30 | −3.50 | −2.30 | −1.79 |
| | | | | | |

For filling in values in a blank table row consistent with part (b) i — 1 point

## Question 2 (continued)

**Distribution of points**

(c) 4 points




| | |
|---|---|
| For correctly labeling the axes with variables and units consistent with part (b) | 1 point |
| For properly scaling the axes | 1 point |
| For correctly plotting data points consistent with part (b) | 1 point |
| For drawing a line of best fit through linear data | 1 point |

(d) 2 points

Relate the time constant to the slope from the graph above

$\tau = \text{slope}$ (or 1/slope if axes on graph are swapped)

For using points on the best fit line, not data points, to calculate the slope, or performing a linear regression — 1 point

$$\tau = \frac{(52 - 23)\ \text{s}}{-1.6 - (-4.0)}$$

For calculating a correct time constant — 1 point

$\tau = 12.1\ \text{s}$

### Question 2 (continued)

**Distribution of points**

(e)

i. 2 points

For a correct expression relating the time constant to the resistance — 1 point

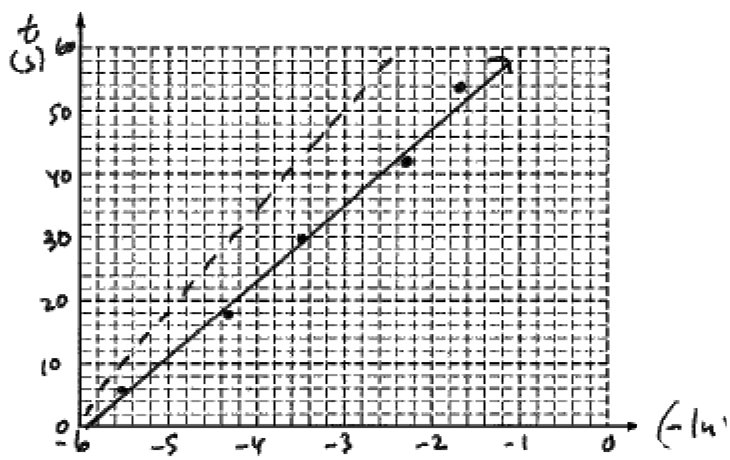
$\tau = RC$

For correctly substituting into the correct equation — 1 point

$$R = \frac{\tau}{C} = \frac{(12.1\ \text{s})}{(1.5\ \mu\text{F})}$$

$R = 8.1\ \text{M}\Omega$

ii. 2 points

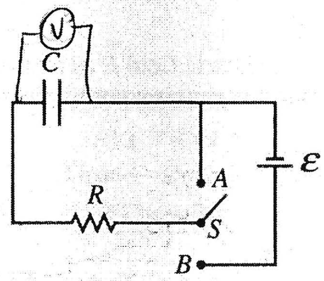



For a dashed straight line with the same voltage at $t = 0$ as that of the solid line and a slope that is: — 1 point

- *steeper* than that of the solid line, if $t$ is graphed as a function of $\pm \ln V$ , as shown above
- *shallower* than that of the solid line, if instead $\pm \ln V$ is graphed as a function of $t$

For a correct justification for the change in the slope that relates the slope to either the time constant or its reciprocal — 1 point

E2 A1




E&M 2.

In a lab, you set up a circuit that contains a capacitor $C$, a resistor $R$, a switch $S$, and a power supply, as shown in the diagram above. The capacitor is initially uncharged. The switch, which is initially open, can be moved to positions $A$ or $B$.

(a)

i. Indicate the position to which the switch should be moved to charge the capacitor.

__ $A$ ✓ $B$

ii. On the diagram, draw a voltmeter that is properly connected to the circuit in a manner that will allow the voltage to be measured across the capacitor.

After a long time you move the switch to discharge the capacitor, and your lab partner starts a stopwatch. You collect the following measurements of the voltage across the capacitor at various times.

| t (s) | 6 | 18 | 30 | 42 | 54 |
|---|---|---|---|---|---|
| $V(V)$ | 252 | 74 | 33 | 10 | 6 |
| ln V (dimensionless) | 5.53 | 4.30 | 3.50 | 2.30 | 1.79 |
| | | | | | |

You wish to determine the time constant $\tau$ of the circuit from the slope of a linear graph.

(b)

i. Indicate two quantities you would plot to obtain a linear graph.

$t$ and $\ln V$

$v = v_0 e^{-t/\tau}$

Because $V = V_0 e^{-t/\tau}$, a graph of $t$ vs $\ln V$ will have a slope of $-\frac{1}{\tau}$

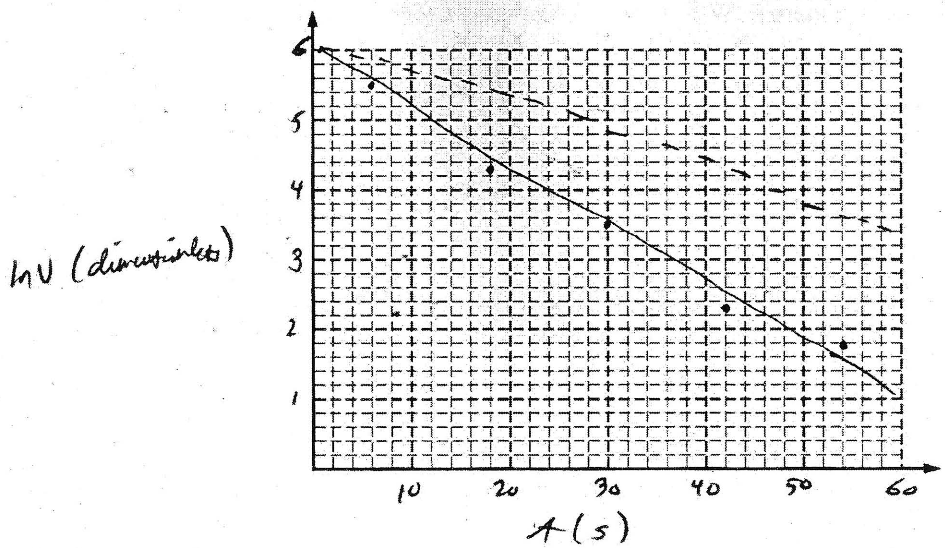
ii. Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.



GO ON TO THE NEXT PAGE.

E2 A2

(c) On the axes below, graph the data from the table that will produce a linear relationship. Clearly scale and label all axes including units, if appropriate. Draw a straight line that best fits your data points.




(d) From your line in part (c), obtain the value of the time constant $\tau$ of the circuit.

$$-\frac{1}{\tau} = \frac{1.79 - 5.53}{54s - 6s}$$

$$\tau = \boxed{12.8\ s}$$

(e)

i. In the experiment, the capacitor $C$ had a capacitance of 1.50 μF. Calculate an experimental value for the resistance $R$.

$$\tau = RC$$

$$12.8s = R(1.50\mu F)$$

$$\boxed{8.53\ M\Omega} = R$$

ii. On the axes in part (c), use a dashed line to sketch a possible graph if the capacitance was greater than 1.50 μF but the resistance $R$ was the same. Justify your answer.

If the capacitance is greater, the time constant will be greater (as $\tau = RC$). Thus, the slope of the graph, $-\frac{1}{\tau}$, will be less negative and the graph will be less steep.



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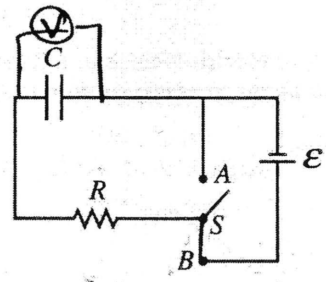
E2 B1




E&M 2.

In a lab, you set up a circuit that contains a capacitor $C$, a resistor $R$, a switch $S$, and a power supply, as shown in the diagram above. The capacitor is initially uncharged. The switch, which is initially open, can be moved to positions $A$ or $B$.

(a)

i. Indicate the position to which the switch should be moved to charge the capacitor.

__ $A$ ✓ $B$

ii. On the diagram, draw a voltmeter that is properly connected to the circuit in a manner that will allow the voltage to be measured across the capacitor.

After a long time you move the switch to discharge the capacitor, and your lab partner starts a stopwatch. You collect the following measurements of the voltage across the capacitor at various times.

$Q = CV$

| t (s) | 6 | 18 | 30 | 42 | 54 |
|---|---|---|---|---|---|
| $V$($V$) | 252 | 74 | 33 | 10 | 6 |
| ln\|V\| | 5.529 | 4.304 | 3.497 | 2.303 | 1.792 |
| | | | | | |

You wish to determine the time constant $\tau$ of the circuit from the slope of a linear graph.

(b)

i. Indicate two quantities you would plot to obtain a linear graph.

$\mathcal{E} - \frac{dQ}{dt}R - \frac{q}{C} = 0 \quad \frac{\mathcal{E}}{R}$

$q(t) = C\mathcal{E}(1 - e^{-t/RC})$

$\ln|q| = 1 \cdot \frac{-t}{RC}$

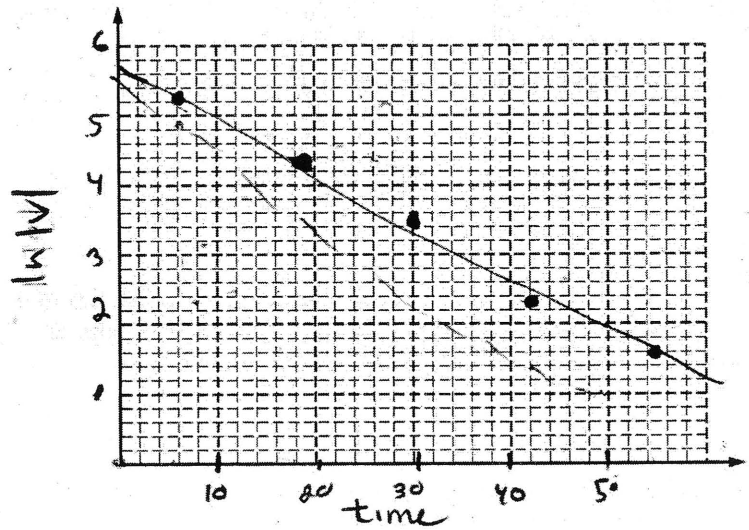
$Q$ is proportional to $V$ because $Q = CV$ (constant capacitor)

$\frac{\mathcal{E}}{R} - \frac{dQ}{dt} - \frac{q}{RC} = 0$

I would plot $\ln|V|$ vs time

ii. Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.

(c) On the axes below, graph the data from the table that will produce a linear relationship. Clearly scale and label all axes including units, if appropriate. Draw a straight line that best fits your data points.




(d) From your line in part (c), obtain the value of the time constant $\tau$ of the circuit.

The time constant is derived from the slope, which is $-1/\tau$

$$\frac{-1}{\tau} = \frac{4.304 - 5.529}{18 - 6} = -0.102$$

$$-\frac{1}{\tau} = -0.102 \qquad \tau = \frac{1}{0.102} = 9.804$$

(e)

i. In the experiment, the capacitor $C$ had a capacitance of 1.50 μF. Calculate an experimental value for the resistance $R$.

$\tau = RC$

$\tau =$

ii. On the axes in part (c), use a dashed line to sketch a possible graph if the capacitance was greater than 1.50 μF but the resistance $R$ was the same. Justify your answer.

bigger C mag



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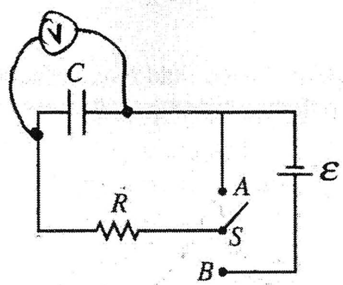
E2 C1




E&M 2.

In a lab, you set up a circuit that contains a capacitor $C$, a resistor $R$, a switch $S$, and a power supply, as shown in the diagram above. The capacitor is initially uncharged. The switch, which is initially open, can be moved to positions $A$ or $B$.

(a)

i. Indicate the position to which the switch should be moved to charge the capacitor.

__ $A$ ✓ $B$

ii. On the diagram, draw a voltmeter that is properly connected to the circuit in a manner that will allow the voltage to be measured across the capacitor.

After a long time you move the switch to discharge the capacitor, and your lab partner starts a stopwatch. You collect the following measurements of the voltage across the capacitor at various times.

| t (s) | 6 | 18 | 30 | 42 | 54 |
|---|---|---|---|---|---|
| $V(V)$ | 252 | 74 | 33 | 10 | 6 |
| $\frac{1}{V}\left(\frac{1}{V}\right)$ | 0.00397 | 0.0135 | 0.0303 | 0.1 | 0.1667 |
| | | | | | |

You wish to determine the time constant $\tau$ of the circuit from the slope of a linear graph.

$\tau = RC = \frac{y}{x} = \frac{V}{I}$

(b)

i. Indicate two quantities you would plot to obtain a linear graph.

$\frac{1}{V}$ and $t$

ii. Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.



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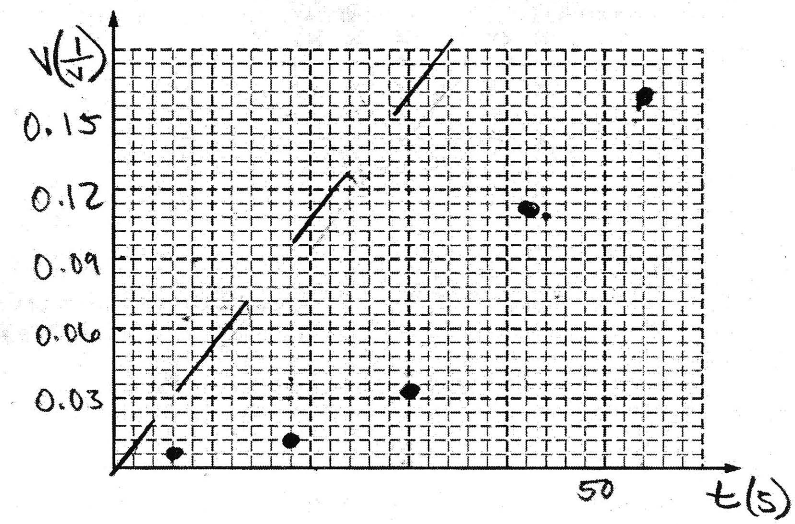
(c) On the axes below, graph the data from the table that will produce a linear relationship. Clearly scale and label all axes including units, if appropriate. Draw a straight line that best fits your data points.




(d) From your line in part (c), obtain the value of the time constant $\tau$ of the circuit.

$$\tau = m = \frac{0.1667 - 0.1}{54 - 42} = 0.00556$$

(e)

i. In the experiment, the capacitor $C$ had a capacitance of 1.50 μF. Calculate an experimental value for the resistance $R$.

$$\tau = RC$$

$$0.00556 = R(1.5 \times 10^{-6}\,F)$$

$$R = 3706\,\Omega$$

ii. On the axes in part (c), use a dashed line to sketch a possible graph if the capacitance was greater than 1.50 μF but the resistance $R$ was the same. Justify your answer.

## Question 2

### Overview

This question examined students' understanding of RC circuits, such as those used in common physics classroom experiments, and evaluated if students could analyze data collected for the discharge cycle to obtain the time constant graphically from linearized data. In part (a), students determined the switch setting that would charge the capacitor from a sketch of an RC circuit. The question also asked students to show proper placement of a voltmeter to measure the potential across the capacitor. In part (b) students tabulated any additional calculated data for the pair of variables identified. In part (c), students graphed the tabulated data and draw a best-fit straight line through it. In part (d), students used the slope of the line to obtain the time constant of the circuit. In part (e)(i), students were given a known capacitance and asked to use the experimental time constant to calculate resistance. In part (e)(ii), students were asked to describe by drawing a dashed line on their graph how the line would change if the capacitance was changed but the resistance was not. Finally, students were asked to justify the placement of their dashed line.

**Sample E2-A**
**Score: 14**

This problem is a strong solution that loses only 1 point. Part (a) correctly indicated position $B$ to charge the capacitor and correctly draws the voltmeter in parallel with the capacitor. Part (b)(i) determines a linear relationship between $t$ and $\ln V$. Part (b)(ii) determines the values for $\ln V$. Part (c) has a well-drawn graph with proper labels. In part (d), the student calculates the time constant correctly from the slope, but does not indicate that the slope is derived from the best-fit line and not the data points, so the slope point was not awarded. In part (e)(i), the student correctly calculates the resistance numerically starting from the time constant equation and substituting the result from the previous step. For part (e)(ii), the dashed line with a flatter slope for a circuit with a larger capacitance is correct in both slope and intercept, and the justification is correct.

**Sample: E2-B**
**Score: 9**

This response earned full credit of 5 total points for parts (a) and (b). In part (c) the student did not receive 1 of the 4 graphing points because the unit of time is not indicated. Credit was not earned in part (d) because the time constant is out of range and there is no evidence that the best-fit line is used to calculate the slope because data points from the table are used. In part (e)(i) the student earned 1 point for the correct time constant equation. No credit was earned in part (e)(ii) because the dashed line should have a flatter slope for this arrangement of axes.

**Sample E2-C**
**Score: 5**

This response earned 2 points for part (a). No points for linearization or use of natural logarithms were earned part (b)(i), but 1 point was awarded in part (b)(ii) for tabulating what was indicated to be graphed. In part (c), no credit is earned. The vertical axis is not properly labeled. Because at least two points are needed to explicitly define a scale, the horizontal axis cannot be assumed to have an appropriate linear scale; thus the data points cannot be presumed to be properly graphed, and, because they are clearly not linear, a straight line curve fit would not be valid. No credit was earned in part (d) for appropriate slope calculation or answer. Part (e)(i) received 2 points because the time constant calculation is consistent with previous numerical answers. No credit was awarded for the dashed line drawn for part (e)(ii), and because the line is incorrect, a justification (had it been provided) would not have been considered.