Question 3

15 points total | **Distribution of points**

(a)

i. 3 points

For using a correct expression of Faraday's law — 1 point

$\mathcal{E} = -d\phi/dt$

For correct substitution of area and magnetic field into Faraday's law — 1 point

$\mathcal{E} = -A[dB/dt] = -A\left[d\left(1.8e^{-0.05t}\right)/dt\right]$

Taking the derivative and substituting values

$\mathcal{E} = -\left(0.25\text{ m}^2\right)(-0.05)(1.8)e^{-0.05t}$

For a correct answer — 1 point

$\mathcal{E} = 0.0225e^{-0.05t}\text{ V}$

Note: The negative sign is not needed in the calculations because the question asked for the magnitude of the emf

ii. 1 point

Use Ohm's law

$I = V/R$

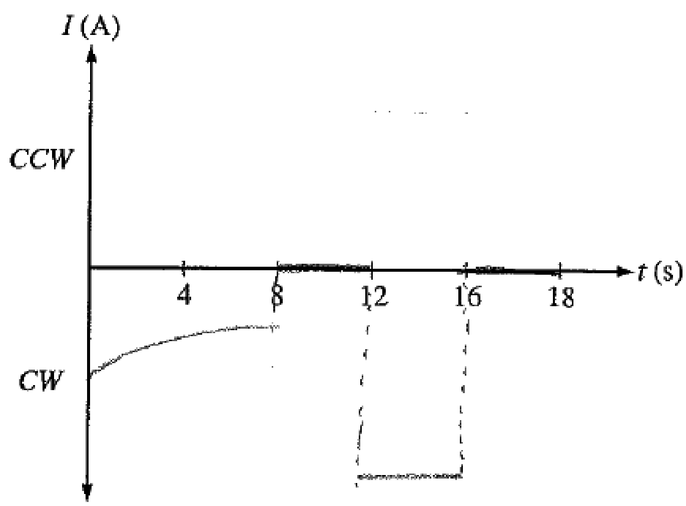
For using the expression from (a) in Ohm's law with the correct time — 1 point

$I = (0.0225)\left(e^{(-0.05)(4\text{ s})}\right)/12\ \Omega$

$I = 0.00154\text{ A or }1.54\text{ mA}$

(b)

i. 4 points



For indicating a current that decays exponentially in the region $0\text{ s} < t < 8\text{ s}$ — 1 point

For indicating a current of zero in the regions $8\text{ s} < t < 12\text{ s}$ and $t > 16\text{ s}$ — 1 point

For indicating a current that is constant and nonzero in the region $12\text{ s} < t < 16\text{ s}$ — 1 point

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Question 3 (continued)

Distribution of points

For indicating a clockwise current in the regions $0\text{ s} < t < 8\text{ s}$ and $12\text{ s} < t < 16\text{ s}$ — 1 point

ii. 3 points

For indicating that magnetic field is decreasing — 1 point
For indicating that an induced magnetic field opposes the change — 1 point
For indicating that the induced current must be clockwise to produce the induced magnetic field — 1 point

Example:
The magnetic field is into the page and decreasing. From Lenz's law, the new current must create a magnetic field to oppose this change. So the new current must create its own magnetic field that is into the page. Thus, according to the right hand rule, the current in the loop must be clockwise.

Note: 1 earned point was deducted for any incorrect statements made in conjunction with correct statements.

(c) 4 points

For indicating that energy is the integral of power over time — 1 point

$$E = \int P\,dt$$

For using a correct expression of power — 1 point

$$E = \int \frac{\mathcal{E}^2}{R}\,dt$$

For a correct substitution from part (a) — 1 point
For using the proper limits on the integration or correctly evaluating a constant of integration — 1 point

$$E = \int_0^8 \frac{\left(0.0225e^{-0.05t}\right)^2}{(12\ \Omega)}\,dt = \frac{(0.0225)^2}{(12\ \Omega)}\int_0^8 e^{-0.10t}\,dt$$

$$E = \frac{\left(4.22\times10^{-5}\right)}{(-0.10)}\left[e^{-0.10t}\right]_0^8$$

$$E = \left(-4.22\times10^{-4}\right)\left(e^{-(0.10)(8\text{ s})} - e^0\right)$$

$$E = 2.32\times10^{-4}\ \text{J}$$

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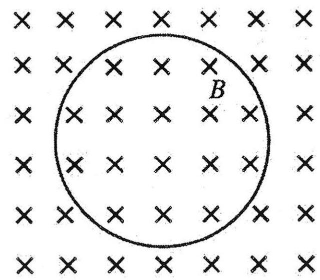
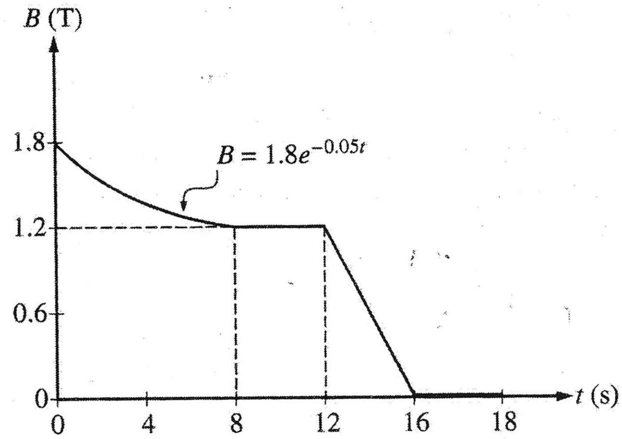
E3 A1



E&M 3.

The figure above shows a circular loop of area $0.25\ m^2$ and resistance $12\ \Omega$ that lies in the plane of the page. A magnetic field of magnitude B directed into the page exists in the area of the loop. The field varies with time t, as shown in the graph below.



(a)

i. Derive an expression for the magnitude of the induced emf in the loop as a function of time for the interval $t = 0$ s to $t = 8$ s.

$$\mathcal{E} = \frac{-d\phi_m}{dt}$$

$$= \frac{dB}{dt} \cdot A$$

$$= -.09e^{-0.05t} \cdot 0.25 = -0.0225e^{-0.05t} \checkmark$$

ii. Calculate the magnitude of the induced current I in the loop at time $t = 4$ s.

$$\mathcal{E} = IR$$

$$I = \frac{-0.0225e^{-0.05t}}{12} = -.001535\ A$$

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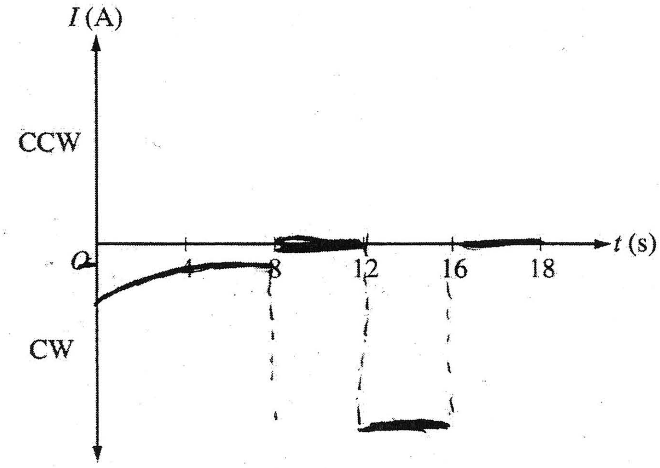
(b)

i. Sketch a graph of the induced current I in the loop as a function of time t from $t = 0$ s to $t = 18$ s on the axes below, assuming that a counterclockwise (CCW) current is positive.



ii. For the time interval 12 s to 16 s, justify the direction of the current you have indicated in your graph.

B-field is decreasing, pointing into the page. According to Lens' Law, a current will be induced that generates flux into the page, thus a clockwise current.

(c) Calculate the total energy dissipated in the loop during the first 8 s shown.

$$P = \frac{V^2}{R}$$

$$E = \int_0^8 \frac{V^2}{R}\, dt$$

$$= \int_0^8 \frac{(-0.0225e^{-0.05t})^2}{12}\, dt$$

$$= 2.32 \times 10^{-4} \text{ J}$$

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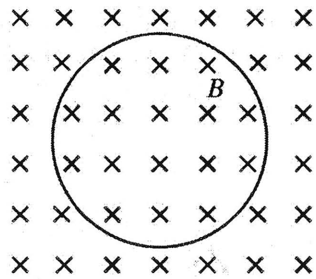
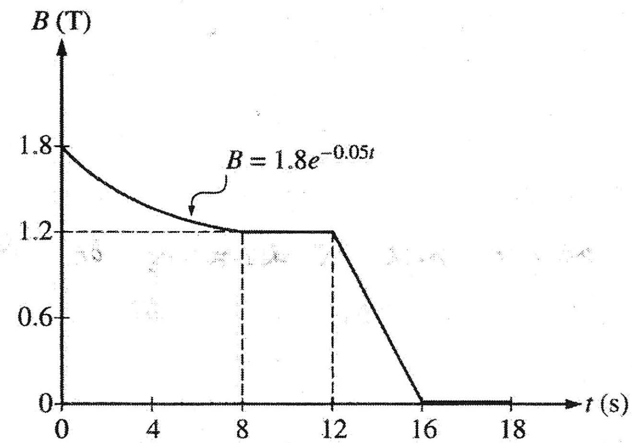


E&M 3.

The figure above shows a circular loop of area $0.25\ \text{m}^2$ and resistance $12\ \Omega$ that lies in the plane of the page. A magnetic field of magnitude B directed into the page exists in the area of the loop. The field varies with time t, as shown in the graph below.



(a)

i. Derive an expression for the magnitude of the induced emf in the loop as a function of time for the interval $t = 0$ s to $t = 8$ s.

$\varepsilon = \frac{-d\Phi}{dt}$ $\Phi = \int B \cdot dA = \int 1.8e^{-.05t}(.25)\, dt$

$\boxed{\varepsilon = .45(.951)^t}$

ii. Calculate the magnitude of the induced current I in the loop at time $t = 4$ s.

$I = \frac{\varepsilon}{R} = \frac{.45(.951)^4}{12} = \boxed{.0307\ A}$

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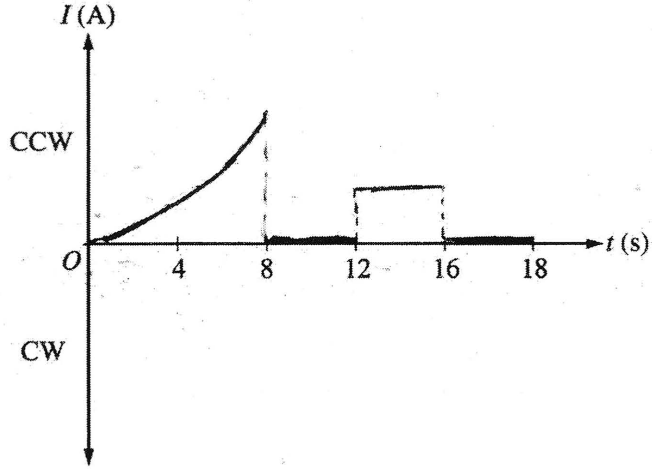
(b)

i. Sketch a graph of the induced current I in the loop as a function of time t from $t = 0$ s to $t = 18$ s on the axes below, assuming that a counterclockwise (CCW) current is positive.



ii. For the time interval 12 s to 16 s, justify the direction of the current you have indicated in your graph.

The magnetic field is decreasing on the interval 12 to 16. Thus, to maintain the same flux, the current induced must be CCW.

(c) Calculate the total energy dissipated in the loop during the first 8 s shown.

$$P = \frac{dW}{dt} \rightarrow W = \int P\,dt$$

$$P = \frac{\varepsilon^2}{R} = \frac{(.45(.951)^t)^2}{12} = .0169(.951)^{2t}$$

$$W = E = \int_0^8 .0169(.951)^{2t}\,dt = \boxed{.0929\text{ J}}$$

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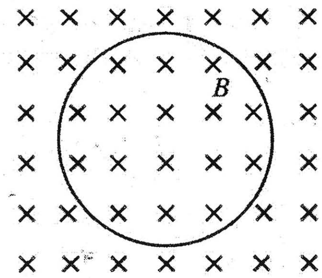
E3 C1



E&M 3.

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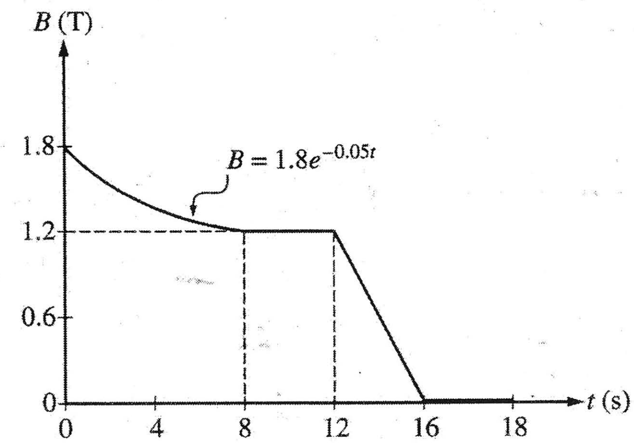

(a)

i. Derive an expression for the magnitude of the induced emf in the loop as a function of time for the interval $t = 0$ s to $t = 8$ s.

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\int B\,dA}{dt}$$

ii. Calculate the magnitude of the induced current I in the loop at time $t = 4$ s.

$$V = IR$$

$$I = \frac{V}{R}$$

$$I = \frac{\frac{\int B\,dA}{dt}}{R}$$

$$I = \frac{1.8e^{-.05(4)}\,.25}{12}$$

$$I = .03\ \text{A}$$

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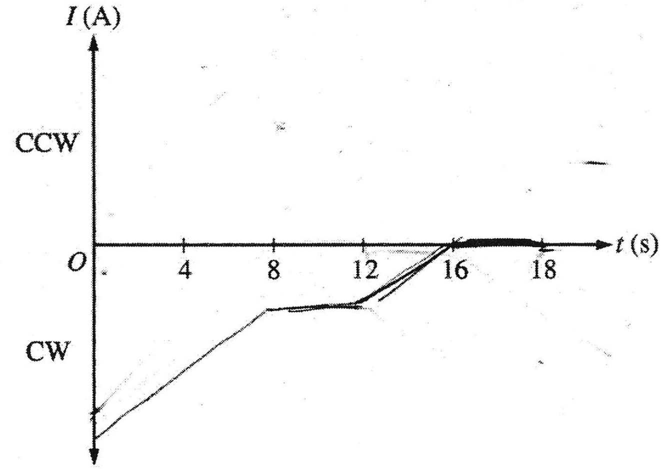
(b)

i. Sketch a graph of the induced current I in the loop as a function of time t from $t = 0$ s to $t = 18$ s on the axes below, assuming that a counterclockwise (CCW) current is positive.



ii. For the time interval 12 s to 16 s, justify the direction of the current you have indicated in your graph.

The direction of the current is clockwise because since B is decreasing, the induced current produces field coming into the page

(c) Calculate the total energy dissipated in the loop during the first 8 s shown.

$P = IV$

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Question 3

Overview

This question probed students' understanding of electromagnetic induction and assessed students ability to calculate energy dissipation in a circuit with changing current. In part (a), students were given an equation for magnetic field B as a function of time t, and asked to calculate the electric potential induced in a circular circuit of known area and resistance. Students used derived potential functions to calculate the current at a specific time. Because B was exponentially decaying in the time region for which the derivation was requested, calculus was required. In part (b), students sketched a graph of current as a function of time for the entire time period, which included regions where B was exponentially decreasing, constant, decreasing linearly, and zero. Students then justified their graphs by correctly describing the magnetic field behavior and explaining why this behavior resulted in the indicated current direction in terms of the production of the induced magnetic field. In part (c), students calculated the energy dissipated during the time period when the magnetic field was decreasing exponentially. Successful completion of this part of the problem required integration of an expression for power over time, with appropriate substitution of previously derived expressions for current or potential.

Sample: E3-A
Score: 15

This excellent response received full credit. Part (a) shows a clear derivation of the emf as a function of time, starting with Faraday's law in part (a)(i) and the subsequent derivation of the current in the loop using Ohm's law in part (a)(ii). The graph in part (b)(i) received full credit, as does the explanation in part (b)(ii), which is succinct but contains the three necessary statements (field is decreasing, induced field will be into the page, produced by a clockwise current). The small error that flux (a scalar) is "into the page" is ignored. Part (c) is a clear example of the appropriate calculation of the total energy dissipated.

Sample: E3-B
Score: 10

This response received 2 points in part (a)(i) for using Faraday's Law and for correct substitution of magnetic field and area. The response loses the third point for failing to derive the correct final expression for emf as a function of time. Full credit was earned in part (a)(ii) for using Ohm's law with substitution of the value obtained in part (a)(i). Because the student is asked to calculate rather than derive, a completely numeric approach was acceptable. The graph in (b)(i) received 2 out of 4 possible points; 1 for a constant field from 12-16 s and 1 point for zero current where appropriate. The concavity from 1-8 s does not show a decay and the positive current values do not show a clockwise current. The justification in (b)(ii) received 1 point for identifying a decreasing magnetic field, but fails to clearly explain in sufficient detail how this results in the production of the current. Full credit was earned in part (c) for an integral with correct limits of integration and an answer consistent with previous parts.

Sample E3-C
Score: 5

Part (a)(i) earned 1 point for starting with Faraday's law and using it in a subsequent step. Part (a)(ii) earned 1 point for substituting the result from part (a)(i) into Ohm's law. The graph in part (b)(i) earned 1 point for being negative, indicating that the induced current, where it exists, is clockwise. No other points are earned by the graph. In part (b)(ii) the student earned 1 point for identifying that the magnetic field is decreasing, and a second point for indicating that a clockwise current is necessary to produce a

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Question 3 (continued)

field into the page. Because there is no explanation that the induced field must point in that direction to support the diminishing magnetic field, the third point is not earned. No credit was earned for part (c) because although a correct formula for power is written, it is not used in a subsequent calculation.

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