Question 2

15 points total **Distribution of points**

(a) 2 points

	Position of Right End of Loop			
Speed of Loop	$L < x < 2L$	$2L < x < 3L$	$3L < x < 4L$	$4L < x < 5L$
Increases				
Decreases	✓		✓	
Stays the same		✓		✓

For selecting "Decreases" in the 1st and 3rd columns — 1 point

For selecting "Stays the same" in the 2nd and 4th columns — 1 point

(b) 4 points

For using a correct equation to solve for the emf — 1 point

$$\mathcal{E} = -\frac{d\phi}{dt}$$

For a clear indication that the area is changing — 1 point

$$|\mathcal{E}| = B\frac{dA}{dt} \text{ or } |\mathcal{E}| = B\ell v$$

Note: since the question asks for a magnitude, students are not penalized for excluding the minus sign for the emf.

For relating emf and current — 1 point

$$I = \frac{V}{R} = \frac{\mathcal{E}}{R}$$

For a correct expression for the current — 1 point

$$I = \frac{BLv_0}{4R}$$

(c) 2 points

For selecting "Counterclockwise" — 1 point

For a correct justification — 1 point

Examples:

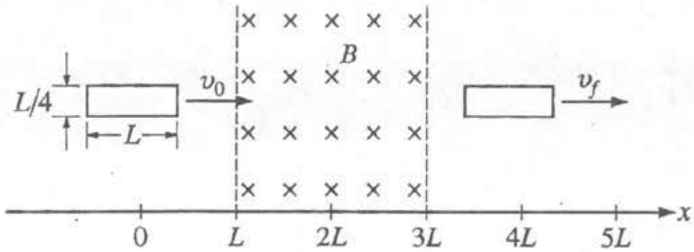
As the loop enters the magnetic field, more of its area is in a magnetic field directed into the page. According to Lenz' law, this increase in flux will create a current with an opposing magnetic field that will be out of the page. Thus, the current must be counterclockwise.

As the loop enters the magnetic field, the combination of magnetic and electric forces on the charges in the right side of the loop will create a potential difference at the right side with the top being positive. This will cause the current to flow in a counterclockwise direction.

Note: If the wrong choice is selected, then no credit can be earned.

Question 2 (continued)

	Distribution of points
(d) 4 points	
For a correct statement of Newton's second law	1 point
$F = ma$	
For using a correct expression for the magnetic force	1 point
$F = I\ell B$	
For substitution of variables into a correct expression for the net force, including a substitution for I consistent with the answer from part (b)	1 point
$a = -\dfrac{B\left(\dfrac{BL\upsilon}{4R}\right)(L/4)}{m}$	
$a = -\dfrac{B^2L^2\upsilon}{16mR}$	
For substituting $d\upsilon/dt$ for a	1 point
$\dfrac{d\upsilon}{dt} = -\dfrac{B^2L^2\upsilon}{16mR}$	
(e) 3 points	
For selecting "Left"	1 point
For correctly stating that a clockwise current will be induced in the loop	1 point
For correctly applying the right hand rule to the current, field, and resulting force on the loop	1 point
Example: As the loop is leaving the field, the magnetic flux in the loop is decreasing. According to Lenz' law, a clockwise current is induced to oppose the change, which creates a magnetic field into the page. In the left end of the loop the current is up. By the right hand rule, the fingers point up in the direction of the current, the fingers cross into the direction of the magnetic field (into the page), and the thumb points left in the direction of the force.	
Note: If the wrong choice is selected, then no credit can be earned.	



E&M. 2.

The rectangular loop of wire shown on the left in the figure above has mass M, length L, width $L/4$, and resistance R. It is initially moving to the right at constant speed v_0, with no net force acting on it. At time $t = 0$ the loop enters a region of length $2L$ that contains a uniform magnetic field of magnitude B directed into the page. The loop emerges from the field at time t_f with final speed v_f. Express all algebraic answers to the following in terms of M, L, R, B, v_0, and fundamental constants, as appropriate.

(a) Let x represent the position of the right end of the loop. Place a check mark in the appropriate box in each column in the table below to indicate whether the speed of the loop increases, decreases, or stays the same as the loop moves to the right.

	Position of Right End of Loop			
Speed of Loop	$L < x < 2L$	$2L < x < 3L$	$3L < x < 4L$	$4L < x < 5L$
Increases				
Decreases	✓		✓	
Stays the same		✓		✓

(b) Derive an expression for the magnitude of the current induced in the loop as its right edge enters the field.

$$\mathcal{E} = -\frac{d\phi_B}{dt} \leftarrow \phi_B = BA\cos\theta \leftarrow A = \left(\frac{L}{4}\right)(x)$$

↑ width entered B

$$\Rightarrow \phi_B = \frac{BLx}{4} \qquad \cos\theta = 1$$

$$\mathcal{E} = -\frac{d\left(\frac{BLx}{4}\right)}{dt} = -\frac{BL}{4}\frac{dx}{dt} = -\frac{BLv_0}{4}$$

$$i = \frac{\mathcal{E}}{R} = \boxed{\frac{BLv_0}{4R}}$$

(c) What is the direction of the induced current determined in part (b) ?

____ Clockwise   X Counterclockwise

Justify your answer.

The amount of inwards magnetic flux is increasing, so by Lenz's Law, the loop wants to create an opposing outwards flux, requiring a CCW current.

(d) Write, but do not solve, a differential equation for the speed v as a function of time as the loop enters the field.

$$\mathcal{E} = -\frac{d\phi_B}{dt} = -\frac{d\left(\frac{BLx}{4}\right)}{dt} = -\frac{BLv}{4} \Rightarrow i = \frac{\mathcal{E}}{R} = \frac{BLv}{4R}$$ (see part b for details)

Now we calculate the leftwards force acted on the wire by the magnetic field:

$$F_B = Bil\sin\theta = B\left(\frac{BLv}{4R}\right)\left(\frac{L}{4}\right) = \frac{B^2L^2v}{16R}$$ [L]

$$\Rightarrow a = \frac{F_B}{M} = \frac{B^2L^2v}{16MR}$$ [L]

We can now use $a = \frac{dv}{dt}$:

$$\boxed{-\frac{B^2L^2v}{16MR} = \frac{dv}{dt}} \Rightarrow \boxed{\int_0^t -\frac{B^2L^2}{16MR}\,dt = \int_{v_0}^{v(t)} \frac{1}{v}\,dv}$$

Note the negative sign is added since $\vec{a}$ points opposite to $\vec{v}$

(e) What is the direction of the acceleration of the loop just before its left edge leaves the field?

_✓_ Left ____ Right ____ Up ____ Down

Justify your answer.

The amount of inwards magnetic flux is decreasing so the loop wants to create more inwards flux by Lenz's Law, thus inducing a CW current. On the left side of the loop, the current is thus directed upwards. By the right hand rule, a leftwards force is acted on this wire, so the acceleration is directed to the left.

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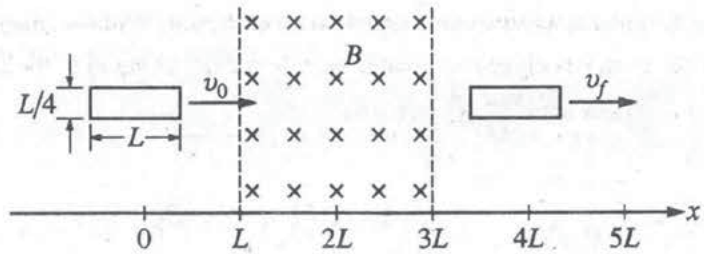

E&M. 2.

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(a) Let x represent the position of the right end of the loop. Place a check mark in the appropriate box in each column in the table below to indicate whether the speed of the loop increases, decreases, or stays the same as the loop moves to the right.

	Position of Right End of Loop			
Speed of Loop	$L < x < 2L$	$2L < x < 3L$	$3L < x < 4L$	$4L < x < 5L$
Increases	✓			
Decreases			✓	
Stays the same		✓		✓

(b) Derive an expression for the magnitude of the current induced in the loop as its right edge enters the field.

$$\mathcal{E} = \frac{-d\Phi_B}{dt} = B\frac{dA}{dt} = Bw\frac{dL}{dt} = \frac{BL}{4}V_0$$

$$\mathcal{E} = iR = \frac{BLV_0}{4}$$

$$\boxed{i = \frac{BLV_0}{4R}, \text{ in amperes}}$$

(c) What is the direction of the induced current determined in part (b) ?

____ Clockwise ✓ Counterclockwise

Justify your answer.

The flux is increasing into the page because a large area of the loop is going through the field. Therefore an increasing magnetic field will be induced out of the page By the right hand rule the induced current is therefore counterclockwise.

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(d) Write, but do not solve, a differential equation for the speed v as a function of time as the loop enters the field.

$\mathcal{E} = \frac{BL}{4}\frac{dL}{dt} = -\frac{d\Phi}{dt}$

$\frac{BL}{4} v = -\frac{d\Phi}{dt}$

$v\,dt = \frac{-d\Phi(4)}{BL}$

$F = ma = m\frac{dv}{dt}$

$F = qvB = m\frac{dv}{dt}$

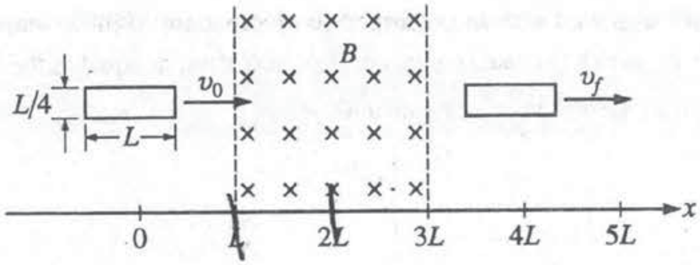
$$\frac{qB}{m} v = \frac{1}{v} dv$$

(e) What is the direction of the acceleration of the loop just before its left edge leaves the field?

__✓__ Left ____ Right ____ Up ____ Down

Justify your answer.

The loop's velocity will decrease as less of the area is intersected by the magnetic field. Thus, since the loop is moving to the right the acceleration must be to the left.



E&M. 2.

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(a) Let x represent the position of the right end of the loop. Place a check mark in the appropriate box in each column in the table below to indicate whether the speed of the loop increases, decreases, or stays the same as the loop moves to the right.

$\Phi = BA$

	Position of Right End of Loop			
Speed of Loop	$L < x < 2L$	$2L < x < 3L$	$3L < x < 4L$	$4L < x < 5L$
Increases	✓		✓	
Decreases				✓
Stays the same		✓		

(b) Derive an expression for the magnitude of the current induced in the loop as its right edge enters the field.

$\mathcal{E} = BlV$

$-\frac{d\Phi}{dt} = BA$ $\quad BA = BlV$ $\quad I = \frac{dQ}{dt}$

$IR = BlV$

$$I = \frac{Blv_0}{R}$$

(c) What is the direction of the induced current determined in part (b) ?

____ Clockwise ✓____ Counterclockwise

Justify your answer.

Lenz's law/right hand rule as B field increases I opposes it

GO ON TO THE NEXT PAGE.

(d) Write, but do not solve, a differential equation for the speed v as a function of time as the loop enters the field.

$\frac{d\Phi}{dt} = B\ell v$ $B\ell v = \oint E\,dS \rightarrow L\left(\frac{L}{4}\right)dl$

$\int B \cdot dA$ $v = \int \frac{\frac{ka}{r^2}\frac{L^2}{4}\,dL}{B} = \int \frac{kaL}{4r^2B}\,dL$

$v = \int \frac{kaL}{4r^2B}\,dL + V_0$

(e) What is the direction of the acceleration of the loop just before its left edge leaves the field?

✓ Left ____ Right ✓ Up ____ Down

Justify your answer.

$qvB = ma$

right-hand rule

B field goes in, current perpendicular, so force is up, force is directly related to acceleration

Question 2

Overview

This question was a standard loop moving into a constant magnetic field. The students are expected to know that there will be an induced emf, resulting current, and force on the moving loop. The question gives the student multiple opportunities to demonstrate this knowledge first by just making correct selections from choices, then using equations, then describing in words what is happening.

Sample: E2 A
Score: 15

This response earned full credit for all parts. The solution in part (b) clearly shows that the emf depends on the changing area. In part (d) the solution was complete with the differential equation. The separation of variables to integrate with limits was not required. The justifications in parts (c) and (e) are clear.

Sample: E2 B
Score: 10

Part (a) earned only 1 point because the speed was incorrectly identified as increasing as the loop moves into the field. Parts (b) and (c) earned full credit. In part (d) the solution begins with a statement of Newton's second law and recognizes that $a = dv/dt$, earning 2 points. Part (e) earned 1 point for the correct selection of the direction of the acceleration as left. The justification does not identify that a current is induced in the loop, resulting in a force on the loop.

Sample: E2 C
Score: 4

This response has only one correct speed selected, so it earned no points for part (a). Part (b) earned 3 points, since the final solution does not include the correct side length $L/4$. Part (c) earned 1 point. The justification does not indicate that it is a change in magnetic flux that gives rise to the current. Part (d) earned no points. None of the work links the magnetic force to the acceleration. Part (e) also earned no credit.