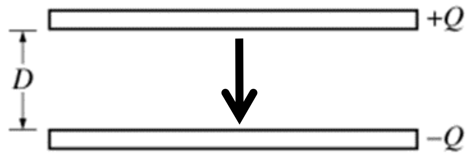
Question 1

15 points total | **Distribution of points**

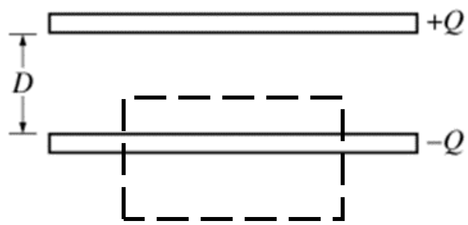
(a)

i. 1 point



For at least one arrow between the plates pointing downward from the positive plates toward the negative plate and no extraneous arrows pointing in any other direction — 1 point

ii. 1 point



For drawing an appropriate Gaussian surface (enclosing at least the inner edge of one of the plates) that can be used to determine the electric field between the plates — 1 point

iii. 3 points

For using a correct statement of Gauss's law — 1 point

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q}{\varepsilon_0}$$

For applying Gauss's law, using an enclosed charge and surface area consistent with the surface drawn in part (a)-ii — 1 point

$E(A_{GS}) = \dfrac{q_{enc}}{\varepsilon_0}$ (A_{GS} is the area of the end of the Gaussian surface between the plates)

$$E = \frac{q_{enc}}{\varepsilon_0 A_{GS}} \quad \left(\text{using } \sigma = \frac{q_{enc}}{A_{GS}}\right)$$

$$E = \frac{\sigma}{\varepsilon_0} \quad \left(\text{using } \sigma = \frac{Q}{A}\right)$$

For a correct answer with work shown — 1 point

$$E = \frac{Q}{\varepsilon_0 A}$$

Question 1 (continued)

Distribution of points

(b) 1 point

Comparing the equation for electric field between parallel plates to the given equation:

$$E = \frac{Q}{\kappa \varepsilon_0 A} = \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$$

For an answer consistent with part (a)-iii — 1 point

$$\kappa = \kappa_0 e^{-x/D}$$

(c)

i. 1 point

Using the equation relating the electric field to potential difference

$$E = -\frac{dV}{dx}$$

For a correct differential equation — 1 point

$$\frac{dV}{dx} = -\left(-\frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\right)$$

$$\frac{dV}{dx} = \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$$

Alternate Solution: — *Alternate Point*

Using the equation relating the electric field to potential difference:

$$\Delta V = -\int \vec{E} \bullet d\vec{r}$$

For a correct differential equation — *1 point*

$$\Delta V = -\int -\frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A} dx$$

$$\Delta V = \int \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A} dx$$

Question 1 (continued)

Distribution of points

(c) (continued)

ii. 4 points

Separating the variables in the differential equation from part (c)(i):

$$\frac{dV}{dx} = \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$$

$$dV = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) e^{x/D} dx$$

For using the correct limits of integration in attempting to integrate the equation above — 1 point

$$\int_{V_0}^{V_D} dV = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) \int_0^D e^{x/D} dx$$

For correctly integrating the equation — 1 point

$$\left[V\right]_{V_0}^{V_D} = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) \left[De^{x/D}\right]_0^D$$

$$\left(V_D - V_0\right) = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right) \left(e^{D/D} - e^0\right)$$

For an expression that gives the correct absolute value of the potential difference between the plates — 1 point

For having the potential difference be positive — 1 point

$$\Delta V = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right)(e - 1)$$

$$\Delta V = \frac{1.72QD}{\varepsilon_0 \kappa_0 A}$$

(d) 1 point

Using the equation for capacitance:

$$C = \frac{Q}{\Delta V} = \frac{Q}{\left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right)(e - 1)}$$

For an answer consistent with part (c)-ii — 1 point

$$C = \frac{\varepsilon_0 \kappa_0 A}{D(e - 1)}$$

$$C = \frac{\varepsilon_0 \kappa_0 A}{1.72D}$$

Question 1 (continued)

Distribution of points

(e) 3 points

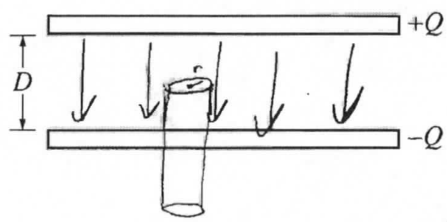
For selecting $U_V > U_C$	1 point
For correctly comparing the capacitance or the potential difference with the varying dielectric constant to the capacitance or the potential difference with the uniform dielectric constant	1 point
For correctly comparing the two stored energies consistent with the comparison of the capacitances or potential differences	1 point

Example: According to the equation from part (d), $C_C > C_V$. Since $U = \frac{Q^2}{2C}$, if the charge stored on the two capacitors is the same, then $U_V > U_C$.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M.1.

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

(a)

i. On the figure above, draw an arrow to indicate the direction of the electric field between the plates.

ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.

iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.

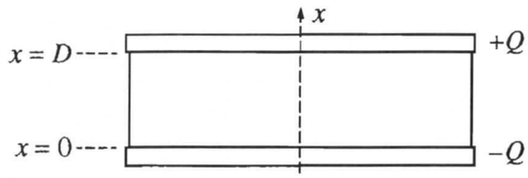
E uniform $\oint \vec{E} \cdot d\vec{A} = \frac{q_{en}}{\varepsilon_0}$

$E \cdot A = \frac{q_{en}}{\varepsilon_0}$

top circle of surface $E \cdot A_G = \frac{\sigma A_G}{\varepsilon_0} = \frac{\frac{Q}{A} A_G}{\varepsilon_0} = \frac{Q A_G}{A \varepsilon_0}$ $A_G \equiv$ area of top circle of Gaussian surface

$$E = \frac{Q}{A\varepsilon_0}$$

GO ON TO THE NEXT PAGE.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant κ as a function of x.

$$E = \frac{Q}{\varepsilon A} \text{ (from a)} = \frac{Q}{\kappa \varepsilon_0 A}$$

$$E = -\frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$$

$$\boxed{\kappa = \kappa_0 e^{-x/D}}$$

(c)

i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

$$V = -\int_{x_1}^{x_2} E \cdot dx$$

$$V = -\frac{Q}{\varepsilon_0 \kappa_0 A}\int_{D}^{0} \frac{1}{e^{-x/D}}\, dx$$

ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

$$V_D - V_0 = -\frac{Q}{\varepsilon_0 \kappa_0 A}\int_{D}^{0} e^{x/D}\, dx \qquad u = x/d,\quad du = \frac{1}{d}dx,\quad dx = d\, du$$

$$= \left(\frac{-Q}{\varepsilon_0 \kappa_0 A}\right)\left(D\left[e^{x/D}\right]_{D}^{0}\right)$$

$$= \frac{-Q}{\varepsilon_0 \kappa_0 A}\left(De^{0} - De^{1}\right)$$

$$= -\frac{Q}{\varepsilon_0 \kappa_0 A}\left(De - D\right) = \boxed{\frac{QD}{\varepsilon_0 \kappa_0 A}(e-1)}$$

Question 1 continues on next page.

GO ON TO THE NEXT PAGE.

(d) Determine the capacitance of the capacitor.

$$C = \frac{Q}{V} = \frac{Q \varepsilon_0 K A}{Q D(e-1)} = \boxed{\frac{K_0 \varepsilon_0 A}{D(e-1)}}$$

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

____ $U_V < U_C$ ✓ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.

The capacitans of the constant K capacitor is $\frac{K_0 \varepsilon_0 A}{d}$ whereas the second one is $\frac{K_0 \varepsilon_0 A}{D(e-1)}$.

Since $C_V < C_C$, and since $U = \frac{Q^2}{2C}$,

$U_V > U_C$

GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

03 - 48

3 Questions

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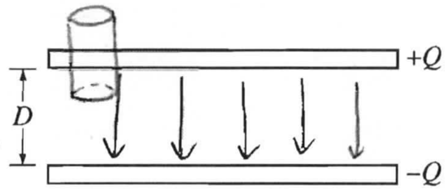

E&M.1.

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(a)

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ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.

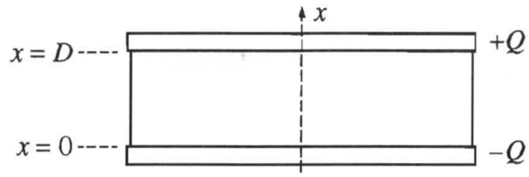
iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.

$$\int E \cdot dA = \frac{q_{encl}}{\varepsilon_0}$$

$$E \cdot \pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{\left(\frac{q}{\pi r^2}\right)}{\varepsilon_0} = \boxed{\frac{Q}{A\varepsilon_0}}$$

GO ON TO THE NEXT PAGE.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant κ as a function of x.

$$K = K_0 e^{-\frac{x}{D}}$$

(c)

i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

$$V_D - V_0 = \Delta V = \int_0^D \frac{Q}{\varepsilon_0 K_0 e^{-\frac{x}{D}} A} dx$$

ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

$$V_D - V_0 = \frac{Q}{\varepsilon_0 K_0 A} \int_0^D e^{\frac{x}{D}} dx$$

$$= \frac{Q}{\varepsilon_0 K_0 A} D e^{\frac{x}{D}} \Big]_0^D$$

$$= \frac{QD}{\varepsilon_0 K_0 A}\left(e^1 - e^0\right) = \frac{QD}{\varepsilon_0 K_0 A}(e - 1)$$

Question 1 continues on next page.

GO ON TO THE NEXT PAGE.

(d) Determine the capacitance of the capacitor.

$C = \frac{Q}{V} = \frac{Q}{\frac{QD}{\varepsilon_0 \kappa_0 A}(e-1)}$

$= \boxed{\frac{\kappa_0 \varepsilon_0 A}{D(e-1)}}$

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

✓ $U_V < U_C$ ____ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.

$U = \frac{1}{2}\frac{Q^2}{E}$

GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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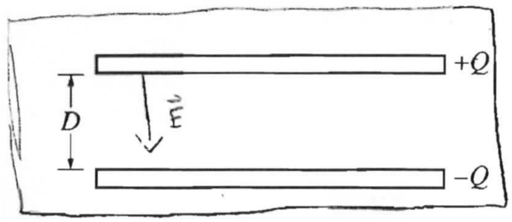

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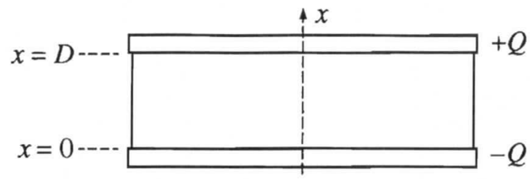
iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.

(iii) $\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$

$EA = \frac{Q}{\varepsilon_0}$

$E = \frac{Q}{A\varepsilon_0}$

GO ON TO THE NEXT PAGE.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant κ as a function of x.

$$\vec{E} = \frac{-Q}{\varepsilon_0 k_0 e^{-x/D} A}$$

$$\boxed{k_0 = \frac{-Q}{\vec{E}\varepsilon_0 e^{-x/D} A}}$$

(c)

i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

$$V = -\int E \cdot dx$$

$$V = -\int\left(\frac{-Q}{\varepsilon_0 k_0 e^{-x/D} A}\right) dx$$

$$\boxed{V = \frac{Q}{\varepsilon_0 k_0 A}\int\left(e^{x/D}\right) dx}$$

ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

$$V = \frac{Q}{\varepsilon_0 k_0 A}\int_0^D \left(e^{x/D}\right) dx$$

$$V = \frac{Q}{\varepsilon_0 k_0 A}\left(De^{1} - e^{0}\right)$$

$$\boxed{V = \frac{Q}{\varepsilon_0 k_0 A}\left(De^{2} - 1\right)}$$

Question 1 continues on next page.

GO ON TO THE NEXT PAGE.

(d) Determine the capacitance of the capacitor.

$C = \frac{K \varepsilon_0 A}{D}$

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

____ $U_V < U_C$ ✓ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.

$\lim_{x \to \infty} e^x = \infty$

$\lim_{x \to \infty} K_0 = K_0$

$\therefore U_V > U_C$

GO ON TO THE NEXT PAGE.

AP® PHYSICS C: ELECTRICITY AND MAGNETISM
2015 SCORING COMMENTARY

Question 1

Overview

This question assessed students' understanding of Gauss's law, dielectrics, and energy storage in capacitors. The dielectric constant varied with height, and calculus was required. While relatively straightforward, the problem challenged students in an unfamiliar way.

Sample: E&MQ1 A
Score: 15

Part (a)(i) and (ii) of this response earned a total of 2 points for clear drawings of the electric field and Gaussian surface. Part (a)(iii) earned all 3 points for the use of Gauss’s law in a detailed solution consistent with the drawing in part (a)(ii). Part (b) earned 1 point for stating a correct expression. Part (c)(i) earned 1 point for stating a correct equation that can be used to find the potential difference between the plates, regardless of the sign. Part (c)(ii) earned all 4 points for a detailed evaluation of the integral that led to a correct positive answer. Part (d) earned 1 point for stating a correct answer consistent with part (c)(ii). Part (e) earned all 3 points for a correct answer selection and statement that compared the capacitance and energy stored in each capacitor.

Sample: E&MQ1 B
Score: 12

Part (a)(i) and (ii) of this response earned a total of 2 points for clear drawings of the electric field and Gaussian surface. Part (a)(iii) earned all 3 points for the use of Gauss’s Law in a detailed solution consistent with the drawing in part (a)(ii). Part (b) earned 1 point for stating a correct expression. Part (c)(i) earned 1 point for stating a correct equation that can be used to find the potential difference between the plates, regardless of the sign. Part (c)(ii) earned all 4 points for a detailed evaluation of the integral that led to a correct positive answer. Part (d) earned 1 point for stating a correct answer consistent with part (c)(ii). Part (e) earned no credit since the incorrect answer is selected, and there are no statements comparing the capacitance or energy for each case.

Sample: E&MQ1 C
Score: 6

Part (a)(i) of this response earned 1 point for a correct arrow drawn in between the plates. Part (a)(ii) earned no credit because the Gaussian surface that is drawn encloses zero net charge. Part (a)(iii) earned just 1 point for the use of Gauss’s law, since the solution uses an enclosed charge not consistent with the surface drawn in part (a)(ii) and has an incorrect answer. Part (b) earned no points. Part (c)(i) earned 1 point for stating a correct equation that can be used to find the potential difference between the plates, regardless of the sign. Part (c)(ii) earned 2 points for the use of the correct limits on the integral and a positive final answer. Part (d) earned no credit. Part (e) earned 1 point for a correct answer selection but no correct statements relative to a justification are included.