Question 2

15 points total | **Distribution of points**

(a)

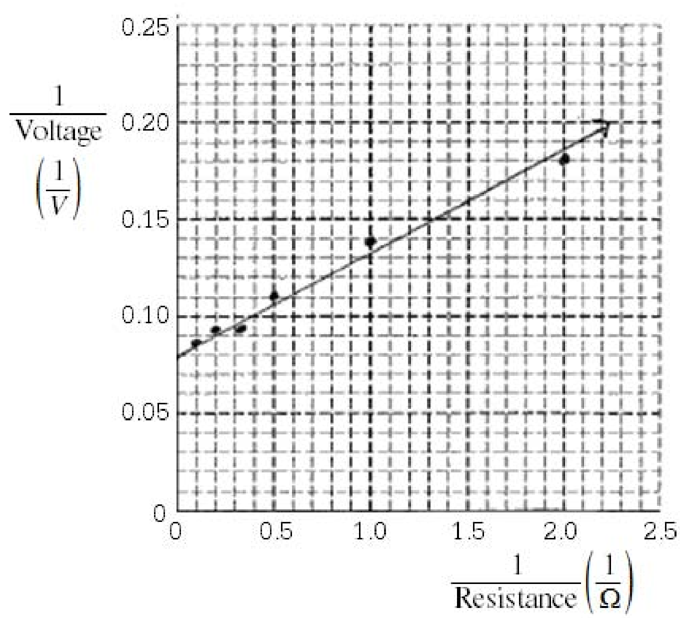
i. 2 points

Using Ohm's law:
$V = IR$
For a correct application of Kirchhoff's loop rule — 1 point
$\mathcal{E} = Ir + IR$
$I = \dfrac{\mathcal{E}}{(r + R)}$
For a correct expression for the measured voltage across the variable resistor — 1 point
$V = \dfrac{\mathcal{E}}{(r + R)}R$

ii. 1 point

For an expression of $1/V$ as a function of $1/R$ consistent with answer from part (a)(i) — 1 point
$\dfrac{1}{V} = \left(\dfrac{r}{\mathcal{E}}\right)\dfrac{1}{R} + \dfrac{1}{\mathcal{E}}$

(b) 4 points



For correctly labeling both axes with variables and units — 1 point
For correctly scaling both axes with an acceptable and appropriate scale — 1 point
For correctly plotting the data points — 1 point
For correctly drawing a straight line that best represents the data — 1 point

Question 2 (continued)

Distribution of points

(c)

i. 2 points

For using a value for the y-intercept consistent with the straight line drawn in part (b) — 1 point

$y = mx + b$

$b = 0.080\ 1/\text{V}$

For a correct substitution into the equation from part (a)(ii) — 1 point

$$\frac{1}{V} = \frac{(r+R)}{\mathcal{E}}\frac{1}{R}$$

$$\frac{1}{V} = \frac{r}{\mathcal{E}}\frac{1}{R} + \frac{1}{\mathcal{E}}$$

$b = 1/\mathcal{E}$

$\mathcal{E} = 1/(0.080\ 1/\text{V})$

$\mathcal{E} = 12.5\ \text{V}$

ii. 2 points

For calculating the slope using the straight line from part (b) and not data points — 1 point

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(0.151 - 0.100)}{(1.50 - 0.40)} = 0.0463\ \Omega/\text{V}$$

For a correct substitution into equation from part (a)(ii) — 1 point

$$m = \frac{r}{\mathcal{E}}$$

$r = m\mathcal{E}$

$r = (0.0463\ \Omega/\text{V})(12.5\ \text{V})$

$r = 0.58\ \Omega$

(d) 2 points

For using Ohm's law — 1 point

$\mathcal{E} = Ir$

For a correct substitution of values from part (c): — 1 point

$(12.5\ \text{V}) = I(0.58\ \Omega)$

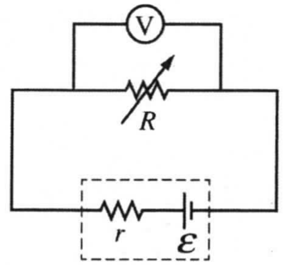
$I = 21.6\ \text{A}$

(e) 2 points

For selecting "The voltmeter with high resistance" — 1 point

For a correct justification — 1 point

Example: The voltmeter acts like a resistor in a circuit with the battery. It will measure the potential difference across its own internal resistance. The higher its internal resistance, the closer its potential difference will be to the emf of the battery.



E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

Trial #	Resistance (Ω)	Voltage (V)	$1/R$ (1/Ω)	$1/V$ (1/V)
1	0.50	5.6	2.00	0.179
2	1.0	7.4	1.00	0.135
3	2.0	9.4	0.50	0.106
4	3.0	10.6	0.33	0.094
5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(a)

i. Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

For the Circuit, $\varepsilon = \cancel{I} V_r + \cancel{IR} V$

Since $V_r = Ir$, $\varepsilon = Ir + V \Rightarrow V = \varepsilon - Ir$

$\cancel{\Rightarrow V = \varepsilon - Rr}$ Since $I = \frac{\varepsilon}{R+r}$,

$$V = \varepsilon - \frac{\varepsilon}{R+r} r = \varepsilon\left(1 - \frac{r}{R+r}\right) = \frac{R}{R+r}\varepsilon$$

ii. Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

$\cancel{V = \frac{R}{R+r}\varepsilon = \varepsilon - \frac{r}{R+r}\varepsilon}$

$$V = \frac{R}{R+r}\varepsilon \Rightarrow \frac{1}{V} = \frac{1}{\varepsilon}\frac{R+r}{R} = \frac{1}{\varepsilon}\left(1 + \frac{r}{R}\right)$$

$$= \frac{1}{\varepsilon} + \frac{r}{\varepsilon}\cdot\frac{1}{R}$$

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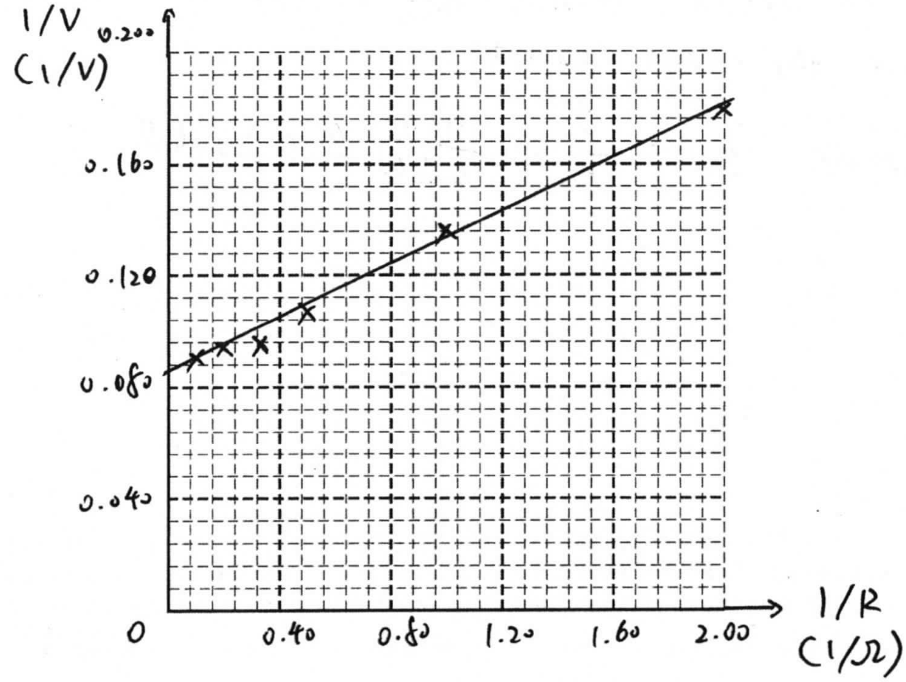
(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(c) Use the straight line from part (b) to obtain values for the following.

i. $\mathcal{E}$ from part (a), $\frac{1}{V} = \frac{r}{\mathcal{E}} \frac{1}{R} + \frac{1}{\mathcal{E}}$

Therefore y-intercept $= \frac{1}{\mathcal{E}} = 0.086\ V^{-1}$

$\mathcal{E} = \frac{1}{0.086} V = 11.6 V$

ii. r slope $= \frac{r}{\mathcal{E}} = \frac{0.144 - 0.086}{1.20 - 0} A^{-1} = 0.0483\ A^{-1}$

Therefore $r = \text{slope} \cdot \mathcal{E} = 0.0483 \times 11.6\ \Omega = 0.562\ \Omega$

Question 2 continues on next page.

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(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

For the circuit, $R_{series} = R + r$ is smallest when $R = 0$, hence the greatest current.

Therefore $I_m = \frac{\varepsilon}{r} = \frac{11.6}{0.562} A = 20.6 A$

(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

____ The voltmeter with low resistance will provide the most accurate measurement.

_✓_ The voltmeter with high resistance will provide the most accurate measurement.

____ The two voltmeters will provide equal accuracy.

Justify your answer.

When ~~the~~ a voltmeter is connected to the battery, it measures the potential difference accross itself.

A voltmeter with higher internal resistance R will yield a $V = \frac{R}{R+r} \varepsilon$ closer to ε.

(equation from part (a))

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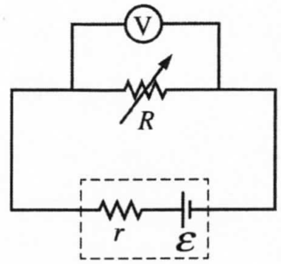

E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

Trial #	Resistance (Ω)	Voltage (V)	$1/R$ $(1/\Omega)$	$1/V$ $(1/\text{V})$
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5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(a)

i. Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

Total $\mathcal{E} = I(r+R) \Rightarrow I = \frac{\mathcal{E}}{r+R}$

$\Rightarrow V = \left(\frac{\mathcal{E}}{r+R}\right)(R)$

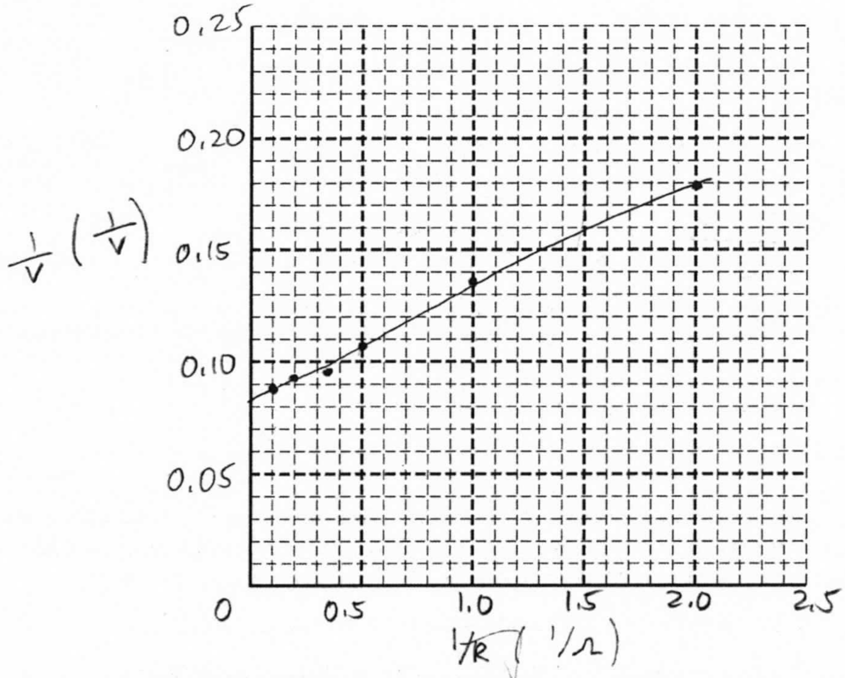
ii. Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

$\frac{1}{V} = \frac{1}{R}\left(\frac{r+R}{\mathcal{E}}\right) = \frac{r+R}{R\mathcal{E}} = \frac{r}{R\mathcal{E}} + \frac{1}{\mathcal{E}}$

$= \frac{1}{R}\left(\frac{r}{\mathcal{E}}\right) + \frac{1}{\mathcal{E}}$

GO ON TO THE NEXT PAGE.

(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(c) Use the straight line from part (b) to obtain values for the following.

i. ε

$$\frac{1}{\varepsilon} = y\text{-intercept} \simeq 0.082\ /V$$

$$\Rightarrow \varepsilon = 12.2\,V$$

ii. r

$$\left(\frac{r}{\varepsilon}\right) = \text{Slope} = 0.048\ \frac{\frac{1}{V}}{\frac{1}{\Omega}} = 0.048\ \frac{\Omega}{V}$$

$$\Rightarrow r = 0.048 \cdot \varepsilon = 0.58\,\Omega$$

Question 2 continues on next page.

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(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

$I = \frac{\mathcal{E}}{r+R} = \frac{12.2V}{0.58\Omega + R}$

R has to be lowest

$\Rightarrow R = 0.5\Omega$ (from exp.)

$\Rightarrow I = \frac{12.2V}{1.08\Omega} = 11.296 A$

(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

_✓_ The voltmeter with low resistance will provide the most accurate measurement.

____ The voltmeter with high resistance will provide the most accurate measurement.

____ The two voltmeters will provide equal accuracy.

Justify your answer.

The low resistance will be most accurate since then voltage will not change much going through the voltmeter. If ΔV is too high, it could cause sig. error when calculating the EMF.

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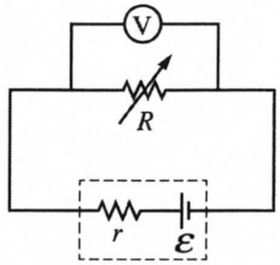

E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

Trial #	Resistance (Ω)	Voltage (V)	$1/R$ ($1/\Omega$)	$1/V$ ($1/\text{V}$)
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5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(a)

i. Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

$V = IR$ $\quad$ $\mathcal{E} - V - Ir = 0$

$\mathcal{E} = IR$

$V = \mathcal{E} - Ir$

ii. Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

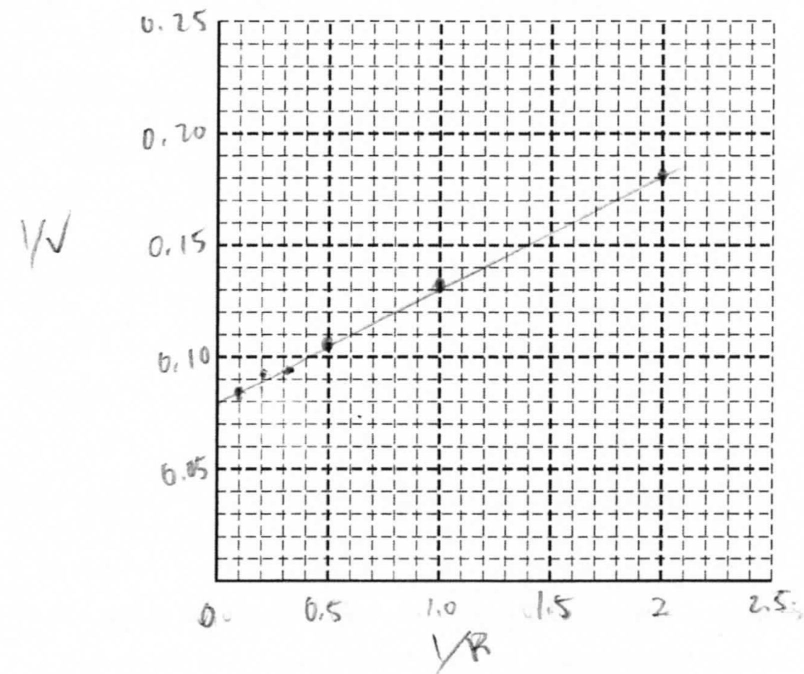
$V = \mathcal{E} - Ir$ $\quad$ $V = IR$

$\frac{1}{V} = \frac{1}{\mathcal{E}} - \frac{1}{Ir}$ $\quad$ $\frac{1}{V} = \frac{1}{I} \cdot \frac{1}{R}$

$\frac{1}{V} = \frac{1}{\mathcal{E}} - \left(\frac{1}{I}\right)\frac{1}{r}$

GO ON TO THE NEXT PAGE.

(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(c) Use the straight line from part (b) to obtain values for the following.

i. $\mathcal{E}$

$(1.0,\ 0.13)$
$1/R \quad 1/V$

$I = 1/R \div 1/V$

$\mathcal{E} = V - Ir = \boxed{7.69 - 7.69r}$

ii. r

$V = IR$

$Ir = \mathcal{E} - V$

$r = \dfrac{\mathcal{E} - V}{I}$

Question 2 continues on next page.

GO ON TO THE NEXT PAGE.

(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

$V = IR$

$I = V/R$

$1/R \div 1/V = V/R$

$2/0.179$

$= 11.17$ A

(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

____ The voltmeter with low resistance will provide the most accurate measurement.

_√_ The voltmeter with high resistance will provide the most accurate measurement.

____ The two voltmeters will provide equal accuracy.

Justify your answer.

Voltage stays the same in parallel, so a high resistance will allow all current to flow through the resistor (current adds in parallel)

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Question 2

Overview

This problem explored the relationship between resistance and voltage in a basic circuit, and explored how to experimentally verify the relationship with measurement, graphing, and straight-line fitting. The concepts stressed are: the relationship between voltage and resistance, the effect of an internal resistance in a battery, the effect of a voltmeter in a circuit, and the experimental means to find the values of the emf and internal resistance of the battery.

Sample: E&MQ2 A
Score: 15

Part (a)(i) of this response earned 2 points. The student correctly combines Kirchhoff's loop rule and Ohm's law to find the correct expression for voltage across the variable resistor. Part (a)(ii) of this response earned 1 point. The student takes the correct inverse of the equation in part (a)(i) and is also able to rearrange that equation so it shows 1/V as a function of 1/R.

Part (b) of this response earned all 4 points. The student makes an appropriate scale and labels the axes correctly with units. The student neatly and correctly plots the six data points and draws an acceptable best-fit straight line.

Part (c)(i) of this response earned 2 points. The student rewrites the equation from part (a)(ii) and states that the y-intercept is the inverse of the emf of the battery, then correctly finds the y-intercept and solves for the emf. Part (c)(ii) of this response earned 2 points. The student uses points from the best-fit line to find the slope and then correctly uses that slope to find the internal resistance of the battery. Part (d) earned 2 points. The student explains that the greatest current will be when the variable resistor is zero and then uses Ohm's law to correctly solve for the current.

Part (e) of this response earned 2 points. The student correctly selects the voltmeter with high resistance and then gives a correct justification. The equation from part (a)(i) is even used to back up the choice.

Sample: E&MQ2 B
Score: 11

Part (a)(i) of this response earned 2 points. The student correctly uses a Kirchhoff loop rule equation to solve for the current in the circuit and then uses that current in Ohm's law to find the correct expression for voltage across the variable resistor. Part (a)(ii) of this response earned 1 point. The student takes the correct inverse of the equation in part (a)(i) and is able to rearrange that equation so it shows 1/V as a function of 1/R.

Part (b) earned all 4 points. The student makes a neat graph that has clearly labeled axes with an appropriate scale and units. The student correctly plots the points and draws an acceptable best-fit line.

Part (c)(i) of this response earned 2 points. The student determines the y-intercept and then correctly relates it to the inverse of the emf of the battery. Part (c)(ii) of this response earned 1 point. The student does have a reasonable value for the slope, but because no work is shown it is not known if the student used data points or points on the best-fit line. The student earned the point for correctly substituting the slope into the equation found in part (a)(ii). Part (d) of this response earned 1 point. The student does not use the correct lowest value of R, i.e., zero.

Part (e) of this response did not earn any points. The student does not check the correct box, and the justification is not correct.

Question 2 (continued)

Sample: E&MQ2 C
Score: 6

Part (a)(i) of this response earned 1 point. The student wrote a correct Kirchhoff loop equation, setting the sum of the voltages around the circuit equal to zero. But the student left I in the equation, which is not one of the given variables, so the second point for the correct expression was not earned. Part (a)(ii) of this response earned no points. The student does not take the proper inverse of the equation in part (a), and the equation does not contain 1/R.

Part (b) of this response earned 3 points. The scale is appropriate, the data points are plotted correctly, and the best-fit line is properly drawn. The student lost a point for not putting units on the axes.

Part (c) of this response earned no points. The student does not find the slope or the y-intercept of the graph. If the student does not find this information the equation cannot be solved for the emf or the internal resistance of the battery. Part (d) of this response earned 1 point. The student makes an attempt to use Ohm's law but is unable to substitute the answers from (c) to find the maximum current.

Part (e) of this response earned 1 point. The student earned the point for checking the correct box that the voltmeter needs a high resistance, but they do not correctly justify the choice.