

**AP<sup>®</sup> PHYSICS C: ELECTRICITY AND MAGNETISM**  
**2015 SCORING GUIDELINES**

**Question 3**

**15 points total**

**Distribution  
of points**

(a) 3 points

For properly using a correct equation to calculate the magnetic flux

1 point

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

For correct substitution of the area and trigonometric function

1 point

$$\Phi_B = BA(\cos\theta) = B\pi r^2(\cos\theta)$$

For correct substitution into the above equation

1 point

$$\Phi_B = 4(1 - 0.2t)\pi(0.10 \text{ m})^2(\cos 60^\circ)$$

$$\Phi_B = 0.063 - 0.013t \quad (\text{or in terms of } a \text{ and } b, \Phi_B = a(1 - bt)(0.016))$$

(b) 2 points

For using a correct equation to solve for the emf

1 point

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Substituting the expression from part (a):

$$\mathcal{E} = \frac{d}{dt}(0.063 - 0.013t)$$

For an answer consistent with part (a)

1 point

$$\mathcal{E} = 0.013 \text{ V}$$

Note: Any sign on the answer is ignored.

(c)

i. 1 point

For a substitution into Ohm's law consistent with the answer from part (b)

1 point

$$V = IR$$

$$I = \frac{(0.013 \text{ V})}{(50 \Omega)}$$

$$I = 2.6 \times 10^{-4} \text{ A}$$

ii. 2 points

The correct choice is "Counterclockwise".

For a justification that incorporates that the original magnetic field is changing

1 point

For a justification that correctly relates the induced current to the direction of a new magnetic field created by that current

1 point

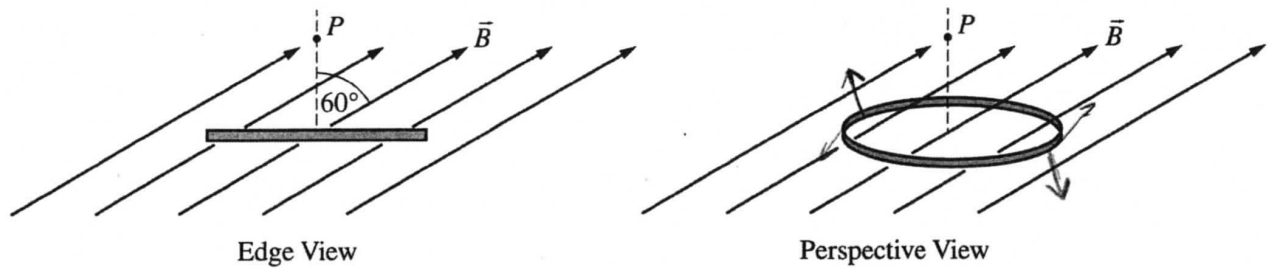
Example: Looking down at the loop from  $P$ , the vertical component of the magnetic field of the loop is upward and decreasing. To oppose this change, the current in the loop must create a magnetic field that is directed upward at point  $P$ .

This requires a counterclockwise current in the loop.

**AP<sup>®</sup> PHYSICS C: ELECTRICITY AND MAGNETISM**  
**2015 SCORING GUIDELINES**

**Question 3 (continued)**

	<b>Distribution of points</b>
(d) 2 points	
For using a correct equation to calculate the energy dissipated $E = Pt = I^2Rt$ or $E = Pt = (\mathcal{E}^2/R)t$	1 point
For a substitution into either of the above equations with the answer from part (c)(i) or part (b)	1 point
$E = (2.6 \times 10^{-4} \text{ A})^2 (50 \Omega)(4.0 \text{ s})$ or $E = \frac{(0.013 \text{ V})^2}{(50 \Omega)}(4.0 \text{ s})$ $E = 1.4 \times 10^{-5} \text{ J}$	
(e) 4 points	
For selecting both “Zero” for the net magnetic force and “Nonzero” for the Net magnetic torque	1 point
For indicating the forces on directly opposite sides of the loop are in opposite directions	1 point
For concluding that the forces cancel & net force is zero	1 point
For indicating that the torques add & net torque is nonzero	1 point
Example: Since the current changes direction relative to the magnetic field as you go around the loop, the magnetic field will exert force of equal magnitude but opposite direction on opposite sides of the loop. These forces all cancel out resulting in zero net force. However, since this force is up on one side of the loop and down on the other side of the loop, this will create torques that rotate the loop in the same direction. Therefore, the net torque is not zero.	
Units 1 point	
For correct units on at least two parts with a calculated numerical answer and no incorrect units	1 point



E&amp;M. 3.

A circular wire loop with radius 0.10 m and resistance  $50 \Omega$  is held in place horizontally in a magnetic field  $\vec{B}$  directed upward at an angle of  $60^\circ$  with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time  $t$  by  $B(t) = a(1 - bt)$ , where  $a = 4.0 \text{ T}$  and  $b = 0.20 \text{ s}^{-1}$ .

(a) Derive an expression for the magnetic flux through the loop as a function of time  $t$ .

$$\begin{aligned}\Phi_m &= \int \vec{B} \cdot d\vec{A} = (a)(1 - bt)(\pi r^2)(\cos 60^\circ) \\ &= \frac{1}{2} a(1 - bt)(\pi)(0.01) = \frac{a\pi(1 - bt)}{200}\end{aligned}$$

(b) Calculate the numerical value of the induced emf in the loop.

$$\mathcal{E} = -\frac{d\Phi_m}{dt} = -\frac{(-b)(a\pi)}{200} = \frac{ba\pi}{200} = \frac{0.2(4)\pi}{200} = 12.6 \text{ mV}$$

(c)

- i. Calculate the numerical value of the induced current in the loop.

$$\begin{aligned} \mathcal{E} &= IR \\ I &= \frac{\mathcal{E}}{R} = \frac{0.0126\text{V}}{50\Omega} = 0.252\text{mA} \end{aligned}$$

- ii. What is the direction of the induced current in the loop as viewed from point P?

Clockwise     Counterclockwise

Justify your answer.

*B* is decreasing based on its function. By Lenz's law the emf opposes the change <sup>in flux</sup>. By the right-hand rule the current flows counterclockwise to induce a magnetic field out of the page, in the direction of *B*.

- (d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.

$$E = Pt = (\mathcal{E}^2/R)t = \frac{(0.0126)^2}{50}(4) = 1.27 \times 10^{-5} \text{ J}$$

- (e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.

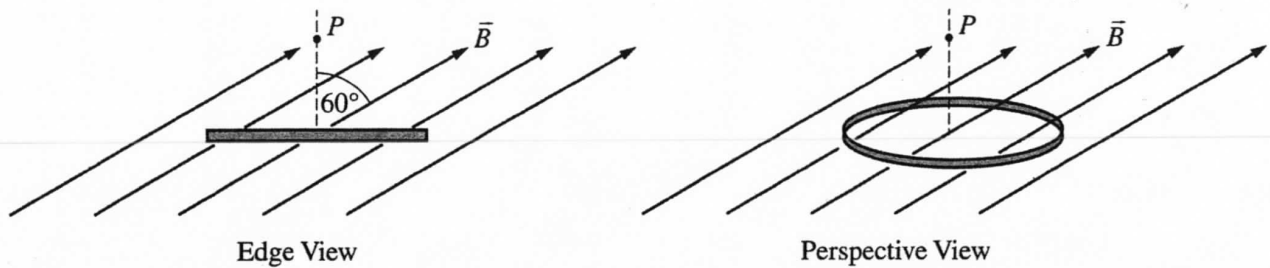
Net magnetic force:  Zero     Nonzero

Net magnetic torque:  Zero     Nonzero

Justify both of your answers.

$$F = Il \times B$$

By the right hand rule, the direction of induced current results in an upward-left force on the left side of the loop and a downward-right force of equal magnitude on the right side. The sum of these forces is zero but they rotate the loop around its axis by providing a torque.



E&amp;M. 3.

A circular wire loop with radius  $0.10 \text{ m}$  and resistance  $50 \Omega$  is held in place horizontally in a magnetic field  $\vec{B}$  directed upward at an angle of  $60^\circ$  with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time  $t$  by  $B(t) = a(1 - bt)$ , where  $a = 4.0 \text{ T}$  and  $b = 0.20 \text{ s}^{-1}$ .

(a) Derive an expression for the magnetic flux through the loop as a function of time  $t$ .

$$\begin{aligned} \Phi &= BA \cos \phi & \Phi &= .02\pi - .004\pi t \\ \Phi &= \pi r^2 a (1 - bt) \cos 60^\circ \\ \Phi &= \pi (.1)^2 4 (1 - .2t) \frac{1}{2} \\ \Phi &= .02\pi (1 - .2t) \end{aligned}$$

(b) Calculate the numerical value of the induced emf in the loop.

$$\begin{aligned} \mathcal{E} &= -N \frac{d\Phi}{dt} \\ \mathcal{E} &= -\frac{d}{dt} (.02\pi - .004\pi t) \\ \mathcal{E} &= -(-.004\pi) \\ \mathcal{E} &= .004\pi \\ \mathcal{E} &= .013 \text{ V} \end{aligned}$$



(c)

- i. Calculate the numerical value of the induced current in the loop.

$$\mathcal{E} = iR$$

$$i = \frac{.013}{50}$$

$$i = \frac{\mathcal{E}}{R}$$

$$i = 2.513 \times 10^{-4} \text{ A}$$

- ii. What is the direction of the induced current in the loop as viewed from point P?

Clockwise     Counterclockwise

Justify your answer.

The induced current in the loop is counterclockwise because the magnetic field is decreasing with time. Due to Lenz's Law, the induced field opposes any change of the original field, so the induced field would be out of the page viewed from point P. Using the 1<sup>st</sup> Right Hand Rule, the induced current would be counterclockwise.

- (d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.

$$P = \frac{dW}{dt}$$

$$\int_0^W dW = \int_0^4 i^2 R dt$$

$$W = (2.513 \times 10^{-4})^2 (50) (4 - 0)$$

$$W = 1.263 \times 10^{-5} \text{ J}$$

$$dW = P dt$$

$$W = i^2 R t \Big|_0^4$$

- (e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.

Net magnetic force:  Zero     Nonzero

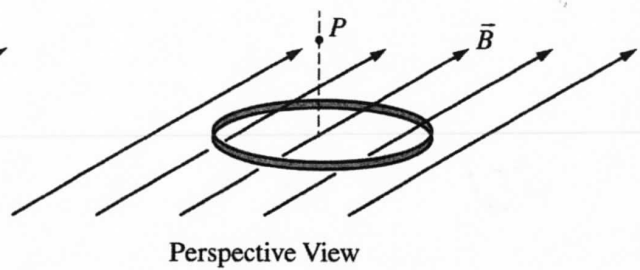
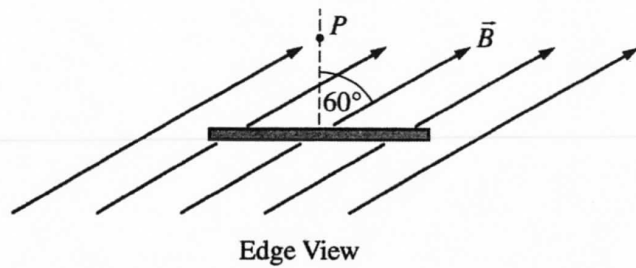
$$F_B = ILB$$

Net magnetic torque:  Zero     Nonzero

$$\tau_B = F_B r_{\perp}$$

Justify both of your answers.

The net magnetic force is nonzero on the loop because there is an induced current and magnetic field present in the loop of wire. The net magnetic torque is nonzero because the net magnetic force is nonzero.



E&amp;M. 3.

A circular wire loop with radius 0.10 m and resistance  $50 \Omega$  is held in place horizontally in a magnetic field  $\vec{B}$  directed upward at an angle of  $60^\circ$  with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time  $t$  by  $B(t) = a(1 - bt)$ , where  $a = 4.0 \text{ T}$  and  $b = 0.20 \text{ s}^{-1}$ .

(a) Derive an expression for the magnetic flux through the loop as a function of time  $t$ .

$$\Phi_B = BA \cos \theta$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\begin{aligned} \Phi &= 4(1 - .20t)(\pi r^2) \cos 60^\circ \\ &= 4(1 - .20t)(\pi(.1)^2) \cos 60^\circ \\ &= 2(1 - .20t)(.0314) \\ \Phi &= (.0628)(1 - .20t) \end{aligned}$$

$$\Phi_B = .0628 - .0126t$$

(b) Calculate the numerical value of the induced emf in the loop.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad \mathcal{E} = -A \frac{dB}{dt} \cos \theta$$

$$\mathcal{E} = -\pi(.1)^2 \cdot (-.20) \cos 60^\circ$$

$$\mathcal{E} = .0126$$



(c)

- i. Calculate the numerical value of the induced current in the loop.

$$I = \frac{\mathcal{E}}{R} = \frac{.0126}{50} = 2.513 \times 10^{-4}$$

- ii. What is the direction of the induced current in the loop as viewed from point  $P$ ?

Clockwise  Counterclockwise

Justify your answer.

The right hand rule produces a counterclockwise motion if  $B$  is viewed out of the page.

- (d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.

$$P = I^2 R = (2.513 \times 10^{-4})^2 \cdot 50$$

$$P = 3.158 \times 10^{-6} \text{ W}$$

- (e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.

Net magnetic force:  Zero  Nonzero

Net magnetic torque:  Zero  Nonzero

Justify both of your answers.

$$F = \int I d\ell \times B$$

$B$  affects the loop



# AP<sup>®</sup> PHYSICS C: ELECTRICITY AND MAGNETISM

## 2015 SCORING COMMENTARY

### Question 3

#### Overview

The question tests concepts of flux and induced current in a loop in a variable magnetic field. Students demonstrate their understanding of flux, voltage, current, energy, net force, and net torque on the loop.

#### Sample: E&MQ3 A

**Score: 15**

Part (a) on this response earned all 3 points. The student uses the correct equation for magnetic flux and neatly substitutes the area and correct trigonometric function. The student leaves the answer in terms of  $\pi$  and the two constants  $a$  and  $b$ . Part (b) earned 2 points. The student uses the correct equation for induced emf and solves for the correct answer.

Part (c)(i) on this response earned 1 point. The student uses Ohm's law and substitutes correctly from part (b). Part (c)(ii) earned 2 points. The student states that the magnetic field is decreasing, and also explains that the induced current will move in the direction to oppose the change in the magnetic field. The student specifically mentions Lenz's law, which is not required but adds to the description.

Part (d) of this response earned 2 points. The student uses a correct equation for energy and substitutes correctly from previous parts.

Part (e) of this response earned 4 points. The student chooses the correct two boxes for the first point and has a clear justification for the choices to earn the remaining points. The student is specific in describing the directions of the forces on the sides of the loop, showing that they cancel each other out because they are equal and opposite, and indicating that they provide a net torque.

This response earned the 1 unit point because all three units are correct, and mV and mA are used correctly.

#### Sample: E&MQ3 B

**Score: 11**

Part (a) on this response earned all 3 points. The student uses a correct equation for flux, which in this case does not need to be in integral form. The work is neat and clear, showing an understanding of use of the proper component of magnetic field (the one parallel to the area vector). The student leaves the answer in terms of  $\pi$ , which is acceptable. Part (b) earned full credit of 2 points because the correct equation for induced emf is used, and the correct answer is given.

Part (c)(i) on this response clearly uses Ohm's law to solve for the induced current in the loop, which earned 1 point. The student apparently used  $0.004\pi$  in the calculation, resulting in an answer that is a bit off from that in the scoring guideline. Part (c)(ii) earned full credit of 2 points because the student explains correctly that the magnetic field is decreasing and explains that the induced current will oppose the change in magnetic field. The student does more than just simply say "because of Lenz's law," which would not have earned full credit.

Part (d) on this response clearly and neatly shows how to find the energy dissipated in the loop during the four seconds. Although calculus is not required to solve this problem, the student is correctly able to do so, which earned 2 points.

Part (e) on this response did not earn any credit. The student did not earn any points for checking the boxes because they both need to be correct. The student did not earn credit for the explanation of why the torque is nonzero because it is not justified correctly.

This response earned the 1 unit point because all three units are correct.

**AP<sup>®</sup> PHYSICS C: ELECTRICITY AND MAGNETISM**  
**2015 SCORING COMMENTARY**

**Question 3 (continued)**

**Sample: E&MQ3 C**

**Score: 6**

Part (a) on this response earned all 3 points. The student uses a correct equation for flux, which in this case does not need to be in integral form. The work is neat and clear, showing an understanding of use of the proper component of magnetic field (the one parallel to the area vector). Part (b) earned full credit of 2 points because the student uses the correct equation for induced emf. This student also shows understanding that it is the changing magnetic field, specifically, that is causing the change in flux and, therefore, the induced emf.

Part (c)(i) on this response earned 1 point because the student uses Ohm's law and substitutes the correct answer from (b) to find the induced current. Part (c)(ii), however earned no credit. The student says that it is counterclockwise because of the right hand rule, but that is not a good enough justification. The student needs to explain the right hand rule in more detail and apply it specifically to this situation.

Part (d) did not earn any credit because the student is solving for power and not energy.

Part (e) on this response did not earn any credit because the boxes are not checked correctly, and the reason for a nonzero torque is not justified.

This response did not earn the unit point because part (b) and (c)(i) do not have units, and the unit for part (d) is not correct for energy, which is what the problem is asking for.