Question 3

15 points total | **Distribution of points**

(a) 1 point

For selecting "Left" — 1 point

(b)

i. 2 points

For selecting "Less than" — 1 point

For a correct justification — 1 point

Example: Because as the bar falls the flux at point C is increasing, the emf generated must create a magnetic field to oppose this change. Therefore, it will create a magnetic field to decrease the flux and thus decrease the magnetic field.

No points are earned if the wrong answer is selected.

ii. 2 points

For selecting "Greater than" — 1 point

For a correct justification — 1 point

Example: The field at point C, which is above the bar, is less than the original magnetic field, and point D is on the other side of the bar. Therefore, the direction of the magnetic field from the bar at point D is the opposite of the direction at point C, so the net magnetic field at D when the bar is falling must be greater than the original magnetic field.

No points are earned if the wrong answer is selected.

(c) 4 points

For correctly applying Newton's second law to the motion of the bar — 1 point

$F_{net} = Mg - F_M = Mg - BIL$

$I = \mathcal{E}/R$

For attempting to use Faraday's law to obtain an expression for the emf in the bar — 1 point

$\mathcal{E} = d\Phi/dt = BL(dx/dt)$

For correctly using the expression for emf to obtain an expression for the current — 1 point

$$I = \frac{BL(dx/dt)}{R} = \frac{BLv}{R}$$

$$Ma = Mg - B\frac{BLv}{R}L$$

$$a = g - \frac{B^2L^2v}{MR}$$

For writing the acceleration as dv/dt — 1 point

$$\frac{dv}{dt} = g - \frac{B^2L^2v}{MR}$$

Question 3 (continued)

Distribution of points

(d) 2 points

For setting the net force at terminal velocity equal to zero — 1 point

$$0 = Mg - \frac{B^2L^2v_T}{R}$$

$$\frac{B^2L^2v_T}{R} = Mg$$

For an answer consistent with part (c) — 1 point

$$v_T = \frac{MgR}{B^2L^2}$$

(e) 1 point

For a correct or consistent substitution in an appropriate power equation — 1 point

$$P = I^2R \quad \text{or} \quad V = IR$$

$$\frac{BLv_T}{R} = I$$

$$P = \frac{B^2L^2{v_T}^2}{R}$$

Alternate Solution — *Alternate points*

For a correct or consistent substitution in an appropriate power equation — *1 point*

$$P = I^2R$$

$$Mg = BIL$$

$$I = \frac{Mg}{BL}$$

$$P = \frac{M^2g^2R}{B^2L^2}$$

Question 3 (continued)

Distribution of points

(f) 3 points

Using the equation from part (c)

$$\frac{dv}{dt} = g - \frac{B^2L^2v}{MR}$$

$$\frac{dv}{dt} = \frac{-B^2L^2}{MR}\left(v - \frac{MRg}{B^2L^2}\right)$$

For attempting separation of variables — 1 point

$$\frac{dv}{v - \frac{MRg}{B^2L^2}} = \frac{-B^2L^2}{MR}dt$$

For attempting to integrate with the correct limits or the correct constant of integration — 1 point

$$\int_{v'=0}^{v'=v(t)} \frac{1}{\left(v' - \frac{MRg}{B^2L^2}\right)}\,dv' = \int_{t'=0}^{t'=t} \frac{-B^2L^2}{MR}dt'$$

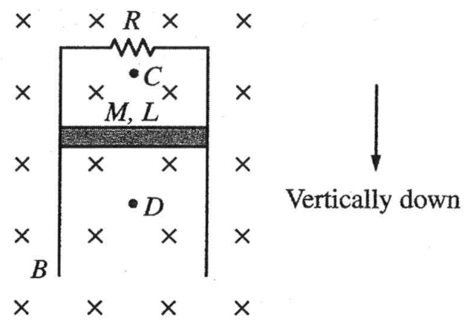
$$\ln\left[v' - \frac{MRg}{B^2L^2}\right]_{v'=0}^{v'=v(t)} = \frac{-B^2L^2}{MR}t$$

$$\ln\left(\frac{v(t) - \frac{MRg}{B^2L^2}}{\frac{MRg}{B^2L^2}}\right) = \frac{-B^2L^2}{MR}t$$

For a correct answer — 1 point

$$v(t) = \frac{MRg}{B^2L^2}\left(1 - e^{-\frac{B^2L^2}{MR}t}\right)$$

Using a trial solution in the differential equation and verifying its correctness is also acceptable.



E&M.3.

A conducting bar of mass M, length L, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

✓ Left ____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

____ Greater than ✓ Less than ____ Equal to

Justify your answer.

The induced current goes counter-clockwise, creating a magnetic field that comes out of the page at point C. Considering the magnetic field B that goes into the page, I conclude that the net magnetic field decreases (or less than before).

ii. While the bar is above point D, is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

✓ Greater than ____ Less than ____ Equal to

Justify your answer.

The current goes to the right in the bar, creating a magnetic field that points in to the page at point D, therefore making the net magnetic field (combining B and B_I of induced current) greater than before.

Express your answers to parts (c) and (d) in terms of M, L, R, B, and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t.

$\Sigma F = mg - F_B$

$Ma = Mg - ILB$, where $I = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$

$Ma = Mg - \frac{B^2L^2v}{R}$

$$\boxed{\frac{dv}{dt} = g - \frac{B^2L^2v}{MR}}$$

(d) Determine an expression for the terminal velocity v_T of the bar.

At v_t, $\frac{dv}{dt} = 0$

$g - \frac{B^2L^2v_T}{MR} = 0$

$v_T = \frac{gMR}{B^2L^2}$

Express your answers to parts (e) and (f) in terms of v_T, M, L, R, B, and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

$I = \frac{BLv_T}{R}$

$P = I^2R = \frac{B^2L^2v_T^2}{R^2} \cdot R = \boxed{\frac{B^2L^2v_T^2}{R}}$

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t.

$\frac{dv}{dt} = g - \frac{B^2L^2v}{MR}$

$\int_0^{v(t)} \frac{dv}{g - \frac{B^2L^2v}{MR}} = \int_0^t dt$

$-\frac{MR}{B^2L^2} \ln\left(\frac{g - \frac{B^2L^2v(t)}{MR}}{g}\right) = t$

$1 - \frac{B^2L^2v(t)}{MRg} = e^{-\frac{B^2L^2}{MR}t}$

$v(t) = \frac{MRg}{B^2L^2}\left(1 - e^{-\frac{B^2L^2}{MR}t}\right)$

$$\boxed{v(t) = v_T\left(1 - e^{-\frac{B^2L^2}{MR}t}\right)}$$

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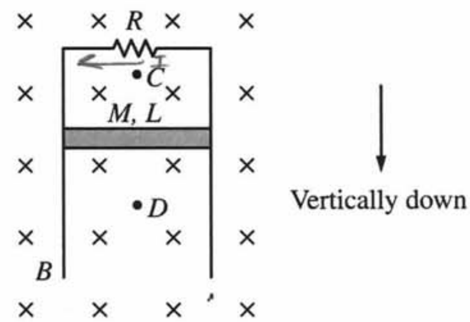


E&M.3.

A conducting bar of mass M, length L, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

_✓_ Left ____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _✓_ Equal to

Justify your answer.

Before the bar is released, there is no change in flux thus no induced current opposing the magnetic field

ii. While the bar is above point D, is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _✓_ Equal to

Justify your answer.

Before the bar is released, ther is no change in flux thus no induced current opposing the magnetic field

GO ON TO THE NEXT PAGE.

Express your answers to parts (c) and (d) in terms of M, L, R, B, and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t.

$$mg - IL \times B = ma$$

$$V = IR$$
$$V = -\frac{d\Phi}{dt}$$
$$I = \frac{-\frac{d\Phi}{dt}}{R}$$

$$\frac{d\Phi}{dt} = B\,dA$$
$$B\ell v$$

$$Mg - \frac{B^2L^2v}{R} = M\frac{dv}{dt}$$

$$\frac{dv}{dt} = g - \frac{B^2L^2v}{MR}$$

(d) Determine an expression for the terminal velocity v_T of the bar.

$$\Sigma F = ILB - mg = 0$$
$$\frac{B^2L^2v_T}{R} = Mg$$

$$v_T = \frac{MgR}{B^2L^2}$$

Express your answers to parts (e) and (f) in terms of v_T, M, L, R, B, and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

$$P = Fv$$
$$ILBv$$

$$P = \frac{B^2L^2v_T^2}{R}$$

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t.

$$dv = \left(g - \frac{B^2L^2v}{MR}\right)dt$$

$$\int_0^{v_t} \frac{dv}{g - \frac{B^2L^2v}{MR}} = \int_0^t dt$$

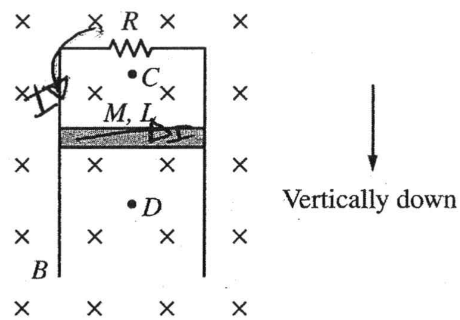
$$\frac{-MR}{B^2L^2}\ln\left(g - \frac{B^2L^2v}{MR}\right)\Big|_0^{v_t}$$

$$\frac{-MR}{B^2L^2}\ln\left(\frac{g - \frac{B^2L^2v_t}{MR}}{g}\right) = t$$

$$\frac{g - \frac{B^2L^2v}{MR}}{g} = e^{-\frac{MRt}{B^2L^2}}$$

$$g - \frac{B^2L^2v}{MR} = ge^{-\frac{MRt}{B^2L^2}}$$

$$V = \frac{MRg}{B^2L^2}\left(1 - e^{\frac{-MRt}{B^2L^2}}\right)$$



E&M.3.

A conducting bar of mass M, length L, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

✓ Left ____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than ✓ Equal to

Justify your answer.

As the bar slides down the rails, magnetic flux increases which increases the magnetic field pointing into the page. According to Lenz's law, the system will create its own magnetic field pointing out of the page to keep the magnetic field constant.

ii. While the bar is above point D, is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than ✓ Less than _____ Equal to

Justify your answer.

Magnetic flux decreases which decreases the magnetic field pointing into the page.

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Express your answers to parts (c) and (d) in terms of M, L, R, B, and physical constants, as appropriate.

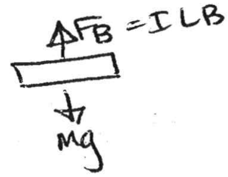
(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t.

$F = ma$

~~$Mg = M\frac{dv}{dt}$~~

~~$Mg - qvB = M\frac{dv}{dt}$~~

~~$Mg - ILR - Mg$~~ I



$R = \frac{V}{I}$

$Mg - ILB = M\frac{dv}{dt}$

(d) Determine an expression for the terminal velocity v_T of the bar.

$\Sigma F = 0$

$Mg = ILB$

Express your answers to parts (e) and (f) in terms of v_T, M, L, R, B, and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

$P = IV = \frac{V^2}{R} = I^2R$

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t.

$Mg - \frac{\varepsilon}{I} LB = M\frac{dv}{dt}$

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Question 3

Overview

The intent of this question was to elicit students' knowledge of how current is induced in response to a changing magnetic flux, of how this induced current can create a magnetic field, and how that induced field can affect the existing magnetic field. The question also assessed the students' knowledge of motional emf, terminal velocity, power dissipated in a resistor, and setting up and integrating Newton's second law in order to find velocity as a function of time.

Sample: E Q3 A
Score: 15

Part (a) earned 1 point for selecting "left." Part (b)(i) earned 2 points for selecting "less than" and explaining how the induced current creates a magnetic field out of the page at point C, which decreases the overall magnetic field. Part (b)(ii) earned 2 points for selecting "greater than" and explaining how the induced current creates a magnetic field into the page at point D, which increases the overall magnetic field. Part (c) earned 4 points for using the correct equation for emf, obtaining an expression for the current, correctly applying Newton's second law, and for writing acceleration as dv/dt. Part (d) earned 2 points for showing that the net force is equal to zero by setting the gravitational force equal to the magnetic force and for having an answer consistent with part (c). Part (e) earned 1 point for having a correct substitution in an appropriate equation for power. Part (f) earned 3 points for separating variables, integrating with the correct limits, and for the correct answer.

Sample: E Q3 B
Score: 10

Part (a) earned 1 point for selecting "left." Parts (b)(i) and (b)(ii) earned no credit for selecting the wrong answers. Parts (c), (d), and (e) earned full credit. Part (f) earned 2 points for separating variables and integrating with the correct limits, but the exponent on the answer is incorrect.

Sample: E Q3 C
Score: 4

Part (a) earned 1 point for selecting "left." Parts (b)(i) and (b)(ii) earned no credit for selecting the wrong answers. Part (c) earned 2 points for correctly applying Newton's second law and writing the acceleration as dv/dt, but there was no attempt to use Faraday's law to find the emf or correct expression for the current. Part (d) earned 1 point for stating that the net force would be zero, but the answer is not consistent with part (c). Part (e) earned no credit because there is no substitution into an equation for power. Part (f) earned no credit because there is no separation of variables, attempt to integrate, and no answer.