2017

AP® CollegeBoard

AP Physics C: Electricity and Magnetism

Sample Student Responses and Scoring Commentary

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- ☑ Free Response Question 1
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- ☑ Student Samples
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General Notes About 2017 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

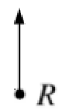
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points total | **Distribution of points**

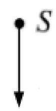
(a)

i. 1 point



For correctly drawing a single vector pointing upward at point *R* — 1 point

ii. 1 point



For correctly drawing a single vector pointing downward at point *S* — 1 point

(b)

i. 2 points

For using Gauss's law to calculate the electric flux — 1 point

$$\Phi = \frac{Q}{\varepsilon_0} = \frac{\rho V}{\varepsilon_0}$$

For a correct answer — 1 point

$$\Phi = \frac{\rho_0 A h}{\varepsilon_0}$$

ii. 3 points

For correctly applying Gauss's law to the slab — 1 point

$$\frac{Q}{\varepsilon_0} = \oint \vec{E} \bullet d\vec{A} = EA_{top} + EA_{bottom}$$

For substituting the flux from part (b)(i) into equation above and for including the contributions of the top and bottom parts of the Gaussian surface — 1 point

$$\frac{\rho_0 A h}{\varepsilon_0} = 2EA$$

For correctly relating *h* to 2*z* — 1 point

$$\frac{\rho_0 A (2z)}{\varepsilon_0} = 2EA$$

$$E = \frac{\rho_0 z}{\varepsilon_0}$$

Question 1 (continued)

Distribution of points

(c)

i. 1 point

Selecting "$z_0 < 0$"

For a correct justification — 1 point

Example: The electric field due to the plates alone is directed toward the top of the page. Therefore, the net electric field can only be zero where the electric field of the slab is directed toward the bottom of the page. So the net electric field can only be zero when $z_0 < 0$.

ii. 3 points

For setting the net electric field between the plates equal to zero and relating it to the sum of the electric fields of the plates and the slab — 1 point

$E_{total} = 0 = E_{plate} + E_{slab}$

For using the correct formula for the electric field between the plates regardless of sign — 1 point

$$\frac{\sigma}{\varepsilon_0} = -\frac{\rho_0 z_0}{\varepsilon_0}$$

$$z_0 = -\frac{\sigma}{\rho_0}$$

Substitute into equation above

$$z_0 = -\frac{\left(2.0 \times 10^{-6}\ \text{C/m}^2\right)}{\left(1.0 \times 10^{-3}\ \text{C/m}^3\right)}$$

For an answer that is consistent with the formula provided by the student above (must include units) — 1 point

$z_0 = -2.0 \times 10^{-3}$ m

Question 1 (continued)

Distribution of points

(d) 4 points

For using the sum of the electric potential of the slab and the plates as the total potential difference between the center and top of the slab — 1 point

$$\Delta V = \Delta V_{slab} + \Delta V_{plate}$$

$$\Delta V = \int E_{total}\, dr = \int \left(E_{slab} + E_{plate}\right) dr$$

$$\Delta V = \int E_{slab} dr + \int E_{plate}\, dr$$

For integrating with proper limits or constants of integration — 1 point

$$\Delta V = \int_{z=0}^{z=0.005} \frac{\rho_0 z}{\varepsilon_0} dz + \int_{z=0}^{z=0.005} \frac{\sigma}{\varepsilon_0} dz$$

For correctly integrating each term regardless of whether they contain the correct expressions — 1 point

$$\Delta V = \frac{\rho_0}{\varepsilon_0}\left[\frac{z^2}{2}\right]_{z=0}^{z=0.005} + \frac{\sigma}{\varepsilon_0}[z]_{z=0}^{z=0.005} = \frac{\rho_0}{2\varepsilon_0}\left(0.005^2 - 0\right) + \frac{\sigma}{\varepsilon_0}(0.005 - 0)$$

$$\Delta V = \frac{\rho_0 z^2}{2\varepsilon_0} + \frac{\sigma z}{\varepsilon_0}$$

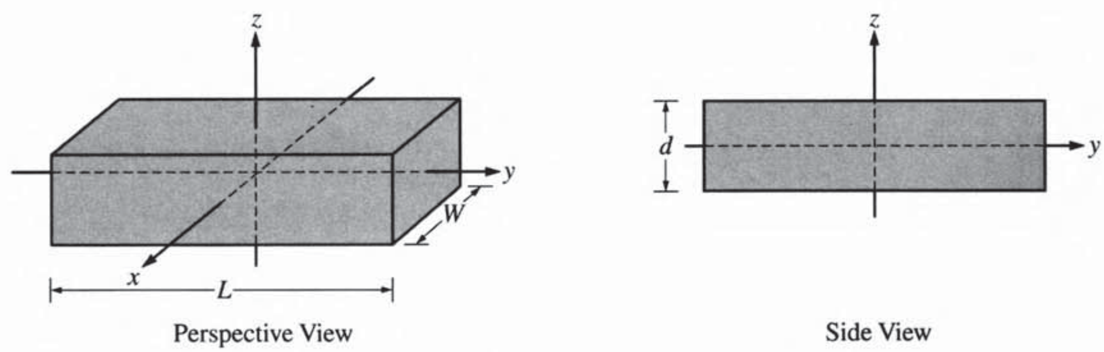
$$\Delta V = \frac{\left(1.0 \times 10^{-3}\ \text{C/m}^3\right)\left(5.0 \times 10^{-3}\ \text{m}\right)^2}{(2)\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)} + \frac{\left(2.0 \times 10^{-6}\ \text{C/m}^2\right)\left(5.0 \times 10^{-3}\ \text{m}\right)}{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)}$$

For a correct answer — 1 point

$\Delta V = 2540\ \text{V}$

PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes
3 Questions

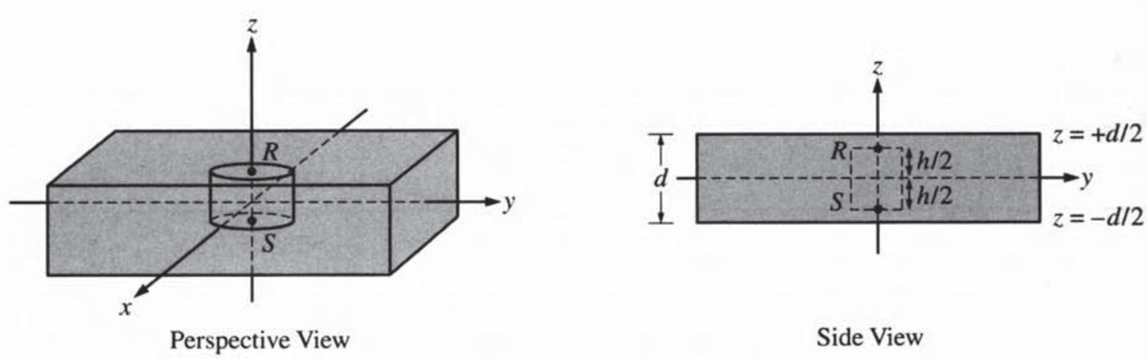
Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figures not drawn to scale.

1. A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the xyz-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L >> d$ and $W >> d$. The large faces of the slab are parallel to the xy-plane.

   Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the z-axis, as shown in the figure below.



Note: Figures not drawn to scale.

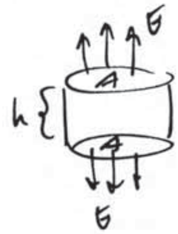
GO ON TO THE NEXT PAGE.

(a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write "$E = 0$" next to the point.

i. ii.

(b) Use Gauss's law to derive expressions for the following. Express your answers in terms of ρ_0, A, d, h, z, and physical constants, as appropriate.

i. Derive an expression for the total flux Φ through the Gaussian surface shown.

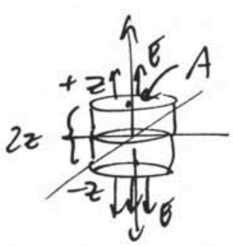


$\phi = \oint E\,dA = q/\varepsilon_0$ Gauss's Law

$= \rho_0 V/\varepsilon_0$

$= \boxed{\rho_0 A h/\varepsilon_0}$

ii. Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \dfrac{\rho_0 z}{\varepsilon_0}$.



Symmetry

~~$E(A) = q/\varepsilon_0$~~

~~$E(A)$~~

$E(2A) = q/\varepsilon_0$ Gauss's Law

$E(2A) = \rho_0 A \cdot 2z/\varepsilon_0$ (V)

$\boxed{E = \rho_0 z/\varepsilon_0}$

E constant across A, always parallel to Area vector

Question 1 continues on the next 2 pages

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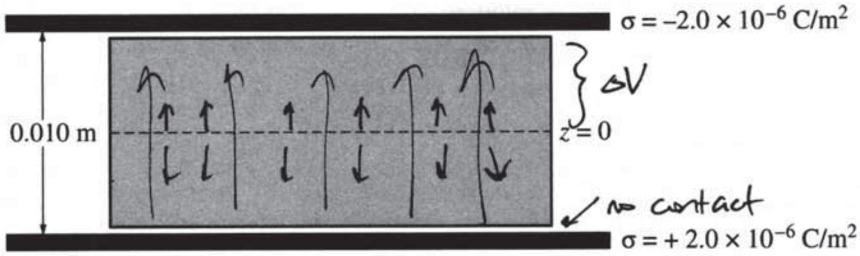

Note: Figure not drawn to scale.

The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6}\ \mathrm{C/m^2}$, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab's volume charge density is $\rho_0 = 1.00 \times 10^{-3}\ \mathrm{C/m^3}$.

(c)

i. The magnitude of the electric field inside the slab is zero on the z-axis at position z_0 . Which of the following correctly indicates the value for z_0 ?

____ $z_0 > 0$ ____ $z_0 = 0$ ✓ $z_0 < 0$

Justify your answer.

$-E_{slab} = E_{plates}$

$E_{slab} = \frac{1.00 \times 10^{-3} z}{\varepsilon_0}$ $-\frac{1.00 \times 10^{-3} z}{\varepsilon_0} = \frac{2.0 \times 10^{-6}}{\varepsilon_0}$

$E_{plates} = \frac{2.0 \times 10^{-6}}{\varepsilon_0} \left(\frac{6}{\varepsilon_0}\right)$ $z = -0.002\ \mathrm{m}$

(below z=0, Eslab points down, balancing out upward E from parallel plates)

ii. Calculate the value z_0 .

$z_0 = -0.002\ \mathrm{m}$

GO ON TO THE NEXT PAGE.

(d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

$$\Delta V_{slab} = -\int_0^{0.005} E\, ds$$

$$= \int_0^{0.005} \frac{1.00 \times 10^{-3} z}{\varepsilon_0}\, dz$$

$$= \frac{1.00 \times 10^{-3}}{\varepsilon_0} \left(\frac{z^2}{2} \Big|_0^{0.005} \right)$$

$$= 1412.4\ V$$

$$\Delta V_{plates} = Ed$$

$$= \left(\frac{2.0 \times 10^{-6}}{\varepsilon_0} \right) (0.005)$$

$$= 1129.9$$

$$\Delta V = \Delta V_s + \Delta V_p = \boxed{2500\ V}$$

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PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

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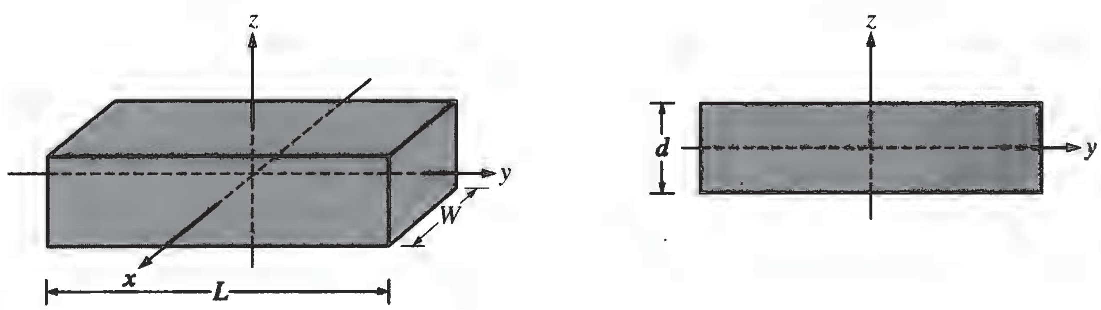
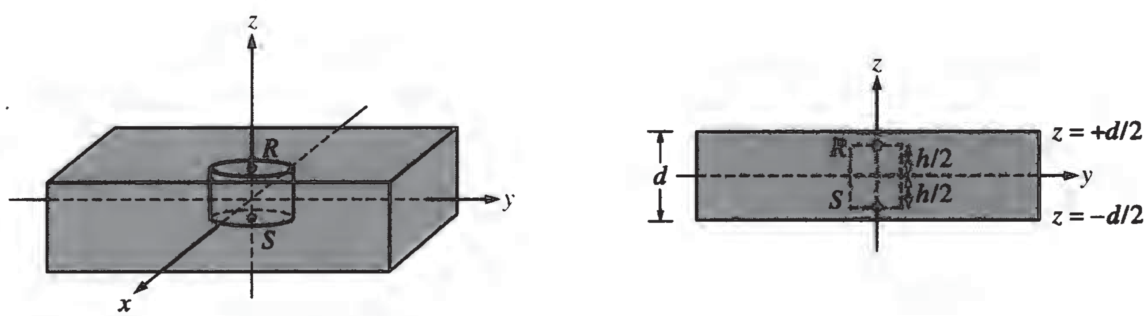


Perspective View   Side View

Note: Figures not drawn to scale.

1. A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is *d*, the length is *L*, and the width is *W*, where $L >> d$ and $W >> d$. The large faces of the slab are parallel to the *xy*-plane.

   Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the *z*-axis, as shown in the figure below.



Perspective View   Side View

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(a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write "$E = 0$" next to the point.

i. ii.

(b) Use Gauss's law to derive expressions for the following. Express your answers in terms of ρ_0, A, d, h, z, and physical constants, as appropriate.

i. Derive an expression for the total flux Φ through the Gaussian surface shown.

$$\oint \vec{E} \cdot dA = \frac{q_{en}}{\varepsilon_0} \qquad \Phi_E = EA$$

$$\Phi = \frac{q_{en}}{\varepsilon_0} \qquad q_{en} = \rho_0 V_{en} = \rho_0 A h$$

$$\boxed{\Phi = \frac{\rho_0 A h}{\varepsilon_0}}$$

ii. Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \dfrac{\rho_0 z}{\varepsilon_0}$.

$$\oint E \cdot dA = \frac{q_{en}}{\varepsilon_0} \qquad q_{en} = \rho_0 A z$$

$$EA = \frac{\rho_0 A z}{\varepsilon_0}$$

$$\boxed{E = \frac{\rho_0 z}{\varepsilon_0}}$$

Question 1 continues on the next 2 pages

GO ON TO THE NEXT PAGE.

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The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6}\ \text{C/m}^2$, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab's volume charge density is $\rho_0 = 1.00 \times 10^{-3}\ \text{C/m}^3$.

(c)

i. The magnitude of the electric field inside the slab is zero on the z-axis at position z_0. Which of the following correctly indicates the value for z_0?

____ $z_0 > 0$ ____ $z_0 = 0$ _✓_ $z_0 < 0$

Justify your answer.

Because the E field due to the slab acts away from the line $z_0 = 0$, and the E field due to the charged plates is up, the E fields cancel when the E slab points downward ($z < 0$).

ii. Calculate the value z_0.

$E_{slab} = \frac{\rho_0 z}{\varepsilon_0}$

$\oint E_{plate} \cdot dA = \frac{Q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$

$E_{plate} A = \frac{\sigma A}{\varepsilon_0}$

$E_{plate} = \frac{\sigma}{\varepsilon_0}$

$E_{slab} = -2 E_{plate}$

$\frac{\rho_0 z}{\varepsilon_0} = \frac{-2\sigma}{\varepsilon_0}$

$z_0 = \frac{-2\sigma}{\rho_0}$

(d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

$$\Delta V = -\int_0^d \frac{2\sigma}{\varepsilon_0} dr = -\frac{2\sigma r}{\varepsilon_0}\Big|_0^d$$

$$|V| = \left|\frac{-2\sigma d}{\varepsilon_0}\right|$$

$$|V| = \boxed{4519.77\ [V]}$$

GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM

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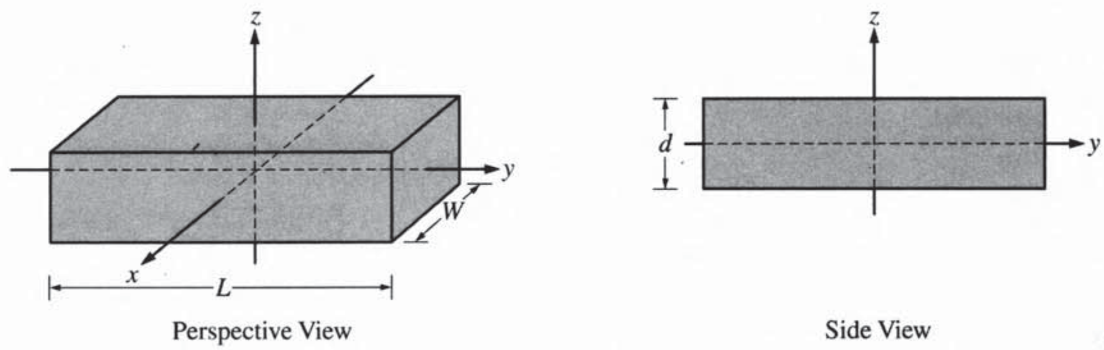

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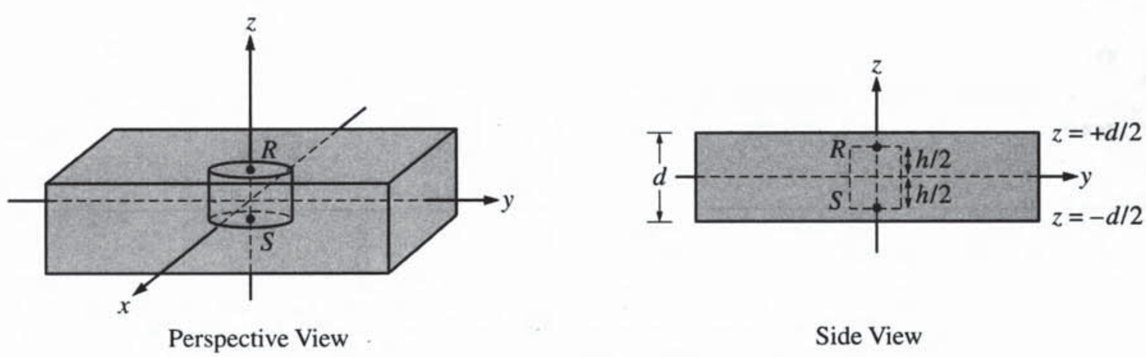

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i. ii.

(b) Use Gauss's law to derive expressions for the following. Express your answers in terms of ρ_0, A, d, h, z, and physical constants, as appropriate.

i. Derive an expression for the total flux Φ through the Gaussian surface shown.

$$\frac{q_{in}}{\epsilon_0} = \oint E \cdot dA$$

$$q_{in} = \rho_0 \times \pi r^2 \times 2 \text{ (2 faces)}, \quad A = \pi r^2$$

$$\boxed{\Phi = \frac{2\rho_0 A}{\epsilon_0}}$$

ii. Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \dfrac{\rho_0 z}{\varepsilon_0}$.

$$E = \frac{\rho_0 z}{\epsilon_0}$$

$$\frac{q_{in}}{\epsilon_0} = \oint E \cdot dA$$

$$\frac{\rho_0 A h}{\epsilon_0 A} = E, \quad E = \frac{\rho_0 z}{\epsilon_0} \text{ where } z = h$$

Question 1 continues on the next 2 pages

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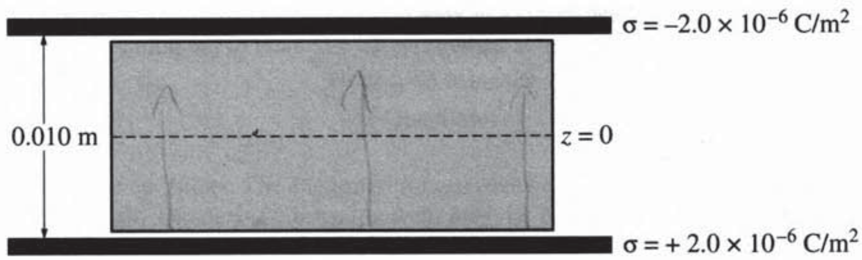

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The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6}\ \mathrm{C/m^2}$, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab's volume charge density is $\rho_0 = 1.00 \times 10^{-3}\ \mathrm{C/m^3}$.

(c)

i. The magnitude of the electric field inside the slab is zero on the z-axis at position z_0. Which of the following correctly indicates the value for z_0?

✓ $z_0 > 0$ ____ $z_0 = 0$ ____ $z_0 < 0$

Justify your answer.

When $E = 0$ at z_0, this occurs between the plates at a value $z_0 > 0$.

ii. Calculate the value z_0.

$$E = \frac{k q_1 q_2}{r} = \frac{q_{in}}{\varepsilon_0 A}$$

$$0 = \frac{2 \times 10^{-6}}{8.85 \times 10^{-12} A}$$

(d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

The magnitude of the ΔV from z=0 to the top is $V = \frac{kq}{r}$

$= \frac{9\times10^{9} e}{(0.05)}$

$V = 2.88\times10^{-27}$

Question 1

Overview

The responses to this question were expected to demonstrate the following:

- The ability to determine the direction of an electric field from a positively charged object.
- The ability to determine the flux through a Gaussian surface.
- The ability to apply Gauss's Law to determine the electric field at a particular location.
- An understanding of the electric field generated by a very large uniformly charged plate.
- An understanding that the total electric field can be found by finding the superposition of component fields.
- The ability to integrate the electric field over a given displacement to determine the electric potential difference between two locations.

Sample: E&M Q1 A
Score: 15

This paper earned full credit for all parts. Part (a) shows vectors that are pointing up at R and down at S. Part (b)(i) uses Gauss's Law to calculate the electric flux. Part (b)(ii) uses this flux to determine the magnitude of the electric field as a function of z. Part (c)(i) has a correct selection and justification. Part (c)(ii) uses the superposition of the electric fields of the slab and plates to determine where the electric field is zero. Part (d) sums the electric potential of the slab and plates, explicitly integrates the electric field for the slab, and uses the correct relationship between E and V for a uniform field.

Sample: E&M Q1 B
Score: 8

Parts (a) and (b)(i) earned full credit. Part (b)(ii) applies Gauss's Law but does not substitute the flux or correctly relate h to z, so 1 point was earned. While the correct final answer is obtained, the solution should have used 2z in the expression for the enclosed charge and 2A in evaluating the flux. Part (c)(i) has a correct selection and justification, so full credit was earned. Part (c)(ii) relates the electric fields from the slab and plates, but the substitution for the plates is incorrect, and the numerical answer is not given, so 1 point was earned. In part (d) the integration is correct for the electric field equation shown, but the sum of the potentials is not indicated, and the limits are not correct, so 1 point was earned.

Sample: E&M Q1 C
Score: 4

Part (a) earned full credit. Part (b)(i) uses Gauss's Law to calculate the electric flux, but the answer is incorrect, so 1 point was earned. Part (b)(ii) applies Gauss's Law but does not substitute the flux or correctly relate h to z, so 1 point was earned. Parts (c) and (d) received no credit. Part (c)(i) has an incorrect selection and justification. Part (c)(ii) has no superposition of the electric field for the slab and plates and has no correct expression for the electric field of the plates. Part (d) has no attempt to sum the electric potentials of the slab and plates, and there is no attempt to integrate.