2017

AP® CollegeBoard

AP Physics C: Electricity and Magnetism

Sample Student Responses and Scoring Commentary

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- ☑ Free Response Question 2
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General Notes About 2017 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Course and Exam Description*.

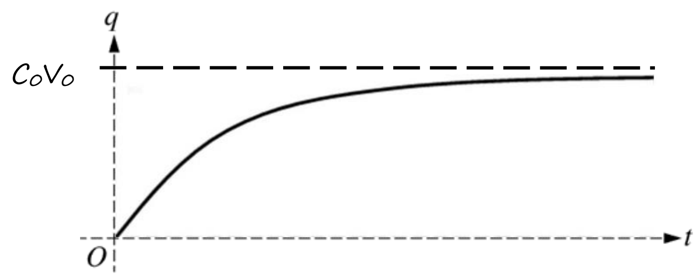
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m}/\text{s}^2$, but use of $10 \text{ m}/\text{s}^2$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 2

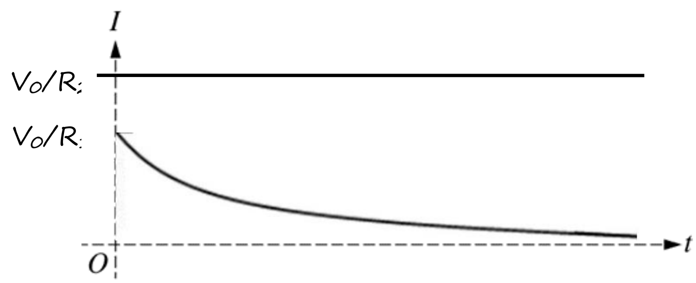
15 points total | **Distribution of points**

(a) 3 points



For a concave down curve starting at the origin — 1 point
For a horizontal asymptote — 1 point
For properly labeling the horizontal asymptote — 1 point

(b) 4 points



For showing that I_2 is horizontal with the correct vertical intercept — 1 point
For showing that I_1 is concave up and asymptotic to I = 0 — 1 point
For correctly labeling the vertical intercept of the I_1 graph — 1 point
For drawing the I_2 graph always above the I_1 graph — 1 point

(c) 2 points

Select "$\Delta t_C < \Delta t_D$"
For indicating that the equivalent resistance during discharging is greater than during charging — 1 point
For a statement relating the greater resistance to the greater time constant, or to a smaller current — 1 point
Example: Because the resistance as the capacitor discharges is greater than when it charges, the time constant is larger for discharging. Therefore, the time to charge to 50% of its maximum charge is less than the time to discharge to 50% of its maximum charge.

Question 2 (continued)

Distribution of points

(d)

i. 2 points

For using a correct statement of Ohm's law or loop rule with the switch open — 1 point

$V = IR$

$I = \dfrac{V_C}{R_{tot}} = \dfrac{V_0}{R_1 + R_2}$

Substitute correct values into equation above

$I = \dfrac{(1.5\ \text{V})}{(100\ \Omega + 150\ \Omega)}$

For a correct answer — 1 point

$I = 6.0\ \text{mA}$ with units, or 0.006 without units

ii. 1 point

For a correct justification — 1 point

Example: Once the capacitor begins to discharge, the charge stored in the capacitor will decrease and the potential difference across the capacitor decreases. Therefore, the current through resistor R_2 decreases.

(e)

i. 1 point

For correctly substituting into an equation for the energy stored in a capacitor, including units for numerical values — 1 point

$U = \dfrac{1}{2}CV^2 = \dfrac{1}{2}(80\ \mu\text{F})(1.5\ \text{V})^2 = 90\ \mu\text{J}$

ii. 2 points

For setting up a valid equation or argument to calculate the energy dissipated by resistor R_1 — 1 point

$E_{dis} = Pt = I^2Rt \therefore E_{dis} \sim R \therefore E_{dis,1} = E_{dis,tot}\dfrac{R_1}{(R_1 + R_2)}$

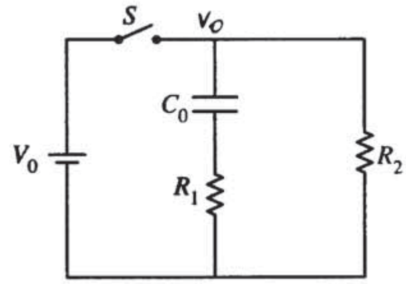
For correctly substituting into the equation above — 1 point

$E_{dis,1} = (90\ \mu\text{J})\dfrac{(150\ \Omega)}{(150\ \Omega + 100\ \Omega)} = 54\ \mu\text{J}$

Alternate Solution on next page.

Question 2 (continued)

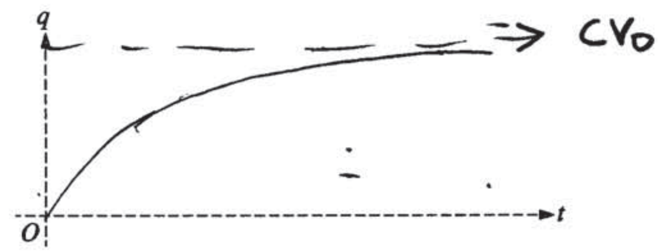
	Distribution of points
(e)(ii) continued	
Alternate Solution	*Alternate Points*
Derive an equation for the current through resistor R_1 as a function of time	
$I(t) = I_{MAX}e^{-t/\tau} = \dfrac{V_{MAX}}{(R_1 + R_2)}e^{-t/(R_1+R_2)C} = \dfrac{(1.5\ V)}{(250\ \Omega)}e^{-t/(250\ \Omega)(80\ \mu F)}$	
$I(t) = (6.0\ mA)e^{-t/(0.02)}$	
For using a correct formula with integral calculus to calculate the energy dissipated by resistor R_1	1 point
$E = \int Pdt = \int I^2Rdt = \int I(t)^2 R_1dt = \int\left((6.0\ mA)e^{-t/(0.02)}\right)^2(150\ \Omega)dt$	
For integrating the above equation with correct limits or constants of integration	1 point
$E = (0.0054)\int_{t=0}^{t=\infty}\left(e^{-t/(0.02)}\right)^2dt = (0.0054)\int_{t=0}^{t=\infty}e^{-100t}dt = (0.0054)\left[\dfrac{e^{-100t}}{(-100)}\right]_{t=0}^{t=\infty}$	
$E = \left(-5.4\times10^{-5}\right)\left(e^{-\infty} - e^0\right) = 54\ \mu J$	



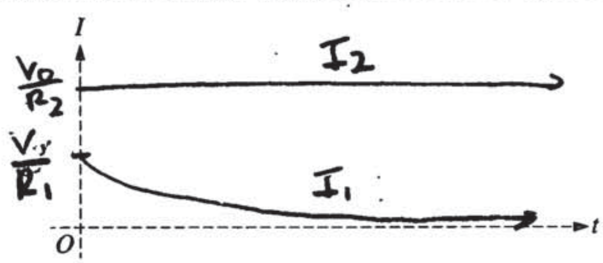
Q=CV

2. In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

(a) The switch is closed at time $t = 0$. On the axes below, sketch the charge q on the capacitor as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(b) On the axes below, sketch the current I through each resistor as a function of time t. Clearly label the two curves as I_1 and I_2, the currents through resistors R_1 and R_2, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



The circuit is constructed using an ideal 1.5 V battery, an 80 μF capacitor, and resistors $R_1 = 150\ \Omega$ and $R_2 = 100\ \Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

(c) The time it takes to charge the capacitor to 50% of its maximum charge is Δt_C. The time it takes for the capacitor to discharge to 50% of its maximum charge is Δt_D. Which of the following correctly relates the two time intervals?

____ $\Delta t_C > \Delta t_D$ ✓ $\Delta t_C = \Delta t_D$ ____ $\Delta t_C < \Delta t_D$

Justify your answer.

$Q_f(1-e^{-t/RC})$ models the rate of charging the capacitor, while $Qe^{-t/RC}$ models the rate of discharge. The time constant RC is constant (same circuit) so the rate of charging and discharging is equal

(d)

i. Calculate the current through resistor R_2 immediately after the switch is opened.

V=IR ohm's law

$I = \frac{V}{R} = \frac{1.5}{100+150} = \frac{1.5}{250} = 0.006\text{ A}$

↑ in series now with R_1

ii. Is the current through resistor R_2 increasing, decreasing, or constant immediately after the switch is opened?

____ Increasing ✓ Decreasing ____ Constant

Justify your answer.

The capacitor and resistors are no longer connected to the voltage source and so the current must decrease as the capacitor is discharging

(e)

i. Calculate the energy stored in the capacitor immediately after the switch is opened.

$E = \frac{1}{2}CV^2 = \frac{1}{2} \cdot 80\mu F \cdot (1.5)^2$

$= \boxed{9\times10^{-5}\text{ J}}$

ii. Calculate the energy dissipated by resistor R_1 as the capacitor completely discharges.

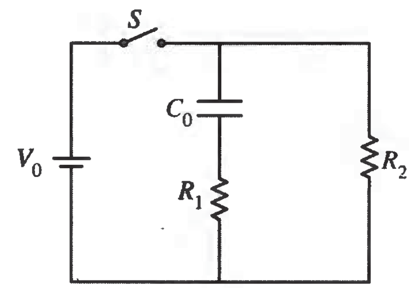
$9\times10^{-5} \cdot \frac{150}{250} = \boxed{5.4\times10^{-5}\text{ J}}$

$P = I^2R$

↳ amount of energy dissipated ∝ resistance

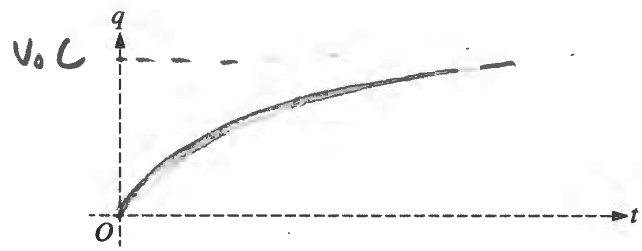
↑ proportion

E&M Q2 B1



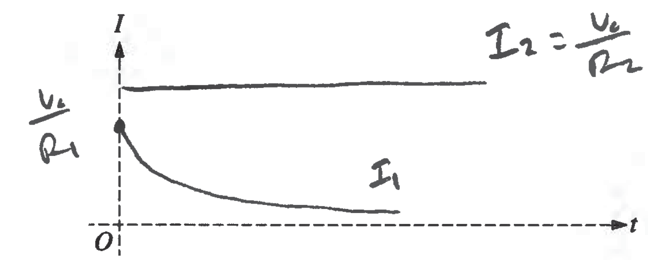
2. In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

   (a) The switch is closed at time $t = 0$. On the axes below, sketch the charge q on the capacitor as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



   (b) On the axes below, sketch the current I through each resistor as a function of time t. Clearly label the two curves as I_1 and I_2, the currents through resistors R_1 and R_2, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

$V = IR$
$I = \frac{V}{R}$



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The circuit is constructed using an ideal 1.5 V battery, an 80 μF capacitor, and resistors $R_1 = 150\ \Omega$ and $R_2 = 100\ \Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

(c) The time it takes to charge the capacitor to 50% of its maximum charge is Δt_C. The time it takes for the capacitor to discharge to 50% of its maximum charge is Δt_D. Which of the following correctly relates the two time intervals?

✓ $\Delta t_C > \Delta t_D$ ____ $\Delta t_C = \Delta t_D$ ____ $\Delta t_C < \Delta t_D$

Justify your answer.

Since the capacitor discharges through the resistor instead of one energy is dissipated more quickly, meaning it will take less time to discharge the capacitor making $\Delta t_C > \Delta t_D$

(d)

i. Calculate the current through resistor R_2 immediately after the switch is opened.

V=1.5

R_1 and R_2 in series, $R_{eq} = 250\ \Omega$

$I = \frac{V}{R} = \frac{1.5}{250} = .006A = I_{R_2}$

ii. Is the current through resistor R_2 increasing, decreasing, or constant immediately after the switch is opened?

____ Increasing ✓ Decreasing ____ Constant

Justify your answer.

As the switch is opened, charge flows off the capacitor plates. Energy is dissipated in the resistors, and since no charge is added to the capacitor, the current must decrease

(e)

i. Calculate the energy stored in the capacitor immediately after the switch is opened.

$U_C = \frac{1}{2}CV^2 = \frac{1}{2}1.5\times10^{-6}(1.5) = 7.5\times10^{-7}J$

ii. Calculate the energy dissipated by resistor R_1 as the capacitor completely discharges.

Total E dissipated $= \frac{1}{2}$ ... $7.5\times10^{-7}J$

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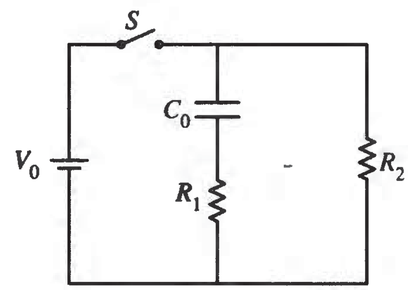
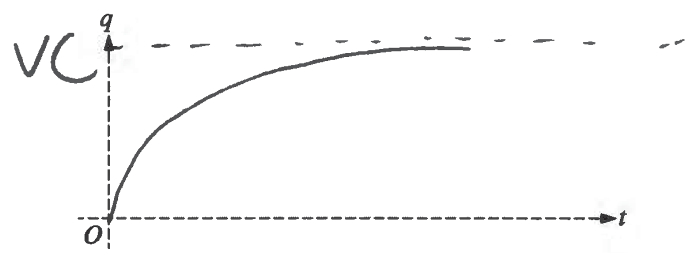


2. In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

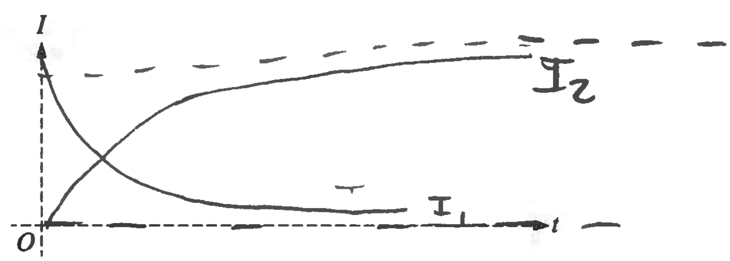
(a) The switch is closed at time $t = 0$. On the axes below, sketch the charge q on the capacitor as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

$V = \frac{Q}{C}$

$Q = VC$



(b) On the axes below, sketch the current I through each resistor as a function of time t. Clearly label the two curves as I_1 and I_2, the currents through resistors R_1 and R_2, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



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The circuit is constructed using an ideal 1.5 V battery, an 80 μF capacitor, and resistors $R_1 = 150\ \Omega$ and $R_2 = 100\ \Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

(c) The time it takes to charge the capacitor to 50% of its maximum charge is Δt_C. The time it takes for the capacitor to discharge to 50% of its maximum charge is Δt_D. Which of the following correctly relates the two time intervals?

____ $\Delta t_C > \Delta t_D$ __✓__ $\Delta t_C = \Delta t_D$ ____ $\Delta t_C < \Delta t_D$

Justify your answer.

Because the time constant of the circuit is the same for discharging and charging $\Delta t_C = \Delta t_D$.

(d)

i. Calculate the current through resistor R_2 immediately after the switch is opened.

$I = 0\ A$

ii. Is the current through resistor R_2 increasing, decreasing, or constant immediately after the switch is opened?

__✓__ Increasing ____ Decreasing ____ Constant

Justify your answer.

As the capacitor charges, the current in the first branch goes to zero, leaving all of the current to go to R_2

(e)

i. Calculate the energy stored in the capacitor immediately after the switch is opened.

$E = \frac{1}{2} C V^2$

$E = \frac{1}{2} (80\ \mu F)(1.5^2)$

$\boxed{E = 90\ J}$

ii. Calculate the energy dissipated by resistor R_1 as the capacitor completely discharges.

$\frac{150}{250}(.90)$

$E = 54\ J.$

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Question 2

Overview

The responses to this question were expected to demonstrate the following:

- Graphing skills for exponential functions/behavior of RC circuits, especially regarding asymptotes.
- An understanding of charging and discharging behavior of RC circuits, including the importance of the time constant and how it relates to various graphs.
- An understanding of Ohm's Law and Kirchoff's Loop Rule as tools for analyzing RC circuits.
- An understanding of the relationships between charge, voltage, energy, and power for capacitors in an RC circuit.
- An understanding of equivalent resistance for resistors in series versus parallel.
- An understanding of how switches work in circuits.

Sample: E&M Q2 A
Score: 12

Parts (a) and (b) earned full credit. The graph in (a) is concave down and has an appropriate asymptote with an appropriate label. The label "C" is acceptable because there is only one capacitor. The graph in part (b) has the proper shapes and labels, and I_2 is always above I_1. Part (c) has an incorrect selection, and the justification has no indication of the change in resistance between the two phases and no statement relating the higher equivalent resistance to a larger time constant; therefore, no credit was earned. Part (d)(i) uses Ohm's law and has a correct answer, so it earned full credit. Part (d)(ii) has no discussion of the decreasing voltage leading to a decreasing current, so no credit was earned. Part (e) earned full credit. Part (e)(i) correctly substitutes into an equation for energy stored in a capacitor, and part (e)(ii) correctly sets up and substitutes into an equation to calculate the energy dissipated in resistor R_1.

Sample: E&M Q2 B
Score: 10

Parts (a) and (b) earn full credit. Part (c) correctly indicates an increase in resistance for discharging but has an incorrect discussion about the effect of the increased resistance on the time constant, so 1 point was earned. Part (d)(i) earned full credit. Part (d)(ii) does not connect charge to potential difference or use Ohm's law to justify decrease in current, so no credit was earned. Part (e)(i) incorrectly substitutes into the equation, so no credit was earned. Part (e)(ii) does not have a correct formula for the energy dissipated, so no credit was earned.

Question 2 (continued)

Sample: E&M Q2 C
Score: 5

The graph in (a) is concave down and has an appropriate asymptote, but the label "V" is ambiguous, so 2 points were earned. In part (b) I_1 is concave up and asymptotic to the axis, so 1 point was earned. In part (c) the change in resistance is neglected, so there is no discussion of the relationship between the time constant and larger equivalent resistance, so no credit was earned. Part (d)(i) did not calculate the current, so no credit was earned. In part (d)(ii) there is no discussion of discharging or its relationship to the current, so no credit was earned. Part (e)(i) has incorrect units on the answer, so no credit was earned. In part (e)(ii) the proportionality statement and substitution are correct, so full credit was earned.