2018

AP CollegeBoard

AP Physics C: Electricity and Magnetism

Sample Student Responses and Scoring Commentary

Inside:

Free Response Question 1

- ☑ **Scoring Guideline**
- ☑ **Student Samples**
- ☑ **Scoring Commentary**

AP® PHYSICS
2018 SCORING GUIDELINES

General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

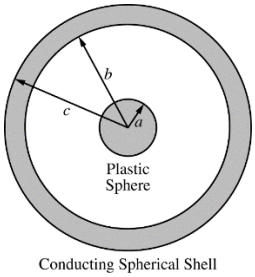
5. The scoring guidelines typically show numerical results using the value $g = 9.8\ \text{m/s}^2$, but the use of $10\ \text{m/s}^2$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

AP® PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 1

15 points total **Distribution of points**



A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

(a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

i. 3 points

$r < a$

For using the integral to determine the charge enclosed		1 point
$q_{enc} = \int \rho dV = \int \beta r dV$		
For correctly substituting for dV into the integration to determine the charge enclosed		1 point
$q_{enc} = \int \left(4\pi r'^2\right)(\beta r')dr' = 4\beta\pi \int_{r'=0}^{r'=r} r'^3 dr' = 4\beta\pi \left[\frac{r'^4}{4}\right]_{r'=0}^{r'=r}$		
For a correct answer		1 point
$q_{enc} = \beta\pi r^4$		

ii. 1 points

$a < r < b$

For an answer consistent with (a)(i)		1 point
$q_{enc} = \beta\pi a^4$		

Question 1 (continued)

Distribution of points

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. 2 points

$r < a$

For correctly substituting the area of a Gaussian sphere into Gauss's law		1 point
$\frac{q_{enc}}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A}$		
$\frac{q(r)}{\varepsilon_0} = EA_{sphere} = E\left(4\pi r^2\right)$		
For correctly substituting the charge from part (a)(i) into the equation above		1 point
$\frac{\beta \pi r^4}{\varepsilon_0} = E\left(4\pi r^2\right)$		
$E = \frac{\beta r^2}{4\varepsilon_0}$		

ii. 1 point

$a < r < b$

For correctly substituting the charge from part (a)(ii) into Gauss's law to calculate the electric field		1 point
$\frac{\beta \pi a^4}{\varepsilon_0} = E\left(4\pi r^2\right)$		
$E = \frac{\beta a^4}{4\varepsilon_0 r^2}$		

Question 1 (continued)

Distribution of points

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. 2 points

Determine the charge on the inner surface of the conducting shell.
Justify your answer.

For an answer consistent with part (a)(ii) (must have opposite sign)		1 point
$q_{inner} = -\beta\pi a^4$		
For a correct justification		1 point
$E_{shell} = 0 = \frac{q_{enc}}{\varepsilon_0} \therefore q_{enc} = 0 = q_{sphere} + q_{inner} \therefore q_{inner} = -q_{sphere}$		

ii. 1 point

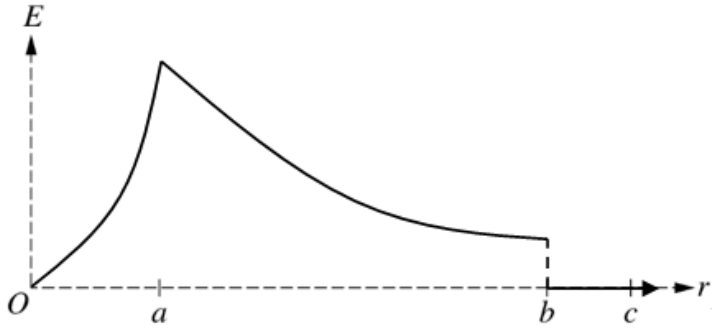
Determine the charge on the outer surface of the conducting shell.

A correct answer		1 point
$q_{outer} = 0$		

(d)

i. 3 points

On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



For concave up graph that begins at the origin and increases to $r = a$		1 point
For a concave up graph that is continuous with the first section and decreases from $r = a$ to nonzero value at $r = b$		1 point
For a graph with a value of $E = 0$ for $r > b$		1 point
Note: Due to scale, discontinuity at point b is not required.		

Question 1 (continued)

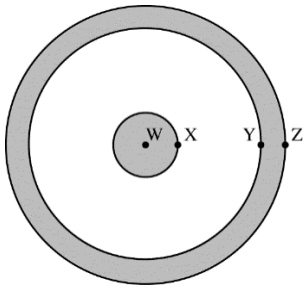
Distribution of points

(d) (continued)

ii. 2 points

The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

__1__ W __2__ X __3__ Y __3__ Z



For any ranking that has Y and Z at the same potential		1 point
For ranking the electric potentials: W > X > Y & Z (because Y = Z is the first point)		1 point

E Q1 A p1

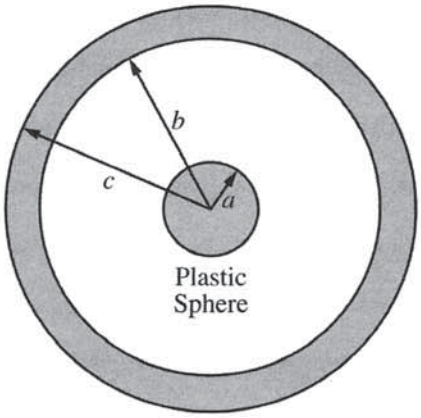
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

   i. $r < a$

$$\int_0^r (\beta)(r)\, 4\pi r^2 dr = \int_0^r \beta 4\pi r^3 dr = \beta 4\pi \left[\frac{r^4}{4}\right]_0^r =$$

$$\frac{\beta 4\pi}{4} r^4 = \beta \pi r^4 = q_{enc}$$

   ii. $a < r < b$

Entire plastic sphere's charge:

$$\int_0^R 4\pi r^2 \beta r\, dr = \beta \pi R^4 = q_{enc}$$

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

($q_{enc} = \beta\pi r^4$, from ai)

$$\frac{q_{enc}}{\epsilon_0} = \oint \vec{E}\cdot d\vec{A} = EA = E4\pi r^2$$

$$\frac{\beta\pi r^4}{\epsilon_0} = E4\pi r^2 \rightarrow \boxed{\frac{\beta r^2}{4\epsilon_0} = E(r)}$$

ii. $a < r < b$

($q_{enc} = \beta\pi R^4$)

$$\frac{q_{enc}}{\epsilon_0} = \oint \vec{E}\cdot d\vec{A} = EA = E4\pi r^2$$

$$\frac{\beta\pi R^4}{\epsilon_0} = 4\pi r^2 E \rightarrow \boxed{E(r) = \frac{\beta R^4}{4r^2\epsilon_0}}$$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

$\boxed{-\beta\pi R^4 = q_{inside}}$

Justify your answer.

~~Draw gaus~~ Any point in conducting shell has $E = 0$. Thus, if one draws a Gaussian surface through the shell, $\oint \vec{E}\cdot d\vec{A}$ must $= 0$, so $\Phi = \frac{q_{enc}}{\epsilon_0} = 0$, and q_{enc} (Net charge enclosed) $= 0$. Net charge enclosed = charge of inner surface + charge of plastic ball $= 0$, so charge of inner shell = $-$charge of plastic ball

ii. Determine the charge on the outer surface of the conducting shell.

Draw gaussian outside the shell. $\oint \vec{E}\cdot d\vec{A} = \frac{q_{net\ enclosed}}{\epsilon_0} = 0$

($\oint \vec{E}\cdot d\vec{A}$ given as 0)

$\rightarrow q_{net\ enclosed} = 0$

$= q_{plastic} + q_{in} + q_{out} \rightarrow q_{out} =$

$q_{plastic}$ + q_{in} same magnitude, different sign, $= 0$.

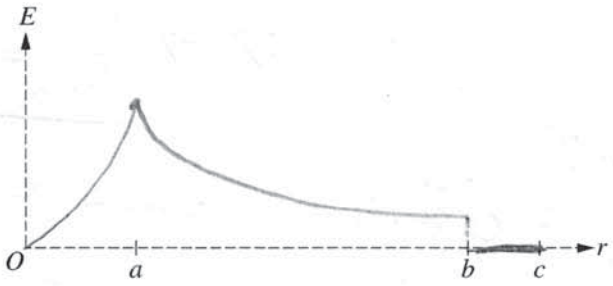
$\boxed{0\ C}$

Question 1 continues on the next page.

GO ON TO THE NEXT PAGE.

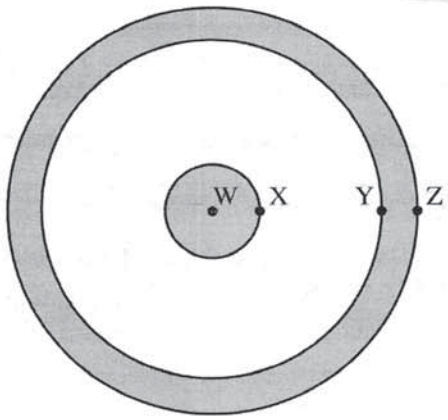
(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

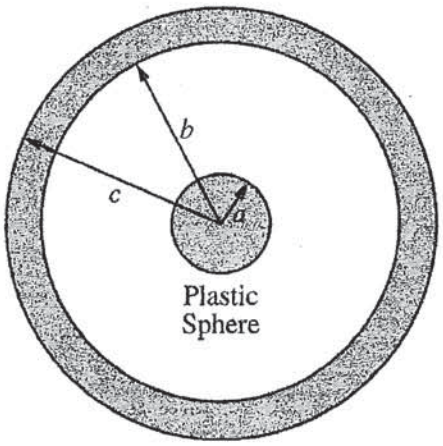
1 W 2 X 3 Y 3 Z



PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

i. $r < a$

$Q_{enc} = \int_0^r \rho \, V$

$\rho = Br$

$V = 4/3 \, \pi r^3$

$Q_{enc} = \int_0^r 4/3 \, B \pi r^4$

$Q_{enc} = 4/15 \, \beta \pi r^5$

ii. $a < r < b$

$Q_{enc} = \int_0^a \rho \, V$

$Q_{enc} = \int_0^a 4/3 \, Br^4$

$Q_{enc} = 4/15 \, \beta \pi a^5$

GO ON TO THE NEXT PAGE.

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

$\oint E \, dA = \frac{Q_{enc}}{\epsilon_0}$

$E(4\pi r^2) = \frac{4\beta\pi r^5}{15\epsilon_0}$

$E = \frac{\beta r^3}{15\epsilon_0}$

ii. $a < r < b$

$\oint E dA = \frac{Q_{enc}}{\epsilon_0}$

$E(4\pi r^2) = \frac{4\beta\pi a^5}{15\epsilon_0}$

$E = \frac{\beta a^5}{15\epsilon_0 r^2}$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

$Q = -4/15 \beta\pi a^5$

Justify your answer.

In order for the electric field to be zero, the enclosed charge at both spheres must sum to zero.

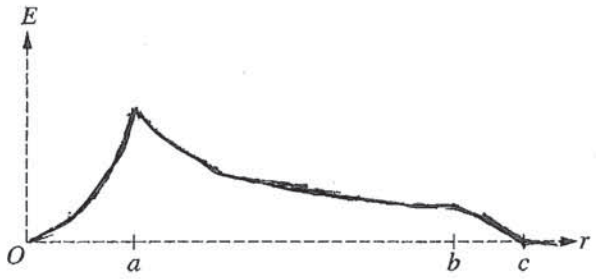
ii. Determine the charge on the outer surface of the conducting shell.

$Q = -4/15 \beta\pi a^5$

Question 1 continues on the next page.

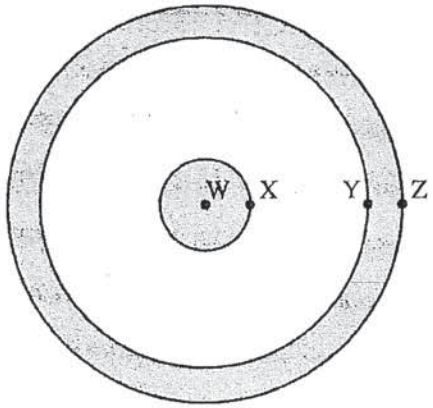
(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

__1__ W __2__ X __3__ Y __4__ Z



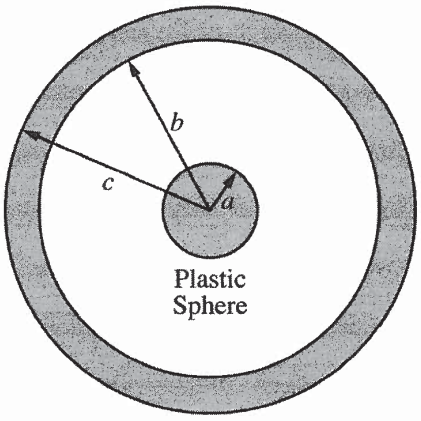
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

   i. $r < a$

$$Q = \rho V = \rho(r)\,\frac{4}{3}\pi r^3 = \beta r\,\frac{4}{3}\pi r^3 = \frac{4\pi}{3}\beta r^4$$

   ii. $a < r < b$

$$Q = \rho V = \rho(a)\,\frac{4}{3}\pi a^3 = \beta a\,\frac{4}{3}\pi a^3 = \frac{4\pi}{3}\beta a^4$$

GO ON TO THE NEXT PAGE.

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

$$\oint E \cdot dA = \frac{Q}{\varepsilon_0} = E \oint dA = E(4\pi r^2) = \frac{4\pi}{3\varepsilon_0} \beta r^4$$

$$E = \frac{\beta r^4}{3\varepsilon_0 r^2} = \boxed{\frac{\beta r^2}{3\varepsilon_0}}$$

ii. $a < r < b$

$$\oint E \cdot dA = \frac{Q}{\varepsilon_0} = E \oint dA = E(4\pi a^2) = \frac{4\pi}{3\varepsilon_0} \beta a^4$$

$$E = \frac{\beta a^4}{3\varepsilon_0 a^2} = \boxed{\frac{\beta a^2}{3\varepsilon_0}}$$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

Justify your answer.

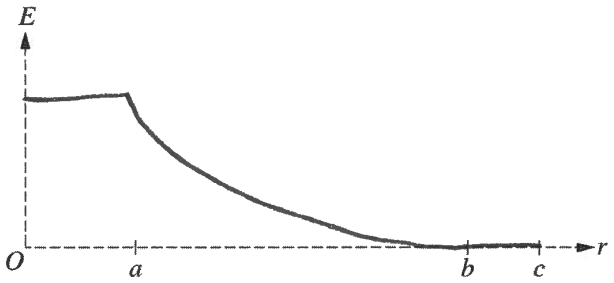
ii. Determine the charge on the outer surface of the conducting shell.

$\boxed{Q = 0\ C}$

Question 1 continues on the next page.

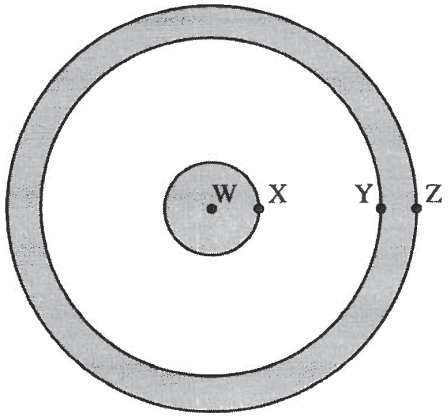
(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

3 W 1 X 2 Y 3 Z



AP® PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING COMMENTARY

Question 1

Overview

The responses to this question were expected to demonstrate the following:

- How to apply Gauss's law for different spherically symmetric distributions of charge.
- An understanding of the magnitude of electric fields at various locations for a given charge distribution.
- The ability to plot the electric field as a function of position and rank the electric potential as a function of position.

Sample: E Q1 A
Score: 14

All parts except part (a)(ii) earned full credit. In part (a)(i) a correct integral is written, the substitution of $4\pi r^2 dr$ for dV is made, and a correct answer is shown, which earned all 3 points. In part (a)(ii) the answer uses R instead of the given radius a, so no points were earned. In part (b)(i) the answer from part (a)(i) is substituted into Gauss's law with a correct expression for area, so 2 points were earned. In part (b)(ii) the answer from part (a)(ii) is substituted into Gauss's law, earning 1 point. In part (c)(i) the charge from part (a)(ii) is included in the justification, and the electric field inside the shell is set equal to zero, so 2 points were earned. Part (c)(ii) earned 1 point for stating that $q = 0$. In part (d)(i) all three sections of the graph have appropriate shapes, which earned 3 points. In part (d)(ii) the rankings are correct, so 2 points were earned.

Sample: E Q1 B
Score: 9

Part (a)(i) integrates to derive the expression for charge, but the substitution dV and the answer are incorrect, so 1 point was earned. Parts (a)(ii) has an answer consistent with part (a)(i), so 1 point was earned. Both parts for (b) have answers consistent with part (a), so the full credit of 3 points was earned. In part (c)(i) the charge is consistent with part (a)(ii), but it does not specify that the electric field is zero inside the shell, so 1 point was earned. Part (c)(ii) has an incorrect answer, so no points were earned. In part (d)(i) the graph is correct for $r < a$ and from points a to b, but E should equal zero from points b to c, so 2 points were earned. In part (d)(ii) the ranking has the potential at point W > X > Y & Z, but it does not have the potential at point Y equal to the potential point Z, so 1 point was earned.

Sample: E Q1 C
Score: 6

In part (a)(i) no the integral is shown, there is no clear substitution of $4\pi r^2 dr$ for dV, and the answer is not correct, so no points were earned. Part (a)(ii) is consistent with (a)(i) and earned 1 point. Part (b) earned full credit of 3 points because the incorrect answers from (a) are used correctly. Part (c)(i) is blank, so no points were earned. In part (c)(ii) q is set equal to zero, so 1 point was earned. In part (d)(i) $E = 0$ for $r > b$ but should not be a straight horizontal line for $r < a$, and E should not equal zero when $r = b$, so 1 point was earned. In part (d)(ii) the ranking does not indicate that the potential at point Y is equal to the potential point Z, and the potential at point W is less than the potential at point X, so no points were earned.