2019

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Sample Student Responses and Scoring Commentary Set 2

Inside:

Free Response Question 2

- ☑ Scoring Guideline
- ☑ Student Samples
- ☑ Scoring Commentary

AP® PHYSICS
2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

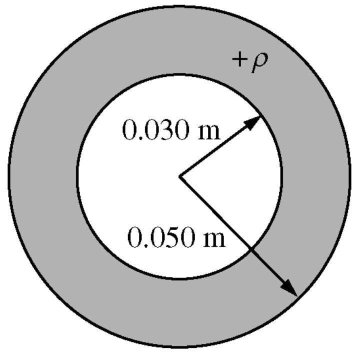
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 2

15 points



A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m}: \ \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m}: \ \rho = b/r \text{, where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m}: \ \rho = 0$$

(a) LO CNV-2.D.a, SP 6.B, 6.C
3 points

Calculate the total charge of the sphere.

For indicating the need to integrate the expression for charge density to determine the total charge on the sphere		1 point
$Q = \int \rho dV$		
For proper substitutions into the integration		1 point
$Q = \int \frac{\left(1.6 \times 10^{-6}\right)}{r}\left(4\pi r^2\right)dr$		
For using the proper limits of integration		1 point
$Q = 4\pi\left(1.6 \times 10^{-6}\right)\int_{r=0.03}^{r=0.05} rdr = 4\pi\left(1.6 \times 10^{-6}\right)\left[\frac{r^2}{2}\right]_{r=0.03}^{r=0.05}$		
$Q = (2\pi)\left(1.6 \times 10^{-6}\right)\left(0.05^2 - 0.03^2\right) = 1.61 \times 10^{-8}$ C		

Question 2 (continued)

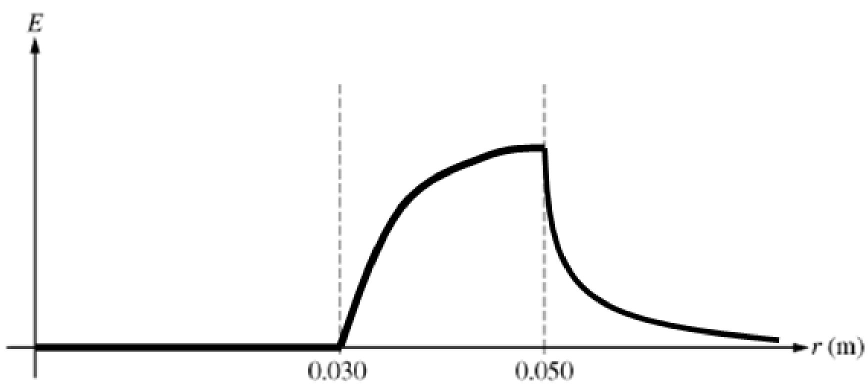
(b) LO CNV-2.D.a, SP 6.C
3 points

Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

For correctly evaluating the surface integral in Gauss's law		1 point
$\oint E \bullet dA = E(4\pi r^2)$		
For correctly substituting the answer from part (a) and correct radius into above equation		1 point
$\frac{Q_{enc}}{\varepsilon_0} = E(4\pi r^2) \therefore E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2} = \frac{(1.61\times 10^{-8}\ \text{C})}{(4\pi)(8.85\times 10^{-12})(0.05)^2}$		
For an answer consistent with part (a) with correct units		1 point
$E = 5.79\times 10^4\ \text{N/C}$		

(c) LO CNV-2.C, SP 3.C
3 points

On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



For clearly showing a graph with a value of $E = 0$ for $r < 0.030$ m		1 point
For a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$		1 point
For a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050$ m		1 point

Question 2 (continued)

(d) LO CNV-1.G.a, SP 6.B, 6.C
2 points

Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

For substituting the total charge from part (a) into a correct expression for electric potential		1 point
For substituting $r = 0.05$ m into a correct expression for electric potential		1 point
$V_R = \dfrac{Q_{\text{tot}}}{4\pi\varepsilon_0 r} = \dfrac{\left(9\times10^9\right)\left(1.61\times10^{-8}\text{ C}\right)}{(0.05\text{ m})} = 2900\text{ V}$		
Alternate Solution		*Alternate Points*
For substituting the total charge from part (a) into an integration for electric potential		*1 point*
$\Delta V_R = V_R - V_\infty = V_R = -\int E dr = -\int \dfrac{Q_{enc}}{4\pi\varepsilon_0 r^2} dr = -\int \dfrac{\left(9\times10^9\right)\left(1.61\times10^{-8}\text{ C}\right)}{r^2} dr$		
For integrating with correct limits of integration		*1 point*
$V_R = -\displaystyle\int_{r=\infty}^{r=0.05\text{ m}} \dfrac{145}{r^2} dr = -145\left[-\dfrac{1}{r}\right]_{r=\infty}^{r=0.05\text{ m}} = 145\left(\dfrac{1}{(0.05\text{ m})} - \dfrac{1}{\infty}\right) = 2900\text{ V}$		

A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

(e)
i. LO ACT-1.D, SP 6.B, 6.C
2 points

Calculate the magnitude of the initial acceleration of the proton.

For using a correct expression of Newton's second law in terms of the electric field		1 point
$F = ma \therefore qE = ma \therefore a = \dfrac{qE}{m}$		
For correctly substituting into equation above		1 point
$a = \dfrac{qE}{m} = \dfrac{\left(1.6\times10^{-19}\text{ C}\right)\left(5.79\times10^{4}\text{ N/C}\right)}{1.67\times10^{-27}\text{ kg}} = 5.55\times10^{12}\text{ m/s}^2$		

Question 2 (continued)

(e) continued
ii. LO CNV-1.E, SP 6.B, 6.C
2 points

Calculate the speed of the proton after a long time.

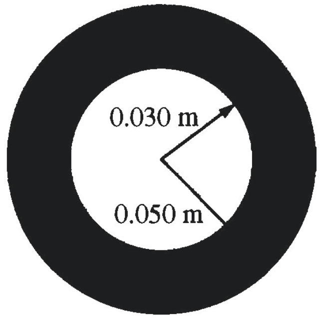
For a correct expression of kinetic energy in terms of the electric potential difference		1 point
$-q\Delta V = \frac{1}{2}mv^2$		
For correctly substituting into equation above		1 point
$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{-2\left(1.6 \times 10^{-19}\ \text{C}\right)(0 - 2900\ \text{V})}{\left(1.67 \times 10^{-27}\ \text{kg}\right)}} = 7.45 \times 10^{5}\ \text{m/s}$		

Learning Objectives

ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.
CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.
CNV-1.G.a: Use the general relationship between electric field and electric potential to calculate the relationships between the magnitude of electric field or the potential difference as a function of position.
CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.
CNV-2.D.a: Using appropriate mathematics (which may involve calculus), calculate the total charge contained in lines, surfaces, or volumes when given a linear-charge density, a surface-charge density, or a volume-charge density of the charge configuration.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.



2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m}: \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m}: \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(a) Calculate the total charge of the sphere.

$\rho = \frac{Q}{V} = \frac{dq}{dV}$

$dV = A\,dr$

$dq = \rho\, dV$

$A = \frac{dV}{dx}$

$Q = \int \rho\, dV$

$A = \frac{d}{dx}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2$

$Q = \int \rho A\, dr = \int_{r=0.03}^{r=0.05} \frac{b}{r} 4\pi r^2\, dr$

$Q = 4\pi b \int_{0.03}^{0.05} r\, dr$

$Q = 4\pi(1.6\times10^{-6}\,C/m^2)\left[\frac{1}{2}r^2\right]_{0.03}^{0.05} = 4\pi(1.6\times10^{-6}\,C/m^2)\left(\frac{(.05)^2}{2} - \frac{(.03)^2}{2}\right) = \boxed{1.61\times10^{-8}\,C}$

GO ON TO THE NEXT PAGE.

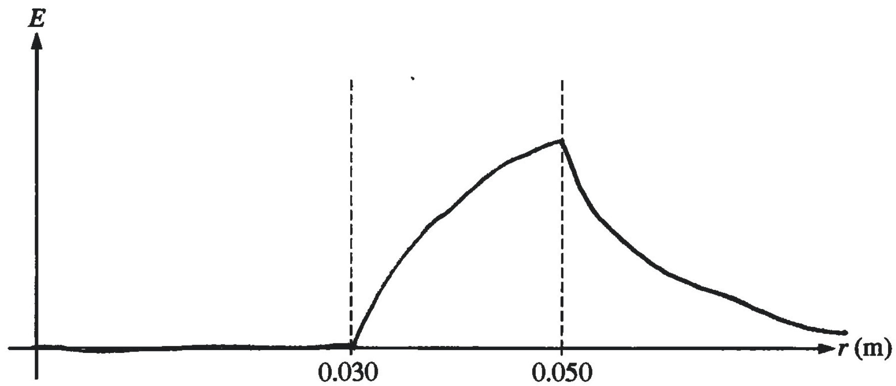
(b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_{enc}}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(1.61\times10^{-8}C)}{(0.05m)^2} = \boxed{5.79\times10^4 N/C}$$

(c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



(d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

$$V = -\int_{\infty}^{r=0.05} E \cdot dr = -\int_{\infty}^{0.05} \frac{kQ_{enc}}{r^2} dr$$

$$V = kQ_{enc}\left[-\frac{1}{r}\right]_{\infty}^{0.05} = -kQ_{enc}\left(\frac{1}{.05} - 0\right) = (9\times10^9)(1.61\times10^{-8}C)\left(\frac{1}{.05}\right)$$

$$= 2898 V$$

$$= \boxed{2.90\times10^3 V}$$

Question 2 continues on the next page.

(e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

i. Calculate the magnitude of the initial acceleration of the proton.

$\Sigma F = F_E$ $\qquad$ $F_E = Eq$

$ma = Eq$

$a = \frac{Eq}{m} = \frac{(5.79 \times 10^{4}\ N/C)(1.6 \times 10^{-19}\ C)}{1.67 \times 10^{-27}\ kg} = \boxed{5.55 \times 10^{12}\ m/s^2}$

ii. Calculate the speed of the proton after a long time.

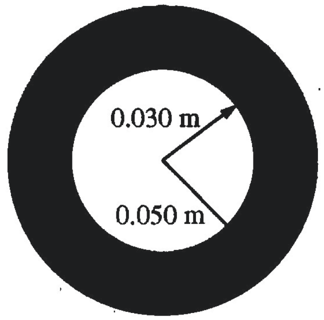
$U_{E_i} + \cancel{K_i} = \cancel{U_{E_f}} + K_f$

$qV = \frac{1}{2}mv^2$

$(1.6 \times 10^{-19} C)(2.90 \times 10^{3} V) = \frac{1}{2}(1.67 \times 10^{-27} kg)(v_p)^2$

$\boxed{v_p = 7.45 \times 10^{5}\ m/s}$

GO ON TO THE NEXT PAGE.



2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030\text{ m}: \quad \rho = 0$$

$$0.030\text{ m} < r < 0.050\text{ m}: \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6}\ \text{C/m}^2$$

$$r > 0.050\text{ m}: \quad \rho = 0$$

(a) Calculate the total charge of the sphere.

$\cancel{V = \frac{4}{3}\pi R^3}$

$\cancel{V = \frac{4}{3}\pi(.05)^3 - \frac{4}{3}\pi(.03)^3 = 4.105\times10^{-4}\ m^3}$

$S = 4\pi R^2$

$4\pi \int_{.03}^{.05} (r)^2\, dr$

$4\pi \int_{.03}^{.05} (b/r)^2\, dr = \boxed{4.29\times10^{-10}\ C}$

GO ON TO THE NEXT PAGE.

(b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

$$EA = \frac{Q}{\varepsilon_0}$$

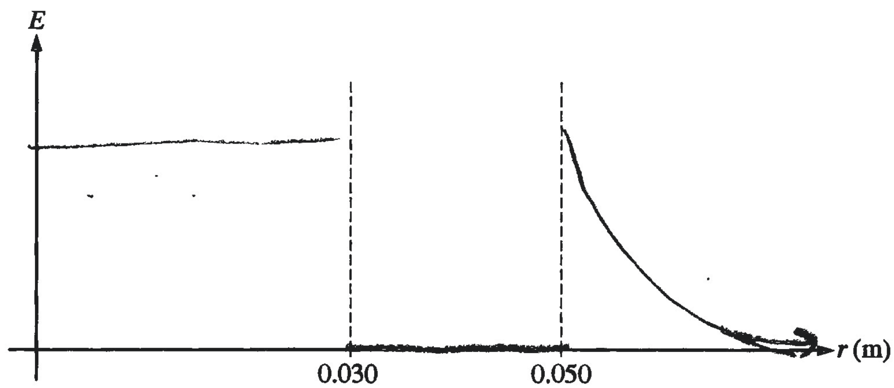
$$Q = 4.29 \times 10^{-10} C$$

$$E = \frac{Q}{A \varepsilon_0}$$

$$A = 4\pi(.05)^2$$

$$\boxed{E = 1543 \frac{N}{C}}$$

(c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



(d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

$$V = k\frac{q}{r}$$

$$q = 4.29 \times 10^{-10} C$$

$$r = .05$$

$$\boxed{V = 77.22 V}$$

Question 2 continues on the next page.

GO ON TO THE NEXT PAGE.

(e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

i. Calculate the magnitude of the initial acceleration of the proton.

$F = Eq = ma$

$1543(e) = (1.67 \times 10^{-27})\, a$

$\boxed{a = 1.48 \times 10^{11}\ \text{m/s}^2}$

ii. Calculate the speed of the proton after a long time.

$U_E = k$

$U_E = qV \qquad k = \frac{1}{2} m v^2$

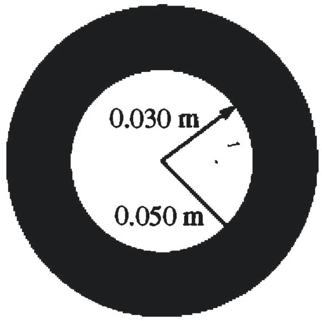
$qV = \frac{1}{2} m v^2$

$e(77.22) = \frac{1}{2}(1.67 \times 10^{-27})\, v^2$

$v^2 = \cancel{1.4815 \times 10^{11}}$

$\boxed{v = 121717.5\ \text{m/s}}$

GO ON TO THE NEXT PAGE.



2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m:} \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m:} \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m:} \quad \rho = 0$$

(a) Calculate the total charge of the sphere.

$\rho = \frac{\Delta Q}{\Delta V}$

$\int_0^Q dQ = \int_{.03}^{.05} dV \left(\frac{b}{r}\right)$

$Q = \int_{.03}^{.05} \frac{4\pi r^2 b}{\cancel{r}} dr$

$Q = 2\pi r^2 b \Big|_{.03}^{.05} = \boxed{1.61 \times 10^{-8} \text{ C}}$

(b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

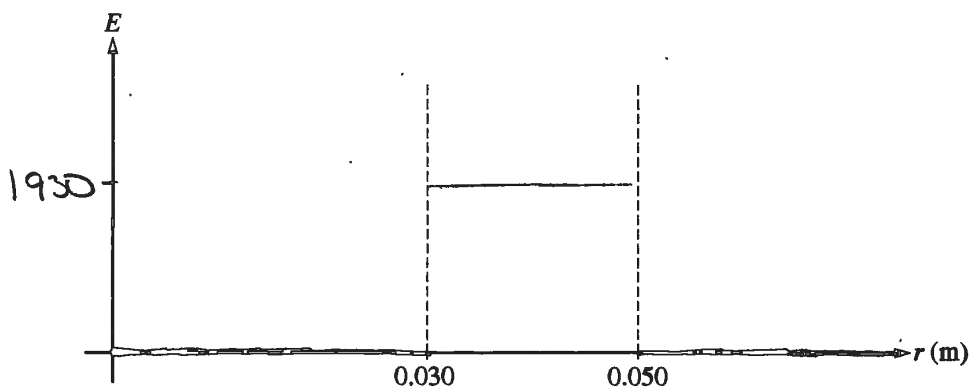
$$E = \frac{Q}{4\pi (.r)^2 \varepsilon_0} =$$

$$E = \int_{.03}^{.05} \frac{Q}{4\pi r^2 \varepsilon_0} dr$$

$$E = \int_{.03}^{.05} \frac{KQ}{r^2} dr = -\frac{KQ}{r}\Big|_{.03}^{.05} = \frac{KQ}{r}\Big|_{.05}^{.03} =$$

$$\boxed{1930 \frac{N}{C}}$$

(c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



(d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

$$VE = -\frac{dV}{dx}$$

$$1930 r\Big|_{.05} = V = \boxed{-57.9 V}$$

Question 2 continues on the next page.

GO ON TO THE NEXT PAGE.

(e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

i. Calculate the magnitude of the initial acceleration of the proton.

$P_{e^-} = KE$

$\frac{kqq}{r} = \frac{1}{2}mv^2$

$\sqrt{\frac{2kqq}{rm}} = \frac{dv}{dt} = a =$

ii. Calculate the speed of the proton after a long time.

$|v| = \left|\frac{2kqq}{rm}\right| = \left|\frac{2ke(1.6\times10^{-8})}{(.05)(1.6\times10^{-22})}\right| =$

$\boxed{3.44\times10^{24}}$

Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses to this question were expected to demonstrate the following:

- Understand and apply Gauss's law to situations with variable charge density and unusual geometry
- Use calculus in determining the total charge of an object with variable charge density
- Interpret quantitative results and illustrate their functional behavior through sketching
- Understand and apply the concept of electric potential and electric potential difference
- Apply Newton's laws and Conservation of Energy concepts to the behavior of electrostatic charge
- Calculate values of E and V and use appropriate units

Sample: E Q2 A
Score: 15

All parts of this response earned full credit. Part (a) has correct substitutions of ρ and dV and integrates with appropriate limits, so 3 points were earned. Part (b) correctly substitutes into Gauss's law and has an answer consistent with part (a), so 3 points were earned. Part (c) has a correct graph, so 3 points were earned. Part (d) substitutes the correct charge and radius into an equation for potential difference, so 2 points were earned. Part (e)(i) correctly substitutes into an appropriate expression for Newton's second law, so 2 points were earned. Part (e)(ii) correctly substitutes into an appropriate expression for potential difference, so 2 points were earned.

Sample: E Q2 B
Score: 9

Parts (b), (d), and (e)(i) earned full credit, 3 points, 2 points, and 2 points, respectively. Part (a) uses correct limits on the integration but uses an incorrect equation and insufficient substitution, so 1 point was earned. Part (c) does not set $E = 0$ for $r < 0.030$ m, does not have a concave down curve for $0.030 \text{ m} < r < 0.050$ m, and has no continuity at $r = 0.050$ m, so no points were earned. Part (e)(ii) correctly substitutes into an appropriate equation but does not indicate the use of a potential difference, so 1 point was earned.

Sample: E Q2 C
Score: 5

Part (a) earned full credit, 3 points. Part (b) uses a correct area, but incorrectly evaluates Gauss's law, so 1 point was earned. Part (c) sets $E = 0$ for $r < 0.030$ m, but the curve is not concave down for $0.030 \text{ m} < r < 0.050$ m and is not concave up for $r = 0.050$ m, so 1 point was earned. Part (d) uses an incorrect method for calculating the electric potential, so no points were earned. Part (e)(i) has no expression for Newton's second law, so no points were earned. Part (e)(ii) uses an incorrect equation, so no points were earned.