2021

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Sample Student Responses and Scoring Commentary

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Question 3: Free-Response Question — 15 points

(a) For using Faraday's law to relate emf to change in flux — **1 point**

$$\mathcal{E} = \left|\frac{d\Phi}{dt}\right| = \left|\frac{d(B \cdot A)}{dt}\right|$$

For a correct expression for the emf — **1 point**

$$\mathcal{E} = \left|A\frac{dB}{dt}\right| = \left|A\frac{d}{dt}\left(\frac{\beta}{t}\right)\right| = \left|-\frac{A\beta}{t^2}\right| = \frac{A\beta}{t^2}$$

For correctly substituting into Ohm's law — **1 point**

$$I = \frac{\Delta V}{R} = \frac{\mathcal{E}}{R} = \frac{A\beta}{Rt^2}$$

Scoring note: Full credit is earned if the negative sign is included.

Total for part (a) — 3 points

(b) For using a correct equation to calculate the energy dissipated in the ring — **1 point**

$$E = \int P dt = \int I^2 R dt$$

For correctly substituting into the equation above — **1 point**

$$E = \int\left(\frac{A\beta}{Rt^2}\right)^2 R dt = \int \frac{A^2\beta^2}{R^2t^4} R dt$$

For indicating the correct limits or constant of integration — **1 point**

$$E = \frac{A^2\beta^2}{R}\int_1^2 \frac{1}{t^4} dt$$

For a correct answer with units — **1 point**

$$E = \frac{A^2\beta^2}{R}\left[-\frac{1}{3t^3}\right]_1^2 = \frac{\left(0.50 \text{ m}^2\right)^2 (0.50 \text{ T}\bullet\text{s})^2}{(3)(2.00\ \Omega)}\left(-\frac{1}{2^3} - \left(-\frac{1}{1^3}\right)\right) = 0.0091 \text{ J}$$

Total for part (b) — 4 points

(c) For selecting "Less than" and attempting a relevant justification — **1 point**

For a correct justification — **1 point**

Example response for part (c)

After the ring rotates, fewer magnetic field lines pass through the ring, and the flux is less than before. Thus, the emf, current, and rate at which energy is dissipated is less.

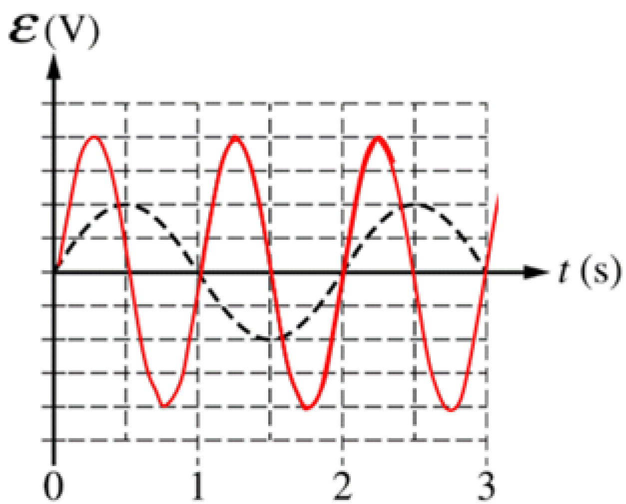
Total for part (c) — 2 points

(d) For using a correct equation to calculate angular speed — **1 point**

$$\omega = \frac{2\pi}{T}$$

	For substituting appropriate values from the graph in the correct equation $\omega = \dfrac{(2\pi)}{(2\text{ s})} = 3.14\ \dfrac{\text{rad}}{\text{s}}$	**1 point**
	Total for part (d)	**2 points**
(e)	For using a correct equation for solving for the emf in terms of the angular speed $\mathcal{E} = \dfrac{d\Phi}{dt} = \dfrac{d(B \cdot A)}{dt} = BA\dfrac{d}{dt}(\cos(\omega t)) = -BA\omega\sin(\omega t)$	**1 point**
	For correctly substituting into the correct equation $\mathcal{E}_{\text{MAX}} = BA\omega = (0.50\text{ T})(0.50\text{ m}^2)\left(3.14\ \dfrac{\text{rad}}{\text{s}}\right) = 0.79\text{ V}$	**1 point**
	Total for part (e)	**2 points**
(f)	For a sine curve with half the period of the dashed curve and a correct justification	**1 point**
	For a sine curve with twice the amplitude of the dashed curve and a correct justification **OR** a sine curve with an amplitude consistent with work shown in part (e)	**1 point**

Example response for part (f)



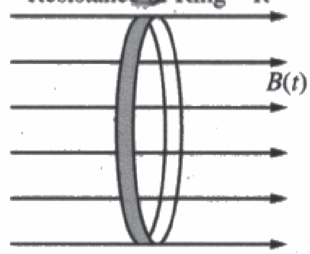
Because the rotational speed is twice the original speed, then, according to the equation $|\mathcal{E}| = B_0 A\omega\sin(\omega t)$ *this will double the amplitude of the curve and cut the period in half.*

Total for part (f) **2 points**

Total for question 3 **15 points**

Begin your response to **QUESTION 3** on this page.

Area of Ring = A
Resistance of Ring = R



3. A thin, conducting ring of area A and resistance R is aligned in a uniform magnetic field directed to the right and perpendicular to the plane of the ring, as shown. At time $t = 0$, the magnitude of the magnetic field is B_0. At $t = 1$ s, the magnitude of the magnetic field begins to decrease according to the equation $B(t) = \frac{\beta}{t}$, where β has units of T·s.

(a) Derive an equation for the magnitude of the induced current I in the ring as a function of t for $t > 1$ s. Express your answer in terms of β, A, R, t, and physical constants, as appropriate.

$$|\mathcal{E}| = \left|\frac{\partial \Phi_B}{\partial t}\right| = \frac{\partial}{\partial t}\left(\int B \cdot dA\right) = \left|\frac{d}{dt} BA\right| = A\frac{dB}{dt} = \left|-A\frac{\beta}{t^2}\right| \doteq A\beta/t^2$$

$$\mathcal{E}/R = i \Rightarrow \frac{A\beta}{Rt^2}$$

Assume $A = 0.50\,\text{m}^2$, $R = 2.0\,\Omega$, and $\beta = 0.50\,\text{T}\cdot\text{s}$.

(b) Calculate the electrical energy dissipated in the ring from $t = 1$ s to $t = 2$ s.

$P = I^2R$ = electrical energy dissipated

$$\int P\,dt = \int I^2 R\,dt = \int \frac{A^2\beta^2}{Rt^4}\,dt$$

$$\frac{A^2\beta^2}{R}\int_{t=1}^{t=2} \frac{1}{t^4}\,dt$$

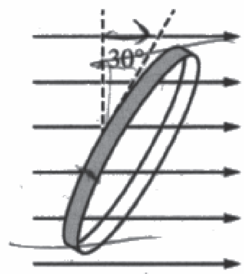
$$\frac{A^2\beta^2}{R}\left(-\frac{4}{t^5}\right)\Big|_1^2$$

0.121 J

$$= \frac{(0.5)^2(0.5)^2}{2}\left(\frac{4}{1} - \frac{4}{32}\right)$$

$$P = \left(\frac{A\beta}{Rt^2}\right)R = \frac{A^2\beta^2}{Rt^4}$$

Continue your response to **QUESTION 3** on this page.



The ring is then rotated so that the plane of the ring is aligned at a 30° angle to the magnetic field, as shown.

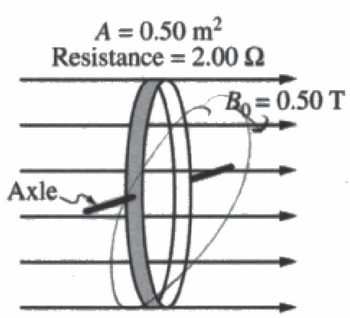
The magnitude of the magnetic field is reset to a magnitude of B_0 at a new time $t = 0$ and again begins to decrease at $t = 1$ s according to the equation $B(t) = \frac{\beta}{t}$, where β has units of T·s.

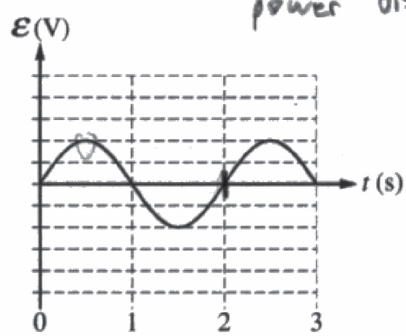
(c) Will the amount of energy dissipated in the ring from $t = 1$ s to $t = 2$ s be greater than, less than, or equal to the energy dissipated in part (b) ?

_____ Greater than __✓__ Less than _____ Equal to

Justify your answer.

Now only a portion of the same magnetic field ($B(t)\sin(30)$, to be exact) goes through the loop. This decreases the EMF, the current, and thus the power dissipated.





The ring is now mounted on an axle that is perpendicular to the magnetic field. The magnitude of the magnetic field is now held at a constant $B_0 = 0.50\,\text{T}$, as shown. The ring rotates about the axle, and the emf ε induced in the ring as a function of time t is shown on the graph.

(d) Calculate the angular speed ω of the rotating ring in rad/s.

Period $T = 2s$ as per 2nd graph $\omega = \frac{2\pi \text{ radians}}{2\text{ s}} = \pi \frac{\text{rad}}{\text{sec}}$

Continue your response to **QUESTION 3** on this page.

(e) Calculate the magnitude of the maximum emf ε_{MAX} induced in the ring.

$\mathcal{E} = \frac{d\Phi_B}{dt} = B\frac{dA}{dt}$

$A = A\cos(\omega t)$

$\frac{dA}{dt} = -A\sin(\omega t)(\omega)$

$\mathcal{E} = BA\omega\sin(\omega t)$

MAX? $\omega t = \pi/2$

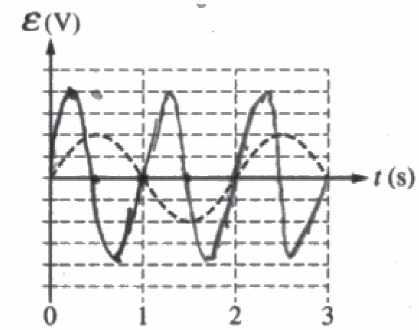
$t = 1/2$

$\omega = \pi$

$0.785\text{ V} = (0.5)(0.5)(\pi)(1) = \pi/4$

The ring now begins to rotate at an angular speed 2ω.

(f) On the graph below, draw a curve to indicate the new induced emf ε in the ring. The dashed curve shows the emf induced under the original conditions.



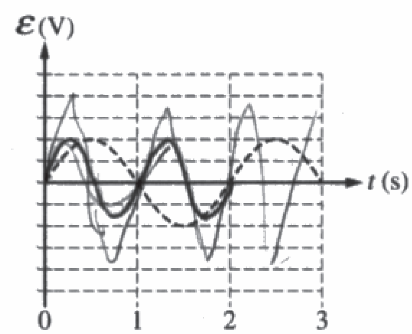
$\mathcal{E} = BA\omega\sin\omega t$

Justify your sketch, specifically identifying and addressing any similarities or differences between the sketch and the original graph.

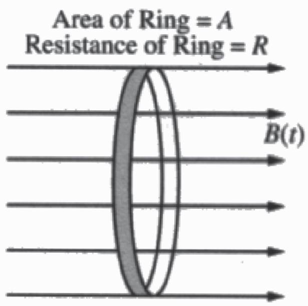
The period will now be halved

The amplitude will double because $\mathcal{E} \propto \omega$. This makes sense because the area will be changing more rapidly, so a greater EMF will be induced

PRACTICE GRAPH - Use the graph below to practice your sketch for part (f). Any work shown on the graph below will NOT be graded.



Begin your response to **QUESTION 3** on this page.



3. A thin, conducting ring of area A and resistance R is aligned in a uniform magnetic field directed to the right and perpendicular to the plane of the ring, as shown. At time $t = 0$, the magnitude of the magnetic field is B_0. At $t = 1$ s, the magnitude of the magnetic field begins to decrease according to the equation $B(t) = \frac{\beta}{t}$, where β has units of T·s.

(a) Derive an equation for the magnitude of the induced current I in the ring as a function of t for $t > 1$ s. Express your answer in terms of β, A, R, t, and physical constants, as appropriate.

$\varepsilon = -\frac{d\phi}{dt}$ $\quad$ $\varepsilon = -\frac{d\frac{\beta}{t}}{dt} A$ $\quad$ $\varepsilon = \frac{\beta}{t^2} A$ $\quad$ $\varepsilon = IR$

$\varepsilon = \frac{d\int \frac{\beta}{t} \cdot dA}{dt}$ $\quad$ t^{-1} $\quad$ $I = \frac{\beta A}{t^2 R}$

Assume $A = 0.50\,\text{m}^2$, $R = 2.0\,\Omega$, and $\beta = 0.50\,\text{T}\cdot\text{s}$.

(b) Calculate the electrical energy dissipated in the ring from $t = 1$ s to $t = 2$ s.

$\Delta E = P\Delta t$ $\quad$ $\Delta E = \frac{\beta^2 A^2}{R^2} \left. -\frac{t^{-3}}{3} \right|_1^2$

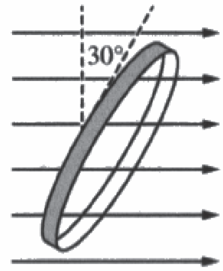
$\Delta E = IV$ $\quad$ ΔE

$\Delta E = I^2 R$

$\Delta E = \int \frac{\beta^2 A^2}{t^4 R^2} R\, dt$

$\Delta E = \frac{\beta^2 A^2}{R^2} \int_1^2 \frac{1}{t^4}\, dt$

Continue your response to **QUESTION 3** on this page.



The ring is then rotated so that the plane of the ring is aligned at a 30° angle to the magnetic field, as shown.

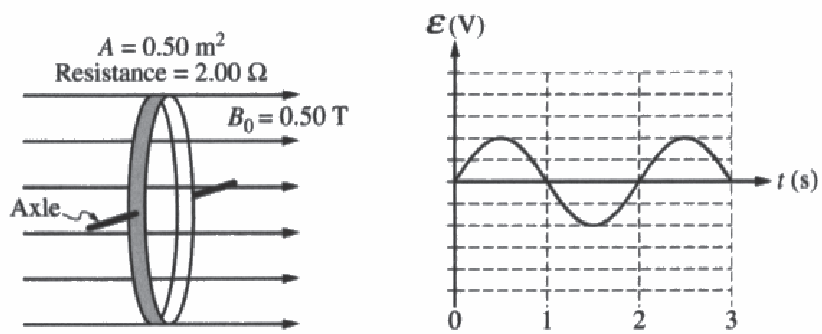
The magnitude of the magnetic field is reset to a magnitude of B_0 at a new time $t = 0$ and again begins to decrease at $t = 1$ s according to the equation $B(t) = \frac{\beta}{t}$, where β has units of T·s.

(c) Will the amount of energy dissipated in the ring from $t = 1$ s to $t = 2$ s be greater than, less than, or equal to the energy dissipated in part (b) ?

____ Greater than   _X_ Less than   ____ Equal to

Justify your answer.

There is less flux, therefore less change in flux so less current so less power dissipated



The ring is now mounted on an axle that is perpendicular to the magnetic field. The magnitude of the magnetic field is now held at a constant $B_0 = 0.50$ T, as shown. The ring rotates about the axle, and the emf ε induced in the ring as a function of time t is shown on the graph.

(d) Calculate the angular speed ω of the rotating ring in rad/s.

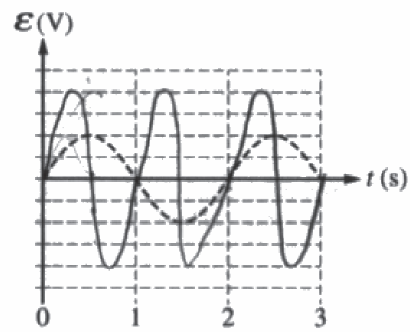
Continue your response to **QUESTION 3** on this page.

(e) Calculate the magnitude of the maximum emf ε_{MAX} induced in the ring.

$\mathcal{E}_{max} = -\frac{d\Phi}{dt}$ $\mathcal{E}_{max} = \frac{d \cos\theta}{dt} \cdot B \cdot A$ $\mathcal{E}_{max} = \frac{d}{dt}\cos\theta \cdot .5 \cdot .5$

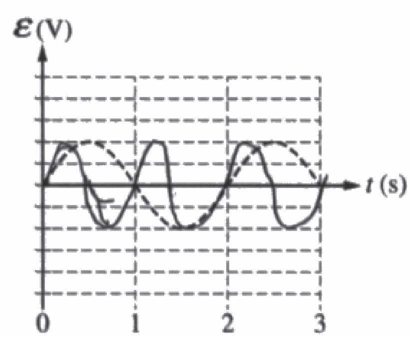
The ring now begins to rotate at an angular speed 2ω.

(f) On the graph below, draw a curve to indicate the new induced emf ε in the ring. The dashed curve shows the emf induced under the original conditions.

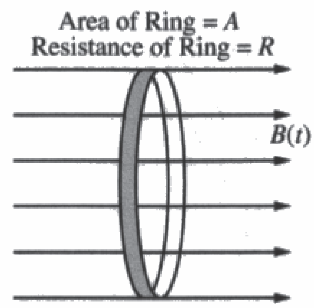


Justify your sketch, specifically identifying and addressing any similarities or differences between the sketch and the original graph.

PRACTICE GRAPH - Use the graph below to practice your sketch for part (f). Any work shown on the graph below will NOT be graded.



Begin your response to **QUESTION 3** on this page.



3. A thin, conducting ring of area A and resistance R is aligned in a uniform magnetic field directed to the right and perpendicular to the plane of the ring, as shown. At time $t = 0$, the magnitude of the magnetic field is B_0. At $t = 1$ s, the magnitude of the magnetic field begins to decrease according to the equation $B(t) = \frac{\beta}{t}$, where β has units of T·s.

(a) Derive an equation for the magnitude of the induced current I in the ring as a function of t for $t > 1$ s. Express your answer in terms of β, A, R, t, and physical constants, as appropriate.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\vec{r}}{r^2}$$

$$B(t) = \frac{\beta}{t} \rightarrow \beta t^{-1}$$

$$\frac{dB}{dt}(-)\beta t^{-2}$$

$$-\beta t^{-2} = \frac{\mu_0}{4\pi}\frac{I}{r}$$

$$r\left(\frac{-\beta(4\pi)}{t^2\mu_0}\right) = I$$

$$\boxed{\frac{-4\pi\beta R}{t^2\mu_0} = I}$$

Assume $A = 0.50\,\mathrm{m}^2$, $R = 2.0\,\Omega$, and $\beta = 0.50\,\mathrm{T\cdot s}$.

(b) Calculate the electrical energy dissipated in the ring from $t = 1$ s to $t = 2$ s.

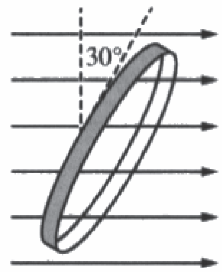
$q >$

$\boxed{10\,\mathrm{J}}$

$$\oint B\cdot d\vec{l} = \mu_0 I$$

$$F_x = -\frac{dV}{dx}$$

Continue your response to **QUESTION 3** on this page.



The ring is then rotated so that the plane of the ring is aligned at a 30° angle to the magnetic field, as shown.

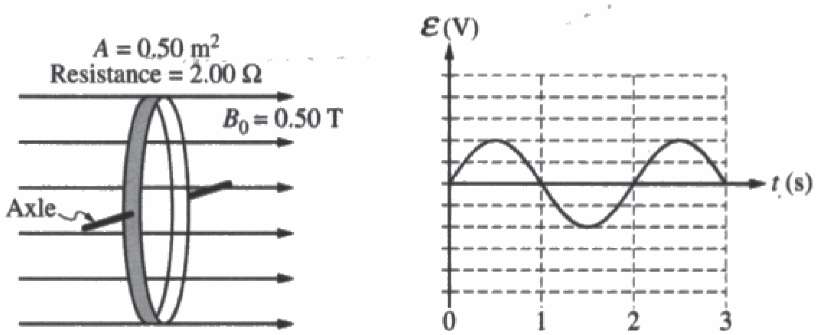
The magnitude of the magnetic field is reset to a magnitude of B_0 at a new time $t = 0$ and again begins to decrease at $t = 1$ s according to the equation $B(t) = \frac{\beta}{t}$, where β has units of T·s.

(c) Will the amount of energy dissipated in the ring from $t = 1$ s to $t = 2$ s be greater than, less than, or equal to the energy dissipated in part (b) ?

_____ Greater than ✓ Less than _____ Equal to

Justify your answer.

The amount of energy dissipated will be less than before because now the B field will be multiplied by the cos 30°.



The ring is now mounted on an axle that is perpendicular to the magnetic field. The magnitude of the magnetic field is now held at a constant $B_0 = 0.50$ T, as shown. The ring rotates about the axle, and the emf ε induced in the ring as a function of time t is shown on the graph.

(d) Calculate the angular speed ω of the rotating ring in rad/s.

$W = \frac{2\pi}{T}$ $W = \frac{2\pi}{2}$ $W = \pi$ rad/s

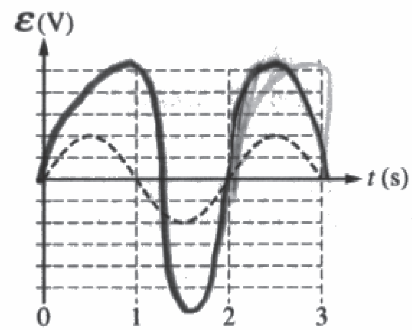
Continue your response to **QUESTION 3** on this page.

(e) Calculate the magnitude of the maximum emf ε_{MAX} induced in the ring.

$\varepsilon_{max} = \boxed{10V}$

The ring now begins to rotate at an angular speed 2ω.

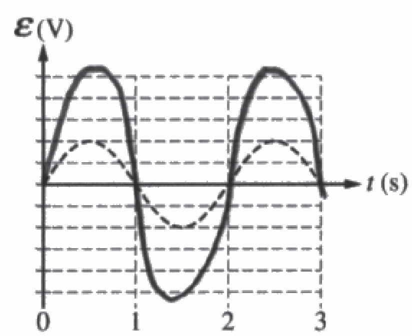
(f) On the graph below, draw a curve to indicate the new induced emf ε in the ring. The dashed curve shows the emf induced under the original conditions.



Justify your sketch, specifically identifying and addressing any similarities or differences between the sketch and the original graph.

The amplitude of this graph is doubled due to the doubling of the angular velocity, but the period and frequency are the same because omega has no effect on the oscillation to shape.

PRACTICE GRAPH - Use the graph below to practice your sketch for part (f). Any work shown on the graph below will NOT be graded.



Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate understanding of:

- The concept of induced emf.
- How to calculate magnetic flux when the magnetic field is time dependent.
- How to calculate energy when power is time dependent.
- How the magnetic flux is affected by a change in relevant variables.
- The relationship among frequency, period, and amplitude for a physical quantity with a sinusoidal time dependence and their graphical representation.

The responses were expected to demonstrate the ability to:

- Support a claim with a correct and complete explanation based on clear physical reasoning.
- Perform calculations involving calculus to provide correct numerical answers with correct units.
- Understand the relationship between parameters in an equation and characteristics of a graph.
- Deduce the effect of changing a variable on the behavior of a physical system.

Sample: E Q3 A
Score: 14

Part (a) earned 3 points. The response earned the first point for using Faraday's law with an expression for the magnetic flux. The response earned the second point for a correct expression for the emf. The response earned the third point for correctly substituting into Ohm's law. Part (b) earned 3 points. The response earned the first point for using a correct equation to calculate the energy dissipated in the ring. The response earned the second point for correctly substituting into the equation relating energy and power. The response earned the third point for indicating the correct limits for the integration. The response did not earn the fourth point because the numerical answer is not correct. Part (c) earned 2 points. The response earned the first point for selecting "Less than" and attempting a relevant justification. The response earned the second point for a correct justification. Part (d) earned 2 points. The response earned the first point for using a correct equation to calculate angular speed. The response earned the second point for substituting an appropriate value from the graph into the correct equation. Part (e) earned 2 points. The response earned the first point for using a correct equation for solving for the emf in terms of the angular speed. The response earned the second point for correctly substituting into the correct equation for the maximum emf. Part (f) earned 2 points. The response earned the first point for a sine curve with half the period of the dashed curve and a correct justification. The response earned the second point for a sine curve with twice the amplitude of the dashed curve and a correct justification.

Question 3 (continued)

Sample: E Q3 B
Score: 7

Part (a) earned 3 points. The response earned the first point for using Faraday's law with an expression for the magnetic flux. The response earned the second point for a correct expression for the emf. The response earned the third point for correctly substituting into Ohm's law. Part (b) earned 3 points. The response earned the first point for using a correct equation to calculate the energy dissipated in the ring. The response earned the second point for correctly substituting into the equation relating energy and power. The response earned the third point for indicating the correct limits for the integration. The response did not earn the fourth point because it does not include the correct answer. Part (c) earned 1 point. The response earned the first point for selecting "Less than" and attempting a relevant justification. The response did not earn the second point because the justification is incomplete. Part (d) earned no points. The response did not earn any points because the response is blank. Part (e) earned no points. The response did not earn the first point because the equation is incorrect. The response did not earn the second point because it does not include a correct substitution into the correct equation. Part (f) earned no points. The response did not earn the first point because it does not include a sine curve with half the period and a correct justification. The response did not earn the second point because it does not include a sine curve with twice the amplitude and a correct justification.

Sample: E Q3 C
Score: 3

Part (a) earned no points. The response did not earn the first point because it does not use Faraday's law to relate emf to change in flux. The response did not earn the second point because there is not a correct expression for the emf. The response did not earn the third point because it does not use Ohm's law to calculate the I. Part (b) earned no points. The response did not earn the first point because it does not use a correct equation to calculate the energy dissipated in the ring. The response did not earn the second point because there is not a substitution into an equation for energy. The response did not earn the third point because there is not an integration where the limits can be identified. The response did not earn the fourth point because it does not find the correct answer. Part (c) earned 1 point. The response earned the first point for selecting "Less than" and attempting a relevant justification. The response did not earn the second point because the justification is incomplete. Part (d) earned 2 points. The response earned the first point for using a correct equation to calculate angular speed. The response earned the second point for substituting an appropriate value from the graph into the correct equation. Part (e) earned no points. The response did not earn the first point because it does not use a correct equation for the emf in terms of angular speed. The response did not earn the second point because it does not correctly substitute into the correct equation for the maximum emf. Part (f) earned no points. The response did not earn the first point because it does not have a sine curve with half the period and a correct justification. The response did not earn the second point because it does not have a sine curve with twice the amplitude and a correct justification.