Chief Reader Report on Student Responses:

2022 AP® Physics C: Electricity and Magnetism Set 1 Free-Response Questions

• Number of Students Scored	19,978		
• Number of Readers	471 (for all Physics exams)		
• Score Distribution	Exam Score	N	%At
	5	6,301	31.5
	4	4,717	23.6
	3	2,855	14.3
	2	3,608	18.1
	1	2,497	12.5
• Global Mean	3.44		

The following comments on the 2022 free-response questions for AP® Physics C: Electricity and Magnetism were written by the Chief Reader, Brian Utter, Teaching Professor, University of California, Merced. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student preparation in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question 1

Task: Short Answer
Topic: Electrostatics; Electric Potential and Field
Max Score: 15
Mean Score: 6.33

What were the responses to this question expected to demonstrate?

The responses were expected to demonstrate the ability to:

- Determine the distribution of charge on the surfaces of a conductor in electrostatic equilibrium in the presence of other charges.
- Use Gauss's law and spherical symmetry to determine the electric field inside and outside an insulator with uniform charge density.
- Use proportional reasoning to relate the magnitude of the electric field at two points based on the functional dependence of E on r.
- Calculate the potential difference between two points by integrating the electric field over a distance.
- Sketch graphs of the electric field and electric potential with respect to radial position for static spherically symmetric charge distributions.

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Most students were able to determine the charge on the outer surface of the conducting shell by understanding how the charge on the inner sphere polarizes the conducting shell.
- Many students did not recognize the need to set up a ratio to find the charge enclosed by the Gaussian surface, instead simply setting q_{enc} to be Q.
- Most students were able to identify the surface area of a sphere of radius r as the proper area to use in Gauss's law, although a few used the surface area of the insulating sphere ($4\pi R^2$) rather than the surface area of the Gaussian surface ($4\pi r^2$).
- A majority of students recognized that outside a spherically symmetric charge distribution, the electric field has an inverse-square relationship with r, which allowed them to correctly divide the electric field by 4 when the distance was doubled.
- Many students recognized the need to integrate the electric field over distance to find the potential difference between two points, $r = R$ and $4R$. A small number of students integrated from infinity rather than from $r = R$ to $4R$. There were two common incorrect expressions for electric field substituted into the integral. Some used the expression for the electric field inside the conductor (for $r < R$) instead of the electric field for $R < r < 4R$. Others used expressions for E found in the previous part, which asked for E at $r = 2R$. These expressions contained the constant R rather than the variable r, leading them to incorrectly assume that E was uniform in the region.
- In an alternate solution for determining the potential difference, some students recognized that the potential outside a spherically symmetric charge distribution could be treated like the potential due to a point charge. Students who did this correctly found the difference in the potential due to the charge on the insulating sphere at two different points, $r = R$ and $r = 4R$. Students who did this incorrectly added the potential due to the insulating sphere (with $q = -Q$ and $r = R$) and an incorrect term that treated the conducting shell as another point-charge source of potential (with $q = 3Q$ and $r = 4R$).
- Many students did not recognize that the opposite signs of the charges of the insulating sphere and the conducting shell would cause the electric field to change direction from Regions I and II ($r < 4R$) to Region III ($r > 4R$). Many students drew the electric field graph as positive in all three regions.
- Some students demonstrated their understanding of the inverse-square dependence of E on r in Regions II and III in the parts of the question that asked for calculations, yet did not represent that dependence graphically, drawing lines that were not asymptotic to the horizontal axis or that were not always decreasing in magnitude.

- Most students were not able to successfully draw the potential graph. It was clear from the variety of responses that many students did not know how to approach this graph.
- Derivations typically start with relationships taken directly from the formula sheet, but to earn credit, students must *use* the equation by substituting appropriate values or expressions. Many low-scoring responses simply listed the starting equation without showing any additional work.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Responses in which an explicit integral over area was not performed when using Gauss's law.	• A Gaussian surface on which the magnitude of E was uniform is chosen so that the integral became the product of E and the surface area of the Gaussian surface.
• Belief that the relationship $V = Ed$ applies in all situations, even when E is not uniform.	• Recognizing that when E depends on r, the electric field must be integrated in order to find the change in electric potential.
• Both R (a given constant value) and r (a variable) were used indiscriminately when finding the potential function using the integral of the electric field.	• Correct responses used the variable r in the integrand and differential element dr and used R and $4R$ as the limits of integration.
• Vertical dashed lines given as part of the graph prompts were provided to separate Regions I, II, and III and were not intended to indicate vertical asymptotes. Many responses clearly reflected student belief that these lines must be vertical asymptotes on the graph.	• In better responses, students realized that the dotted lines did not indicate vertical asymptotes, so they carefully connected their graph from region to region to indicate continuity when appropriate. In other cases, correct graphs had non-zero, non-asymptotic values at the boundaries between regions.
• When sketching graphs, students did not carefully indicate continuity, asymptotic behavior, or increasing/decreasing trends. They did not clearly distinguish between linear graphs and those that were meant to be concave up or concave down.	• For continuous quantities, such as potential, responses should clearly connect the segments of the graphs from region to region. Nonlinear graph segments should be either clearly concave up or concave down, and linear segments drawn with a straightedge to avoid ambiguity.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

- For questions that use the verb "derive," ask students to write an equation from the formula sheet or another fundamental relationship, such as conservation of energy, *before* substituting into that equation.
  - While a student cannot earn credit on the AP Exam for simply writing down an equation from the formula sheet, in-class assignments and assessments may assign point values to this step to encourage students to start with a fundamental principle.
- Linear dependence should be drawn with a ruler. All students are allowed a ruler on the AP Exam for exactly this purpose. Scoring guidelines may require segments to be linear, concave up, or concave down, and these shapes are easy to distinguish when linear segments are drawn with a ruler.
  - Have a supply of rulers readily available in class and require their use on problems that ask for a best-fit line or a sketch of a graph.
  - Talk to your AP Coordinator in the fall about providing an adequate supply of clear rulers for all physics test-takers, giving them time to order supplies if needed.

Question 2

Task: Experimental Design
Topic: RC Circuits
Max Score: 15
Mean Score: 7.25

What were the responses to this question expected to demonstrate?

The responses were expected to demonstrate the ability to:

- Identify the behavior of capacitors in circuits, specifically the properties of charging and discharging RC circuits, including their time dependence.
- Draw a circuit diagram that allows a capacitor to be charged and then discharged through a resistor and ammeter using given circuit elements.
- Use Kirchoff's and Ohm's laws to write a differential equation for a discharging RC circuit that can be integrated to determine the voltage across the capacitor as a function of time.
- Associate the parameters in an equation for an RC circuit with the characteristics of a corresponding graph.
- Use a graph to determine the capacitance of a capacitor using the slope of the line and an equation for the voltage across a discharging capacitor.
- Provide reasoning to justify a claim concerning the changes of the slope and intercept of the graph based on a model for the capacitor circuit.

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Fewer than half of the students were able to draw a circuit that both charged and discharged. Of the *incorrect* circuits, most simply included all the components into a single loop, making a circuit that charged but did not discharge.
- More than half of the students knew that a voltmeter must be placed in parallel with a capacitor or resistor, even if they weren't able to draw an appropriate circuit.
- Most students did not start with a loop rule and Ohm's law when trying to derive a differential equation for the discharge RC circuit. If they did start with Ohm's law, they were usually able to continue with the correct derivation.
- Most students didn't use the slope of the graph to find the desired quantities. Instead, they substituted the values from a point on the line into an equation given for the problem.
- Many of the students didn't understand that the slope of a graph is determined by a combination of parameters. As a consequence, they weren't able to provide reasoning as to what would happen to the slope if one of the experimental parameters changed.
- Most students showed an understanding of the time dependence of RC circuits.
- Most students were able to associate the data given with equations that describe an RC circuit.
- Most students were *not* able to predict how the behavior of the discharge circuit would change if the charging mechanism was changed.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• A capacitor can charge and discharge in the same single-loop circuit.	• Good responses show a switch between the charging and discharging circuit.
• Voltmeters can be connected in series.	• A voltmeter is connected in parallel to the capacitor.
• An experimental value for the unknown capacitance is obtained by plugging a single data point into the physics relationship that relates the values plotted on the axes.	• Good responses used the entire data set by calculating the slope of the best-fit line using two points on the line and relating that slope to the unknown capacitance.
• A common knowledge gap was how to begin the derivation of the voltage as a function of time. Many simply worked backward from the given function.	• Kirchhoff's loop rule can be written for a loop in the circuit. An explicit substitution to relate current to charge on the capacitor, such as $I = dq/dt$, then allows a differential equation to be expressed in terms of a single variable as a function of time.
• The charging parameters of an RC circuit affect the rate of discharge. *(Note: this error could have come from students not carefully reading the problem.)*	• High-scoring responses showed evidence that the student read and understood the experimental setup. They understood that the battery was used to charge the capacitor before the experiment began and that the internal resistance of the battery was not relevant during the capacitor's discharge because the battery was not connected in the circuit at that time. The slope of the line depends on RC, where R is the resistance of the resistor in the discharge circuit.
• A slope consists of the ratio of the two variables in any graph. Many students wrote the slope by dividing the y-axis label by the x-axis label, instead of deriving it from an equation relating the quantities.	• A good response applied a given equation to the graphed quantities and manipulated the equation to obtain the slope in terms of constants, in this case the resistance and capacitance of the circuit.
• An experimental value for the unknown capacitance is obtained by plugging a single data point into the relationship for the values plotted on the axes.	• Better responses calculated the slope of the best-fit line using two points on the line and related that slope to the unknown capacitance.
• The parameters of an RC circuit associated with the charging loop affect the rate of discharge.	• Good responses pointed out that the charging voltage is removed from the circuit before discharge and did not affect discharging.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

- For questions that require data analysis, students need to find a numerical value of the slope from points on the best-fit line and equate that to the appropriate quantity in an appropriate equation relating the variables plotted.
    - At the beginning of the school year, teachers could provide a template for data analysis problems. This template would include a checklist for graphing (labels, units, and linear scales on axes), a section where students use linear regression on a calculator, or an explicit calculation of slope using two clearly marked points on the line to get a numerical value of the slope, and a section where students use a physics relationship to determine a symbolic expression for the slope. The final section would require students to set the numerical slope value to the expression for slope from the equation and solve for the unknown. By the end of the year, students should be able to do this process without the template.
    - If finding a symbolic expression for slope is difficult, encourage students to write down $y = mx + b$ above their physics relationship and rearrange their physics relationship until they can match up each letter in $y = mx + b$ with the appropriate quantities in the problem.
- Points were frequently lost on this problem because students didn't understand the difference between a constant and a variable. Students need to understand this difference.
    - The difference between constants and variables should be pointed out early and often, especially in the context of graphing. When labeling the parts of a linear equation, as described above, point out which quantities are variables and change over the course of the graph and which are constants.
- Drawing circuits is more difficult than analyzing circuits that are already shown on the page. Students should have practice drawing circuits to accomplish specific goals.
    - Have students practice drawing circuits that will complete a given task, such as charging and discharging a capacitor or dimming a lightbulb. Include open-ended labs where students must design and draw a circuit in your class.
    - Exams should have questions that ask students to draw circuits, not just analyze circuits that are provided.

Question 3

Task: Short Answer
Topic: Electromagnetism
Max Score: 15
Mean Score: 4.12

What were the responses to this question expected to demonstrate?

The responses were expected to demonstrate the ability to:

- Solve problems based on the concepts of magnetic induction, including applications of Faraday's law, Lenz's law, magnetic field, and magnetic flux.
- Determine the properties of a magnetic field produced by a current-carrying wire using Ampere's law or by selection of the correct formula.
- Determine the properties of a current induced by a changing magnetic field using Faraday's law and Lenz's law.
- Apply appropriate right-hand rules to determine directions of magnetic forces and fields.
- Use integral calculus to determine the magnetic flux for a stationary loop of wire located in a variable magnetic field.
- Apply Ohm's law.
- Derive expressions by choosing appropriate fundamental equations, substituting relationships specific to the problem, solving for particular variables, and calculating results with correct numerical values from the prompt.
- Make a claim and justify it using physics principles and laws and analyze the effect of sources of error on experimental outcomes.

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Most responses were able to correctly predict the direction of current and induced field for a current loop located in a changing magnetic field, but many responses were unable to provide a justification with sufficient detail to earn justification points from the rubric. In particular, many students failed to distinguish between field and flux when describing the application of Lenz's law. Many students also failed to distinguish between opposition to the field and/or flux and opposition to *changes* in the magnetic flux.
- Most responses demonstrated correct applications of the right-hand rule to find the direction of a magnetic field created by a long current-carrying wire.
- Most responses demonstrated correct applications of the right-hand rule to determine the direction of a magnetic field inside a current-carrying loop of wire.
- Few responses showed the ability to correctly set up an integral version for the determination of magnetic flux in a variable magnetic field. In particular, correctly substituting for the element dA was problematic.
- A number of responses, around half, either did not recall the correct equation for the magnetic field of a long current-carrying wire or were unable to derive it using Ampere's law. The rubric required only that the correct equation be used to set up the integral for flux. Responses did not have to derive the equation, but those that did attempt to derive it often used incorrect substitutions for the element dL in Ampere's law.
- Responses were generally better at applying Faraday's law to the results of the previous flux calculation than in doing the flux calculation itself. Many responses were able to earn all three rubric points for part (c) in spite of having a faulty determination of flux by integration in the previous step.
- In spite of the problems listed above, a relatively large number of responses were able to correctly apply quantitative reasoning to determine the relative effect on the current induced in the loop depending on the rate of change in current in the long current-carrying wire or the orientation of the loop relative to the wire. Many responses correctly stated that a greater rate of change in current induces a greater current in the loop because it causes a greater rate of change in flux. Many responses correctly determined that the same rectangular loop, rotated in a different orientation, causes less induced current in the loop because it results in less flux and a lower rate of change in flux.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• The magnetic field of an induced current always opposes the direction of the magnetic field that caused the induced current.	• The magnetic field of an induced current opposes the <u>*change in magnetic flux*</u> that caused the induced current.
• When using Ampere's law to determine the magnetic field of a current carrying wire, the value of dL is based on the physical dimensions and/or relative positions presented in the prompt stimulus. For example, dL, when integrated, might be the length of a loop of wire.	• Ampere's law involves a path integral such that dL is an infinitesimal part of a closed path. To solve for a magnetic field, the path is chosen at an arbitrary location relative to the current source of the field and aligned in such a way that the integral can be simplified. For a long current-carrying wire, the appropriate path is a circle with an arbitrary radius in a plane normal to the current. The magnetic field about the wire is parallel to this path, and Ampere's law simplifies to: $B(2\pi r) = \mu_0 I$, from which the correct field equation may be determined.
• Determining magnetic flux by integration of BdA can always be simplified to BA.	• In the case of a loop near a wire that carries a changing current, the infinitesimal area element dA should be chosen for a constant value of B, and the value of the integrand B must be represented in terms of an appropriate variable. For example, in this particular question: $B = \mu_0 I/2\pi r$ and $dA = Ldr$.
• Rate of change in flux can be determined by calculating Φ/t .	• Rate of change in flux is determined by $d\Phi/dt$ in all situations. In this particular question, it is also acceptable to equate rate of change with $\Delta\Phi/\Delta t$, because flux varies linearly with respect to time.
• Earth's magnetic field can affect the amount of induced current in a stationary loop.	• Earth's magnetic field undergoes negligible change over time and may be considered to be constant. As such, it does not affect the <u>change</u> in the net magnetic field where some other field is involved. Therefore, it also does not affect the change in magnetic flux for the stationary area of a loop in a scenario such as this. Earth's field does not affect the amount of induced current or emf for a stationary loop or coil of wire.
• The product of magnetic field strength and length of wire determines the amount of induced current in a stationary loop of wire.	• The induced emf and resulting current in a stationary loop of wire is a function of rate of change in magnetic flux for the area bound by the loop. While the area used to determine flux may have a relation to the length of wire, it is not directly proportional to only the length of wire and instead depends also on the shape of the loop, whether it be rectangular, circular, etc.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

Your response should:

- Show work for any integration problem by starting with the most general form of the integral and showing successive substitutions and simplifications step by step.
    - Work lots of examples, explaining step by step each part of the process. Many integrals, like flux, have an infinitesimal element (like dA) that often needs to be put into terms of the relevant variable for the purpose of integration. Stress, with many examples, that it is necessary to identify a single variable for the integrand and then make an appropriate substitution for the infinitesimal element in terms of this same variable. In this question, the magnetic field varies with respect to distance r from the wire, and the element dA is written as Ldr.
    - It is also important to give students adequate opportunities to practice and repeat this process by themselves. The same basic process is used to find center of mass and moment of inertia in mechanics and electric flux, magnetic flux, and several other integrals in electricity and magnetism. Over the course of learning this material, students should become comfortable with the technique. Ask students to evaluate integrals like these during class time and on homework assignments on a regular basis.
    - Emphasize the importance of "setting it up" more than the actual evaluation of the integral. Setting it up correctly requires comprehension and application of physics but also enhances students' appreciation for the necessity of integration and the meaning of the result. Stress an integral is an infinite summation and point out the significance to each scenario. In this particular question, point out that the total magnetic flux through the loop is an infinite sum of quantities of magnetic flux in thin strips across the face of the rectangular coil. It is necessary to integrate because the amount of flux in each of these strips is different due to the variable strength of the magnetic field.
    - Another helpful point of emphasis is to always make a diagram of the infinitesimal element, such as dA.
- Write justifications using complete sentences that incorporate specific physics terms and concepts.
    - Similar to the AP Exam, require that students write an explanation that actually makes sense *as it is written*, rather than what a student "was trying to say." Logical written statements should incorporate correct physics and completely answer the given question at hand.