2022

AP® | CollegeBoard

AP® Physics C: Electricity and Magnetism

Free-Response Questions Set 2

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	1 electron volt,	$1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Speed of light,	$c = 3.00 \times 10^8$ m/s
Avogadro's number,	$N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant,	$G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant,	$R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface,	$g = 9.8\ \text{m/s}^2$
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K		

1 unified atomic mass unit,	$1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\ \text{MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25}\ \text{J}\cdot\text{m} = 1.24 \times 10^{3}\ \text{eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^9\ (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1\text{ atm} = 1.0 \times 10^5\ \text{N/m}^2 = 1.0 \times 10^5\ \text{Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \bullet d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \bullet \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{\max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi \sqrt{\frac{m}{k}}$

$T_p = 2\pi \sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0} \left| \frac{q_1 q_2}{r^2} \right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \bullet d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \bullet d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = N e v_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \bullet d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \bullet d\vec{A}$

$\mathcal{E} = \oint \vec{E} \bullet d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L \frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

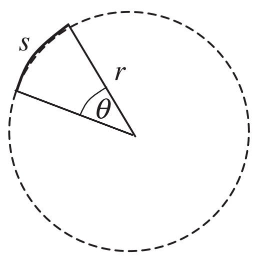
$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle



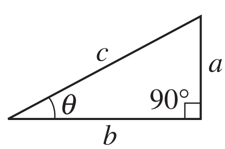
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

Begin your response to **QUESTION 1** on this page.

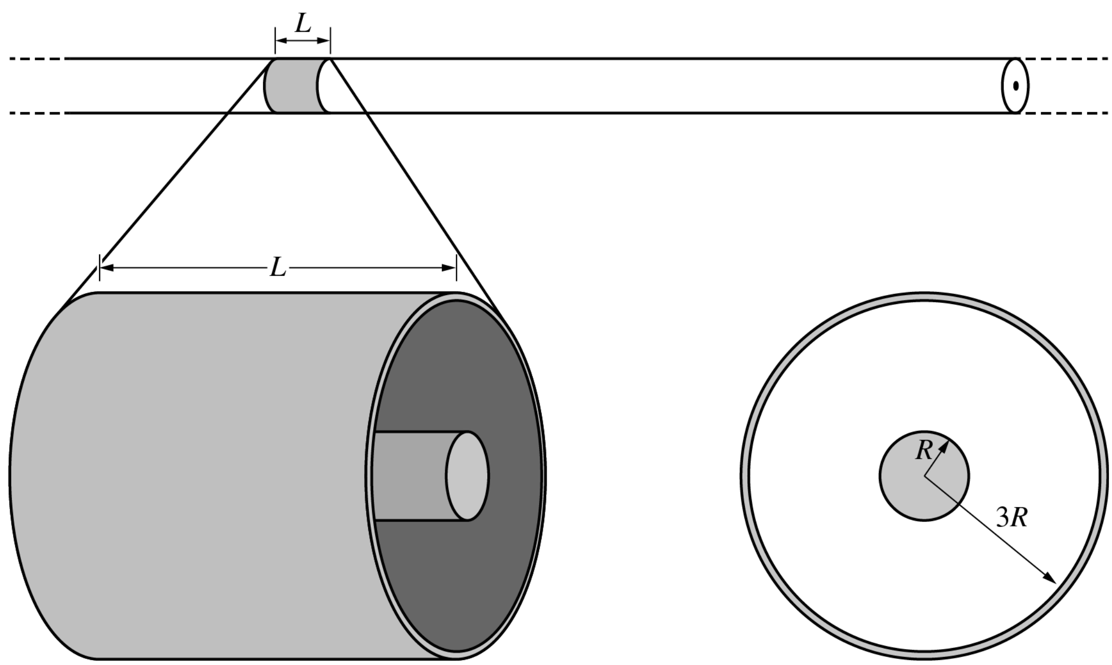
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Cutout View Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

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Continue your response to **QUESTION 1** on this page.

(a) Determine the charge on the outer surface of the cylindrical shell within length L.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q, R, r, L, and physical constants, as appropriate.

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Continue your response to **QUESTION 1** on this page.

(c) The magnitude of the electric field at $r = R$ is $12\ \text{N}/\text{C}$. Calculate the value of the electric field at $r = 2R$.

(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

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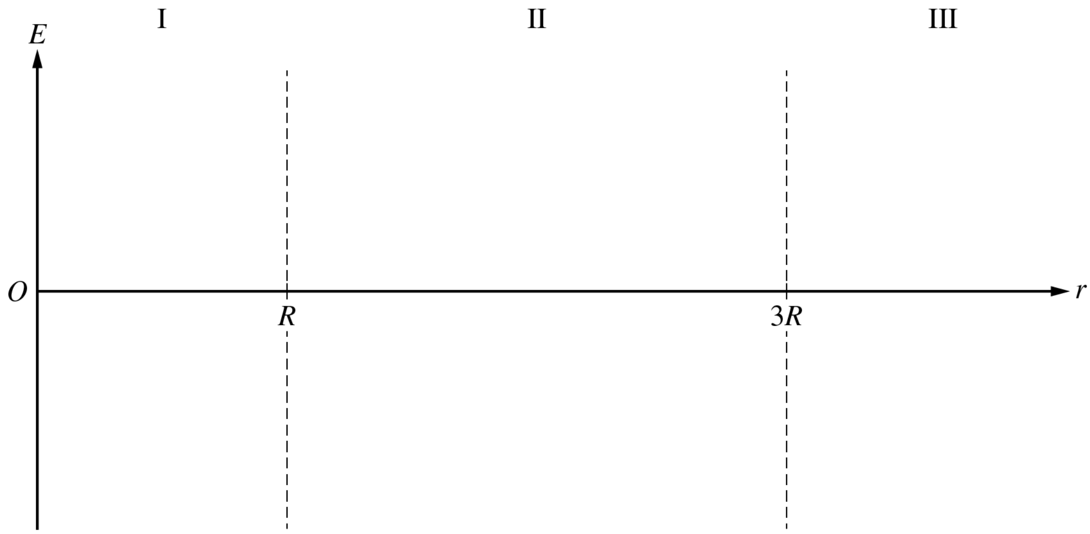
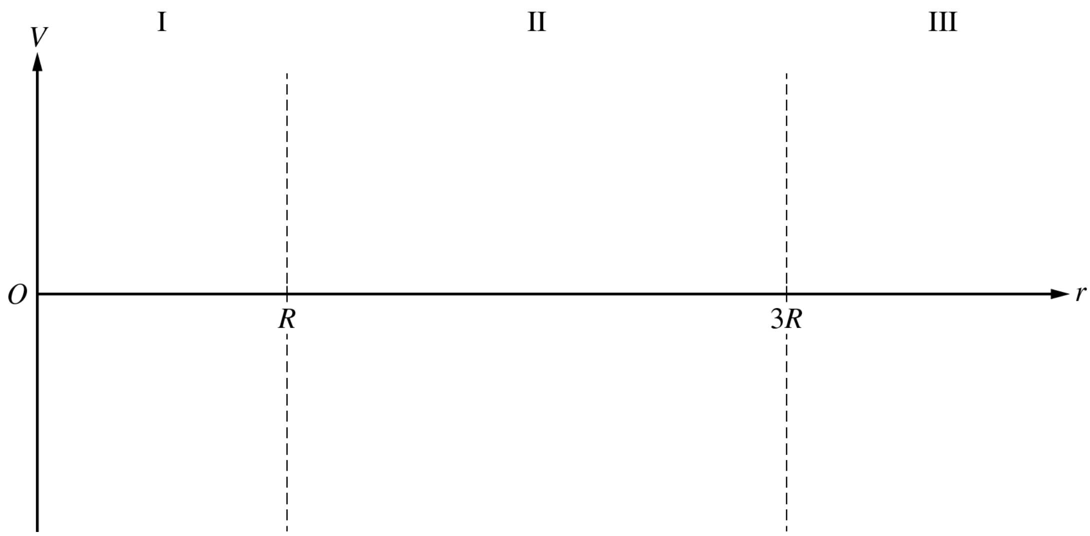
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.



ii. On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.



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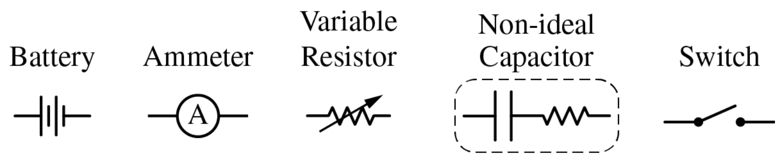
Begin your response to **QUESTION 2** on this page.

2. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the current through the capacitor as it discharges through the resistor.

The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the current I through the capacitor as a function of time t is $I(t) = I_0 e^{\frac{-t}{(R+r_C)C}}$ as the capacitor discharges.

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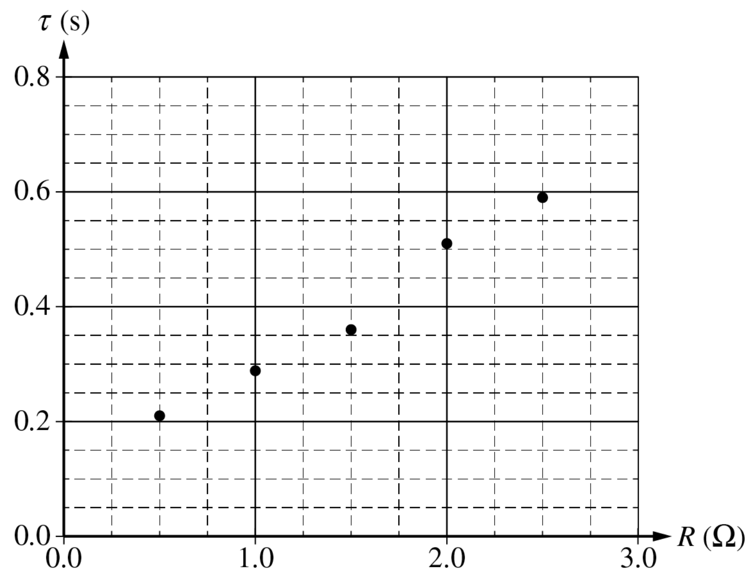
Continue your response to **QUESTION 2** on this page.

(c) The students determine the time constant τ for the circuit as a function of the resistance R. The students' data are shown in the following graph.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the internal resistance r_C of the capacitor.

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Continue your response to **QUESTION 2** on this page.

(d) The ammeter is found to be nonideal. Is the actual value for the internal resistance r_C for the capacitor greater than, less than, or equal to the experimental internal resistance of the capacitor calculated in part (c)?

___ Greater than ___ Less than ___ Equal to

Briefly justify your answer using features of the graph in part (c).

(e) The values of the variable resistor in the original experiment ranged from $0.5\ \Omega$ to $2.5\ \Omega$. The experiment is repeated with values ranging from $3.0\ \Omega$ to $6.0\ \Omega$. Would the slope of the best-fit line be more steep, be less steep, or remain unchanged compared to the graph in part (c)?

___ More steep ___ Less steep ___ Remain unchanged

Briefly justify your answer.

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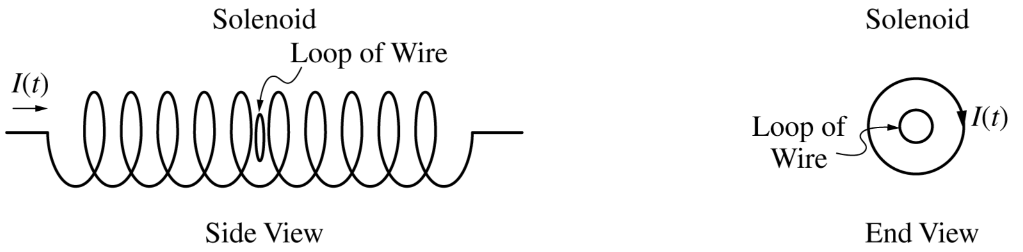
Begin your response to **QUESTION 3** on this page.



Note: Figures not drawn to scale.

3. A single loop of wire with resistance $3.0\ \Omega$ and radius $0.10\ \text{m}$ is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is $0.25\ \text{m}$ long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0\ \text{A/s}$. The direction of the current in the solenoid is clockwise, as shown in the end view.

(a) At time $t = 2.0\ \text{s}$, is the induced current in the loop, as seen from the end view shown, clockwise, counterclockwise, or zero?

___ Clockwise ___ Counterclockwise ___ Zero

Justify your answer.

(b) Calculate the current in the loop of wire at time $t = 2.0\ \text{s}$.

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Continue your response to **QUESTION 3** on this page.

(c) Calculate the total energy dissipated by the loop of wire from time $t = 0$ to time $t = 2.0$ s.

(d) A group of students attempts to verify experimentally the calculation of the current from part (b). The current in the inner circular loop at time $t = 2.0$ s is measured to be less than the current calculated in part (b). Which of the following could explain this discrepancy? Select one answer.

___ The experiment did not account for Earth's magnetic field.
___ The plane of the loop is not perpendicular to the axis of the solenoid.
___ The center of the loop is not on the axis of the solenoid.
___ The resistance of the loop is less than the given value.
___ The radius of the loop is actually larger than 0.10 m.

Justify your answer.

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Continue your response to **QUESTION 3** on this page.

(e) The power supply is now turned off. The original loop of wire is then replaced with a second loop made from wire that has the same thickness and is made from the same material as the original loop of wire. The second loop has radius 0.20 m, is placed in the same orientation as the original loop, and fits completely inside the solenoid. The power supply is turned on, and the current I in the solenoid as a function of t is again given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. Which of the following expressions correctly indicates the ratio $\frac{I_2}{I_1}$ where I_1 represents the current induced in the original loop of wire in part (b) and I_2 represents the current induced in the second loop of wire?

___ $\frac{I_2}{I_1} = 1$ ___ $1 < \frac{I_2}{I_1} < 2$ ___ $\frac{I_2}{I_1} = 2$ ___ $\frac{I_2}{I_1} > 2$

Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM