2022

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Sample Student Responses and Scoring Commentary Set 1

Inside:

Free-Response Question 1

- ☑ Scoring Guidelines
- ☑ Student Samples
- ☑ Scoring Commentary

Question 1: Free-Response Question — 15 points

(a) For correctly determining the charge on the outer surface of the shell — **1 point**

Example Response

$$q_{net} = q_{inner} + q_{outer}$$
$$q_{outer} = q_{net} - q_{inner}$$
$$q_{outer} = +3Q - (+Q)$$
$$q_{outer} = +2Q$$

Scoring Note: A correct response may earn a point even if no work is shown.

Total for part (a) — **1 points**

(b) For using Gauss's law by substituting for either the area or the enclosed charge — **1 point**

Example Response

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \bullet dA = E\left(4\pi r^2\right)$$

For using a correct expression for q_{enc} as a function of r — **1 point**

Example Response

$$q_{enc} = \rho V = \left(\frac{-Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = -Q\frac{r^3}{R^3}$$

Scoring Note: The response may earn this point regardless of the sign of q_{enc}.

For using the correct area — **1 point**

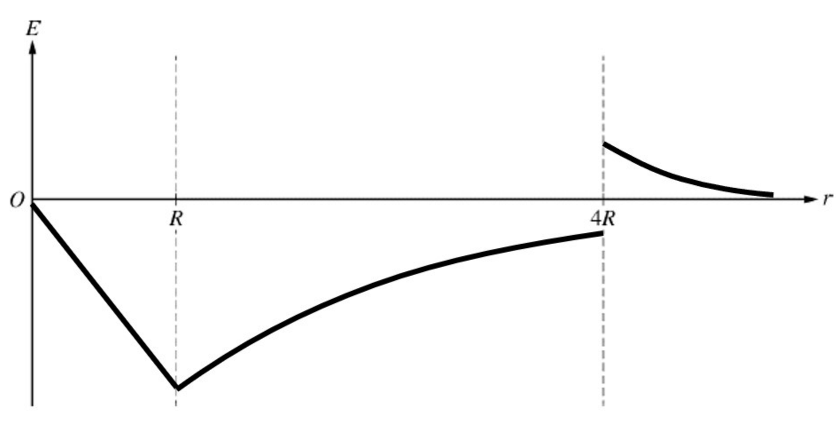
Example Response

$$E = q_{enc}\frac{1}{\varepsilon_0\left(4\pi r^2\right)} = \left(-Q\frac{r^3}{R^3}\right)\frac{1}{\varepsilon_0\left(4\pi r^2\right)}$$
$$E = -Q\frac{r}{4\pi\varepsilon_0 R^3}$$

Scoring Note: The response may earn this point regardless of the sign of E.

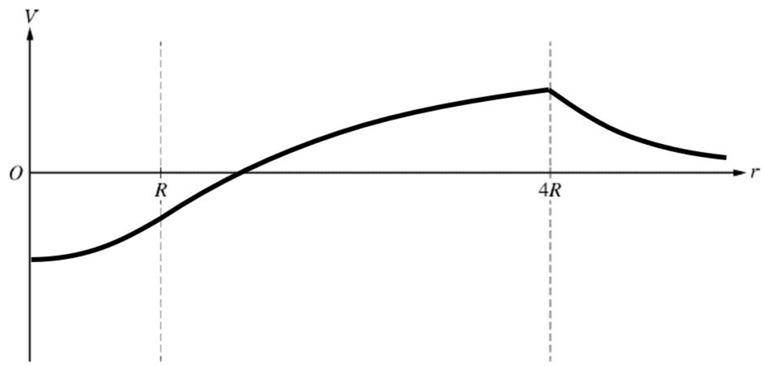
Total for part (b) — **3 points**

(c)	For recognizing the electric field varies as an inverse square of the separation distance from the center of the nonconducting sphere **Example Response** $E \propto \frac{1}{r^2}$	**1 point**
	For the correct magnitude, including units, of the electric field at $2R$ **Example Response** $E_{new} = \frac{E_{old}}{4} = \frac{8\text{ N/C}}{4}$ $E_{new} = 2\text{ N/C}$	**1 point**
	Total for part (c)	**2 points**
(d)	For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral **Example Response** $V_f - V_i = -\int_{s_i}^{s_f} Edr$ $\Delta V = -\int_{r=R}^{r=4R} Edr$	**1 point**
	For substituting the correct expression for the electric field or an expression for E that is consistent with the explicit functional dependence of $E(r)$ from part (c) **Example Response** $\Delta V = -\int_{r=R}^{r=4R} -\frac{Q}{4\pi\varepsilon_0 r^2}dr$ $\Delta V = \frac{Q}{4\pi\varepsilon_0}\int_{r=R}^{r=4R} \frac{1}{r^2}dr$	**1 point**

	For correctly integrating the electric field expression that was substituted in the previous point with correct limits of integration **Example Response** $\Delta V = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r=R}^{r=4R} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r=4R}^{r=R} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{4R}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{4-1}{4R}\right)$ $\Delta V = \frac{3Q}{16\pi\varepsilon_0 R}$	**1 point**
	Alternate Solution	
	For using a difference in potentials treating the inner sphere as a point charge by symmetry $\Delta V = \frac{Q}{4\pi\varepsilon_0 r_f} - \frac{Q}{4\pi\varepsilon_0 r_i}$	**1 Point**
	For using the correct charge on both of the terms; Q *not* Q *and* $3Q$	**1 Point**
	For using the correct values for r $\Delta V = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 4R}$ $\Delta V = \frac{3Q}{16\pi\varepsilon_0 R}$	**1 Point**
	Total for part (d)	**3 points**
(e)(i)	For indicating that E is negative and the magnitude of E increases linearly from 0 to R	**1 point**
	For a curve that is continuous at R and asymptotically approaches zero in the region $R < r < 4R$	**1 point**
	For a positive, concave-up, and decreasing curve for $r > 4R$	**1 point**
	Example Response 	

(e)(ii)	For a potential curve that is always increasing from 0 to $4R$	**1 point**
	For a potential curve that is decreasing for $r > 4R$	**1 point**
	For a continuous graph across all three regions	**1 point**

Example Response



Scoring Note: The intercept of the curve on the vertical axis is irrelevant. The intercept of the curve on the horizontal axis is irrelevant. The curve can, hence, cross the horizontal axis at any location or even be entirely on the negative side of the horizontal axis as long as the other criteria are met.

Total for part (e) 6 points

Total for question 1 15 points

Question 1

Begin your response to **QUESTION 1** on this page.

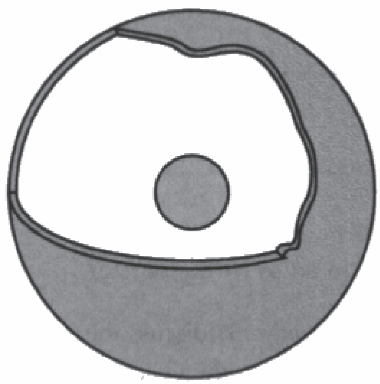
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

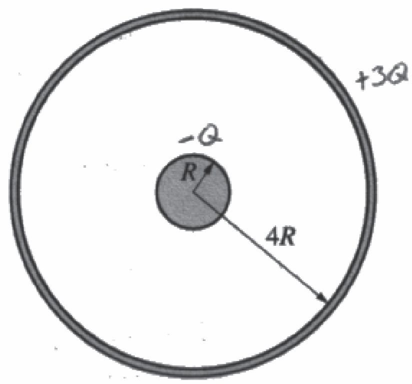
Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Cutout View



Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

   (a) Determine the charge on the outer surface of the shell.

   $-Q + 3Q = 2Q$

Question 1

Continue your response to **QUESTION 1** on this page.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

$$\oint E \cdot dA = \frac{q_{enclosed}}{\varepsilon_0} \qquad \rho = \frac{Q}{V} = \frac{Q_{enc}}{V_{enc}}$$

$$Q_{enc} = \frac{Q V_{enc}}{V}$$

$$\oint E\, dA \cos\theta = \frac{-Q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{Q V_{enc}}{\varepsilon_0 V}$$

$$E(4\pi r^2) = \frac{-Q \frac{4}{3}\pi r^3}{\varepsilon_0 \frac{4}{3}\pi R^3}$$

$$E(4\pi) = \frac{-Qr}{\varepsilon_0 R^3}$$

$$E = \frac{-Qr}{\varepsilon_0 4\pi R^3}$$

(c) The magnitude of the electric field at $r = R$ is $8\,\mathrm{N/C}$. Calculate the value of the electric field at $r = 2R$.

$$E = \frac{kQ}{r^2} \qquad \frac{1}{(2r)^2} \propto \frac{1}{4}E \qquad \frac{1}{4}(8) = 2\,\mathrm{N/C}$$

$$E \propto \frac{1}{r^2}$$

(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

$$E = \frac{-kQ}{r^2}$$

$$\Delta V = -\int E \cdot dr$$

$$\Delta V = -\int \frac{-kQ}{r^2} \cdot dr$$

$$= kQ \int_R^{4R} \frac{1}{r^2} dr$$

$$= kQ\left[\frac{-2}{r}\right]_R^{4R}$$

$$= kQ\left(-\frac{1}{2R} + \frac{2}{R}\right) = kQ\left(\frac{3}{2R}\right)$$

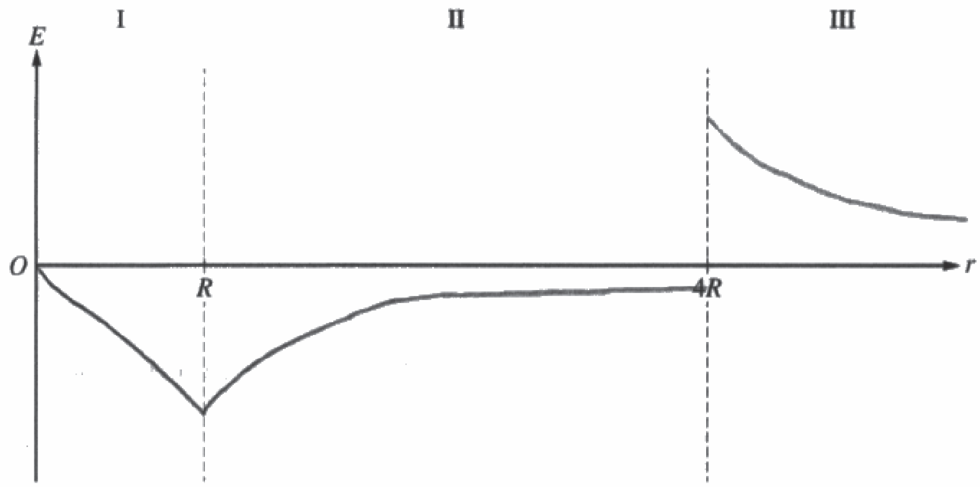
$$\Delta V = \frac{3kQ}{2R}$$

Question 1

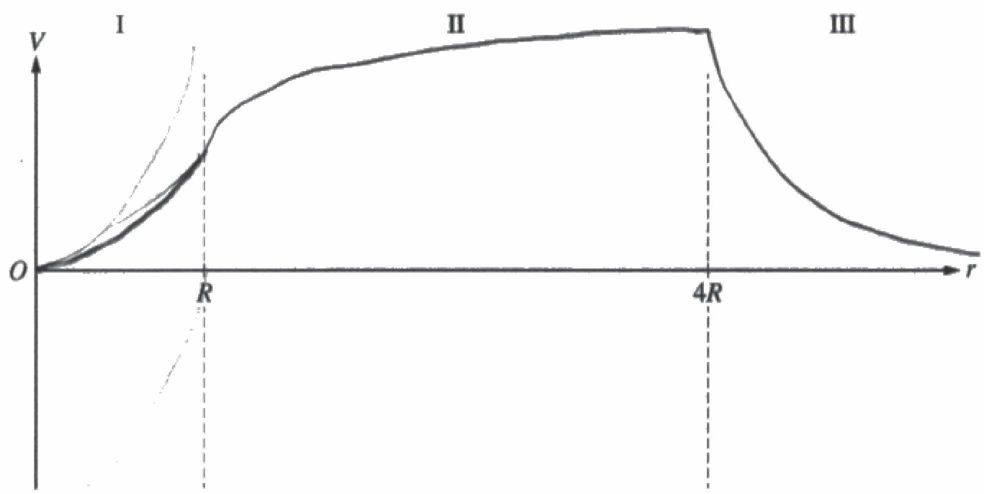
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.



ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.



Question 1

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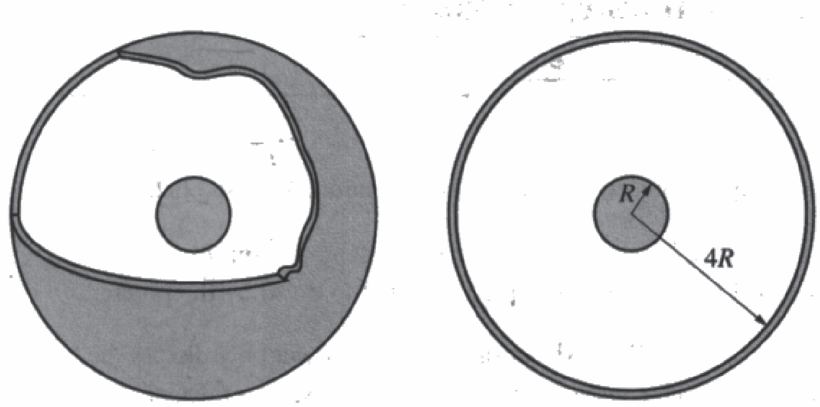
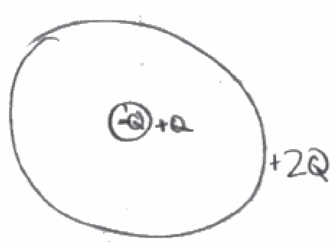


Cutout View Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the shell.



$\boxed{+2Q}$

Question 1

Continue your response to **QUESTION 1** on this page.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

$$\oint E \cdot dA = \frac{q_{enc}}{\epsilon_0} \qquad \rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{q_{enc}}{\frac{4}{3}\pi r^3}$$

$$q_{enc} = \frac{Q \cdot \cancel{\tfrac{4}{3}\pi} r^3}{\cancel{\tfrac{4}{3}\pi} R^3} \qquad q_{enc} = \frac{Q r^3}{R^3}$$

$$\oint E \cdot dA = \frac{Q r^3}{\epsilon_0 R^3}$$

$$E 4\pi \cancel{r^2} = \frac{Q r^{\cancel{3}}}{\epsilon_0 R^3} \qquad \boxed{E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Qr}{R^3}}$$

$$E = \frac{Q r}{4\pi\epsilon_0 R^3}$$

(c) The magnitude of the electric field at $r = R$ is 8N/C. Calculate the value of the electric field at $r = 2R$.

$$E_1 = 8 = \frac{1}{4\pi\epsilon_0} \cdot \frac{QR}{R^3} \qquad E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q 2\cancel{R}}{R^{\cancel{3}}} \qquad E_2 = 2E_1$$

$$8 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^2} \qquad E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Q}{R^2} \qquad \boxed{E_2 = 16 \frac{N}{C}}$$

(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

$$\Delta V = -\int \vec{E} \cdot d\vec{r} \qquad |\Delta V| = \frac{Q}{4\pi\epsilon_0 R} \cdot \frac{r^2}{2}\Big|_R^{4R}$$

$$\Delta V = -\int_R^{4R} \frac{1}{4\pi\epsilon_0} \cdot \frac{Qr}{R^3} \qquad |\Delta V| = \frac{Q}{4\pi\epsilon_0 R}\left[\frac{16R^2}{2} - \frac{R^2}{2}\right]$$

$$|\Delta V| = \int_R^{4R} \frac{1}{4\pi\epsilon_0} \cdot \frac{Qr}{R^3}$$

$$|\Delta V| = \frac{Q}{4\pi\epsilon_0 R}\int_R^{4R} r$$

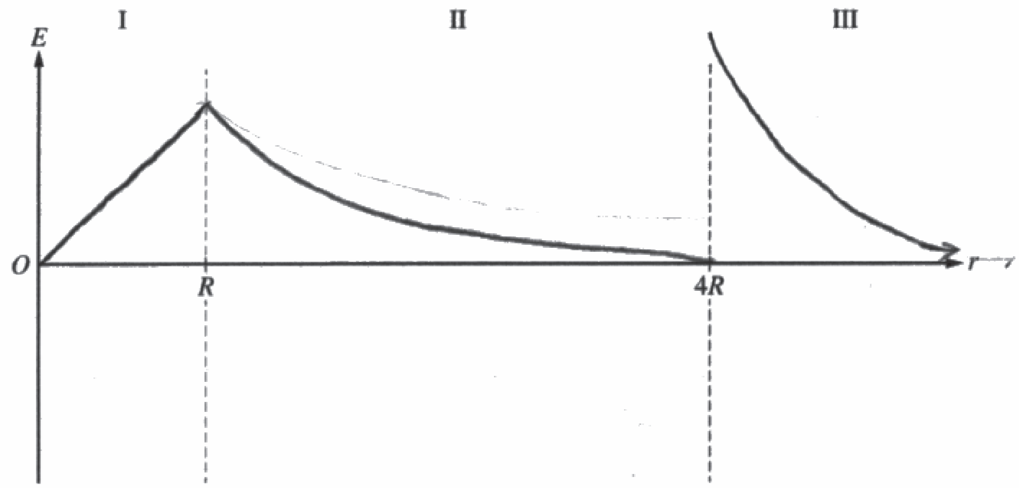
GO ON TO THE NEXT PAGE.

Question 1

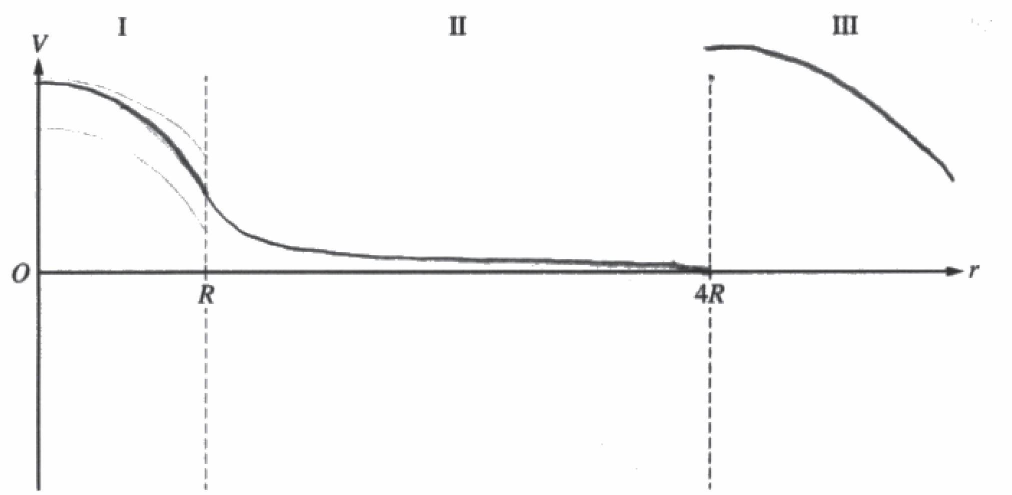
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.



ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.



Question 1

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PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

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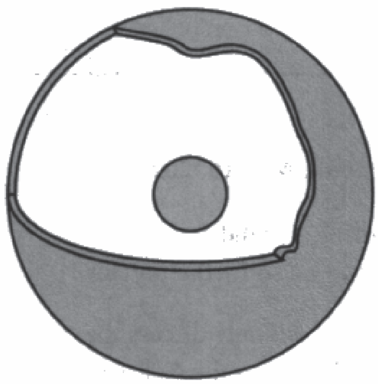
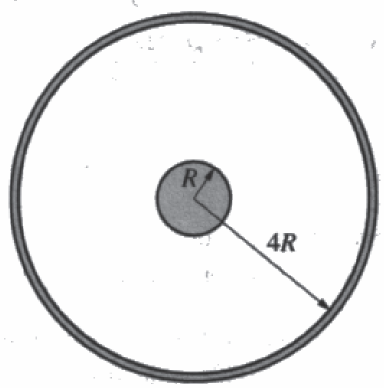


Cutout View



Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

   (a) Determine the charge on the outer surface of the shell.

$+2Q$

Question 1

Continue your response to **QUESTION 1** on this page.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

$$E = \frac{kQ}{r^2}$$

(c) The magnitude of the electric field at $r = R$ is 8 N / C. Calculate the value of the electric field at $r = 2R$.

$$\frac{8 N/C}{(2R)^2} \qquad 2 N/C$$

(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

$$F_e = \frac{1}{4\pi\varepsilon_0} \left| \frac{Q_1 Q_2}{R^2} \right.$$

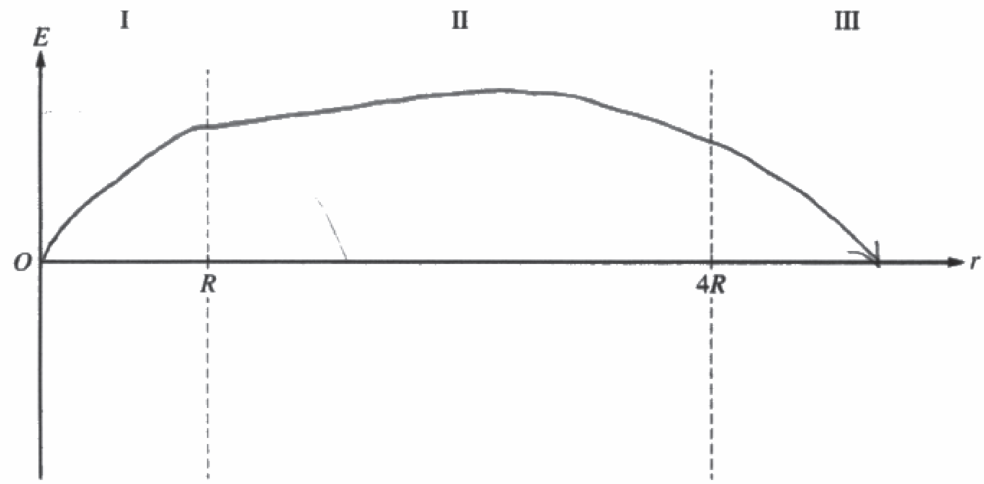
GO ON TO THE NEXT PAGE.

Question 1

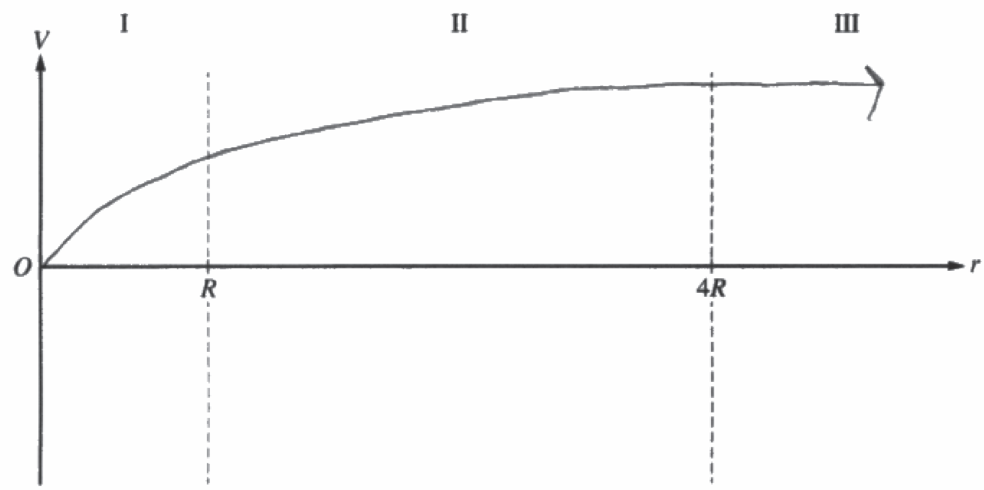
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.



ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.



Question 1

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Determine the distribution of charge on the surfaces of a conductor in electrostatic equilibrium in the presence of other charges.
- Use Gauss's law and spherical symmetry to determine the electric field inside and outside an insulator with uniform charge density.
- Use proportional reasoning to relate the magnitude of the electric field at two points based on the functional dependence of E on r.
- Calculate the potential difference between two points by integrating the electric field over a distance.
- Sketch graphs of the electric field and electric potential with respect to radial position for static spherically symmetric charge distributions.

Sample: 1A
Score: 14

Part (a) earned 1 point because the response correctly indicates the charge of $+2Q$ on the outer surface of the shell. Part (b) earned 3 points. The first point was earned because the response correctly uses Gauss's law, including an attempt at a substitution for area and enclosed charge. The second point was earned because the response indicates and uses the correct enclosed charge. The third point was earned because the response indicates and uses the correct area. Part (c) earned 2 points. The first point was earned because the response correctly indicates that the electric field has an inverse-square dependence. The second point was earned because the response indicates the correct answer of ± 2 N/C. Part (d) earned 2 points. The first point was earned because the response correctly uses the equation relating potential difference to electric field, including an attempt at integration limits and evaluating the integral. The second point was earned because the response substitutes the correct expression for the electric field. The third point was not earned because the response does not correctly integrate the expression for electric field. Part (e)(i) earned 3 points. The first point was earned because the response indicates that E is negative and linearly increasing in magnitude in the region $0 < r < R$. The second point was earned because the response correctly indicates a positive or negative curve that is continuous at R and asymptotically approaches zero in the region $R < r < 4R$. The third point was earned because the response correctly indicates a positive, concave-up, and decreasing curve in the region $r > 4R$. Part (e)(ii) earned 3 points. The first point was earned because the response correctly indicates that the potential is always increasing in the region $0 < r < 4R$. The second point was earned because the response correctly indicates that the potential is decreasing for $r > 4R$. The third point was earned because the response correctly indicates that the potential graph is continuous across all three regions.

Question 1 (continued)

Sample: 1B
Score: 10

Part (a) earned 1 point because the response correctly indicates the charge of $+2Q$ on the outer surface of the shell. Part (b) earned 3 points. The first point was earned because the response correctly uses Gauss's law, including an attempt at a substitution for area and enclosed charge. The second point was earned because the response indicates and uses the correct enclosed charge. The third point was earned because the response indicates and uses the correct area. Part (c) earned 1 point. The first point was earned because the response correctly indicates that the electric field has an inverse-square dependence. The second point was not earned because the response does not indicate the correct numerical answer. Part (d) earned 2 points. The first point was earned because the response correctly uses the equation relating potential difference to electric field, including an attempt at integration limits and evaluating the integral. The second point was not earned because the response substitutes the expression for electric field from part (b) for $r < R$ instead of either the correct expression for the electric field for $R < r < 4R$ or the expression for $E(r)$ from part (c). The third point was earned because the response correctly integrates the expression that was substituted in the previous point using correct integration limits. Part (e)(i) earned 2 points. The first point was not earned because the response indicates that E is positive rather than negative in the region $0 < r < R$. The second point was earned because the response correctly indicates a positive or negative curve that is continuous at R and asymptotically approaches zero in the region $R < r < 4R$. The third point was earned because the response correctly indicates a positive, concave-up, and decreasing curve in the region $r > 4R$. Part (e)(ii) earned 1 point. The first point was not earned because the response does not indicate that the potential is always increasing in the region $0 < r < 4R$. The second point was earned because the response correctly indicates that the potential is decreasing for $r > 4R$. The third point was not earned because the response indicates that the potential graph is not continuous across all three regions.

Sample: 1C
Score: 5

Part (a) earned 1 point because the response correctly indicates the charge of $+2Q$ on the outer surface of the shell. Part (b) earned 0 points. The first point was not earned because the response does not write or use Gauss's law. The second point was not earned because the response does not indicate and use the correct enclosed charge. The third point was not earned because the response does not indicate and use the correct area. Part (c) earned 2 points. The first point was earned because the response correctly indicates that the electric field has an inverse-square dependence. The second point was earned because the response indicates the correct numerical answer with units. Part (d) earned 0 points. The first point was not earned because the response does not write and use an equation relating potential difference to electric field. The second point was not earned because the response does not substitute an expression for electric field. The third point was not earned because the response does not perform an integration. Part (e)(i) earned 0 points. The first point was not earned because the response does not indicate that E is negative and linear in the region $0 < r < R$. The second point was not earned because the response does not indicate a positive or negative curve that asymptotically approaches zero in the region $R < r < 4R$. The third point was not earned because the response does not indicate a positive, concave-up, and decreasing curve in the region $r > 4R$. Part (e)(ii) earned 2 points. The first point was earned because the response correctly indicates that the potential is always increasing in the region $0 < r < 4R$. The second point was not earned because the response does not indicate that the potential is decreasing for $r > 4R$. The third point was earned because the response indicates that the potential graph is continuous across all three regions.