2022

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Sample Student Responses and Scoring Commentary Set 2

Inside:

Free-Response Question 1

- ☑ **Scoring Guidelines**
- ☑ **Student Samples**
- ☑ **Scoring Commentary**

Question 1: Free-Response Question — 15 points

(a) For correctly determining the charge on the outer surface of the shell — **1 point**

Example Response

$q_{net} = q_{inner} + q_{outer}$
$q_{outer} = q_{net} - q_{inner}$
$q_{outer} = +4Q - (-Q)$
$q_{outer} = +5Q$

Scoring Note: A correct response may earn a point even if no work is shown.

Total for part (a) 1 point

(b) For using Gauss's law by substituting for either the area or the enclosed charge — **1 point**

Example Response

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \cdot dA = E(2\pi rL)$$

For using a correct expression for q_{enc} as a function of r — **1 point**

Example Response

$$q_{enc} = \rho V = \left(\frac{Q}{\pi R^2 L}\right)\left(\pi r^2 L\right) = Q\frac{r^2}{R^2}$$

For using the correct area — **1 point**

Example Response

$$E = q_{enc}\frac{1}{\varepsilon_0(2\pi rL)} = \left(Q\frac{r^2}{R^2}\right)\frac{1}{\varepsilon_0(2\pi rL)}$$
$$E = Q\frac{r}{2\pi\varepsilon_0 R^2 L}$$

Total for part (b) 3 points

(c) For recognizing that the E-field varies as the reciprocal of the separation distance from the center of the nonconducting cylinder **1 point**

Example Response

$$E \propto \frac{1}{r}$$

For the correct value, including units, of electric field at $2R$ **1 point**

Example Response

$$E_{new} = \frac{E_{old}}{2} = \frac{12 \text{ N/C}}{2}$$
$$E_{new} = 6 \text{ N/C}$$

Total for part (c) 2 points

(d) For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral **1 point**

Example Response

$$V_f - V_i = -\int_{s_i}^{s_f} E dr$$
$$\Delta V = -\int_{r=R}^{r=3R} E dr$$

For substituting the correct expression for the electric field or an expression consistent with the explicit functional dependence of $E(r)$ from part (c) **1 point**

Example Response

$$\Delta V = -\int_{r=R}^{r=3R} \left(\frac{Q}{2\pi\varepsilon_0 rL} \right) dr = -\frac{Q}{2\pi\varepsilon_0 L} \int_{r=R}^{r=3R} \frac{1}{r} dr$$

For correctly integrating the electric field expression that was substituted in the previous point with the correct limits of of integration **1 point**

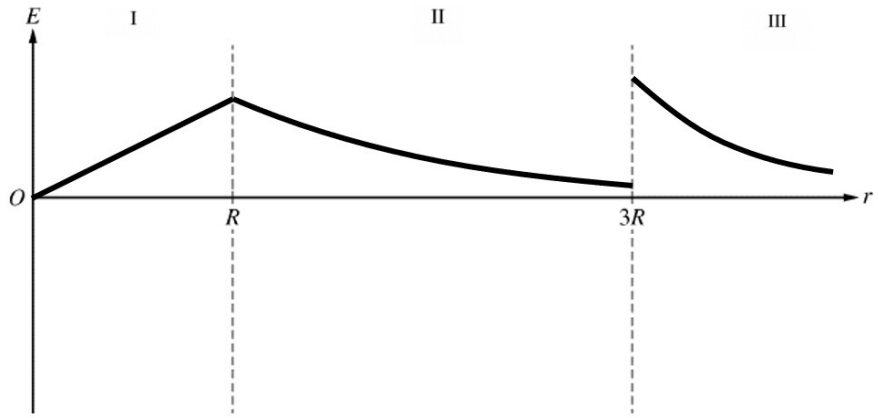
Example Response

$$\Delta V = -\frac{Q}{2\pi\varepsilon_0 L} [\ln(r)]_{r=R}^{r=3R} = -\frac{Q}{2\pi\varepsilon_0 L} [\ln(3R) - \ln(R)]$$
$$\Delta V = -\frac{Q}{2\pi\varepsilon_0 L} \ln(3)$$

Total for part (d) 3 points

(e)(i)	For indicating E is positive and the magnitude increases linearly from 0 to R	**1 point**
	For a positive, decreasing concave up graph for $r > R$	**1 point**
	For a discontinuous increase in magnitude at $r = 3R$	**1 point**

Example Response

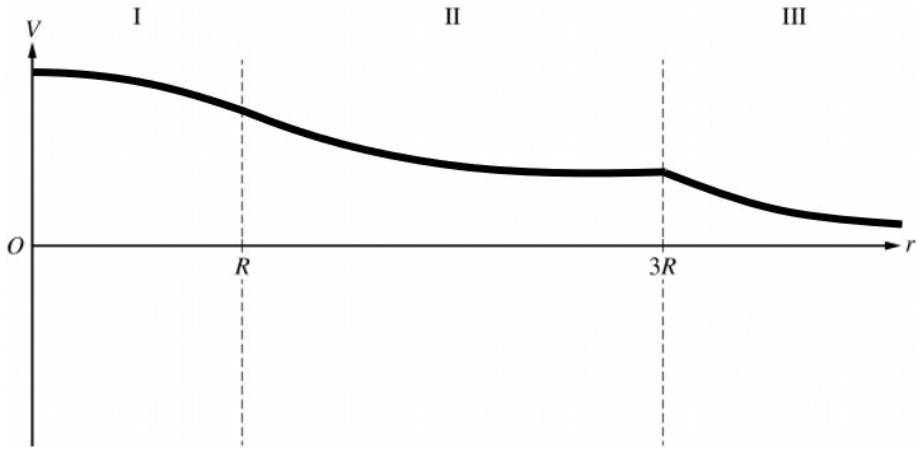


(e)(ii)	For a graph that is concave down and always decreasing as r increases within Region I	**1 point**
	For a graph that is concave up and always decreasing as r increases within each of the Regions II and III	**1 point**
	For a continuous graph across all three regions	**1 point**

Scoring Notes:

- The intercept of the curve on the vertical axis is irrelevant. The intercept of the curve on the horizontal axis is irrelevant. The curve can, hence, cross the horizontal axis at any location or even be entirely on the negative side of the horizontal axis as long as the other criteria are met.
- The response can earn a maximum of 2 points if all three segments are reflected across the horizontal axis.

Example Response



Total for part (e) 6 points

Total for question 1 15 points

Question 1

Begin your response to **QUESTION 1** on this page.

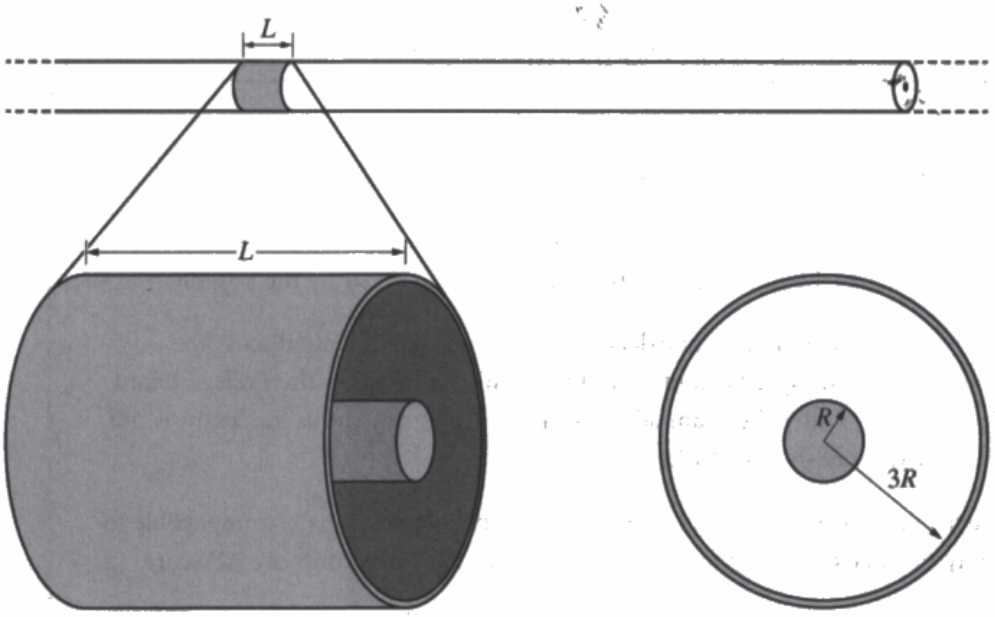
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Cutout View

Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

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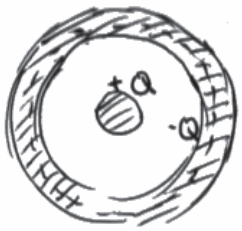
Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1

Continue your response to **QUESTION 1** on this page.

(a) Determine the charge on the outer surface of the cylindrical shell within length L.



electric field inside conductor is zero so total charge on inner surface is $-Q$. therefore charge on outer surface is $+4Q-(-Q)=+5Q$

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q, R, r, L, and physical constants, as appropriate.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E \cdot A = \frac{Q \cdot \frac{r^2}{R^2}}{\varepsilon_0}$$

$$E = \frac{Qr^2}{\varepsilon_0 R^2 \cdot 2\pi r L}$$ [away from axis]

$$= \frac{Qr}{\varepsilon_0 R^2 2\pi L}$$ [away from axis]

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

2001334

Q5133/3

Question 1

Continue your response to **QUESTION 1** on this page.

(c) The magnitude of the electric field at $r = R$ is 12 N / C. Calculate the value of the electric field at $r = 2R$.

$$\oint \vec{E}\, d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi r L = \frac{Q}{\varepsilon_0} \quad E \cdot L$$

$$E = \frac{Q}{\varepsilon_0 2\pi r L} \qquad R < r < 3R$$

$$E(R) = \frac{Q R^2}{\varepsilon_0 R^2 \cdot 2\pi R L} = \frac{Q}{\varepsilon_0 2\pi R L} = 12 \text{ N/C}$$

$$E(2R) = \frac{Q}{\varepsilon_0 2\pi \cdot 2R L} = 6 \text{ N/C}$$

(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

$$E = \frac{Q}{2\pi r L \varepsilon_0} \qquad R < r < 3R$$

$$\Delta V = -\int \vec{E} \cdot d\vec{r} = -\int_{3R}^{3R} \frac{Q}{2\pi r L \varepsilon_0}\, dr = -\frac{Q}{2\pi L \varepsilon_0} \ln|r| \Big]_{R}^{3R}$$

$$= \frac{Q}{2\pi L \varepsilon_0} \ln 3$$

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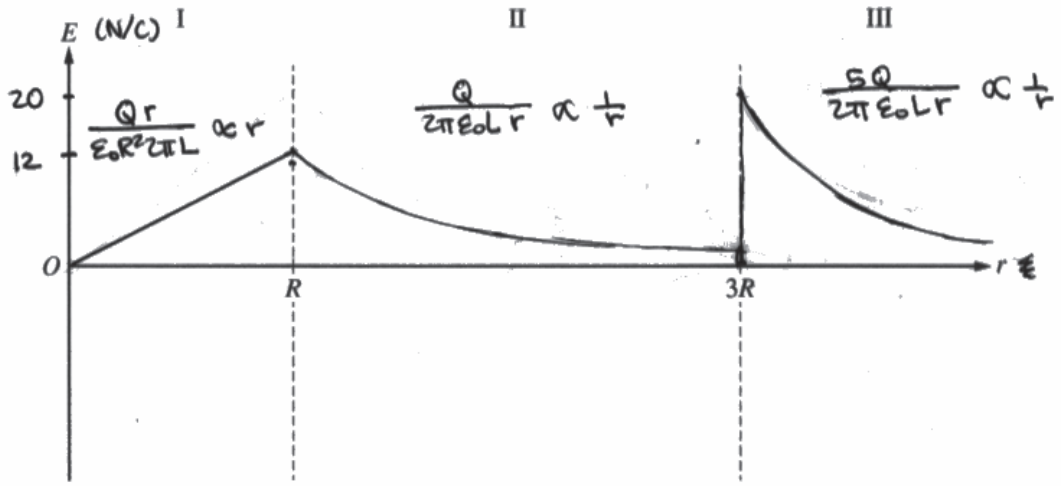
Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1

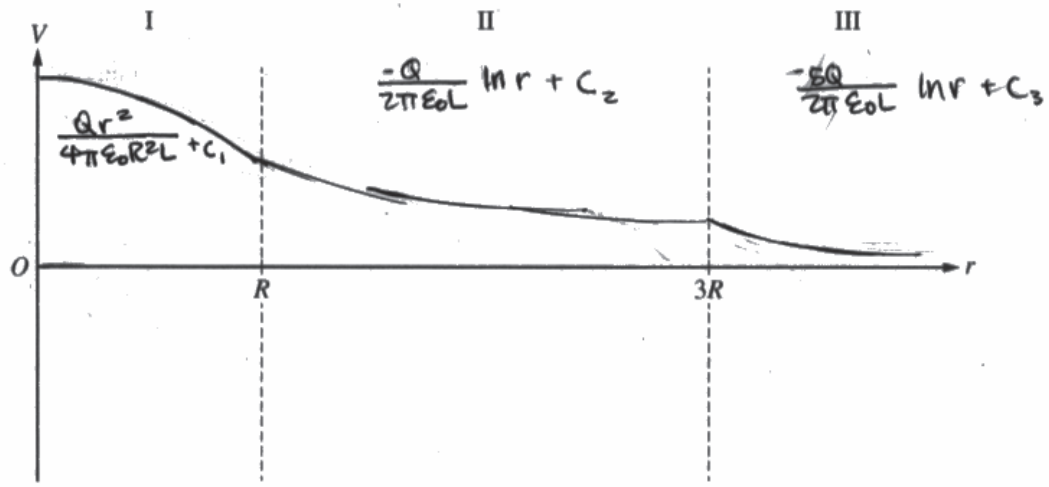
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.



ii. On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.



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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

2001334

Q5133/5

Question 1

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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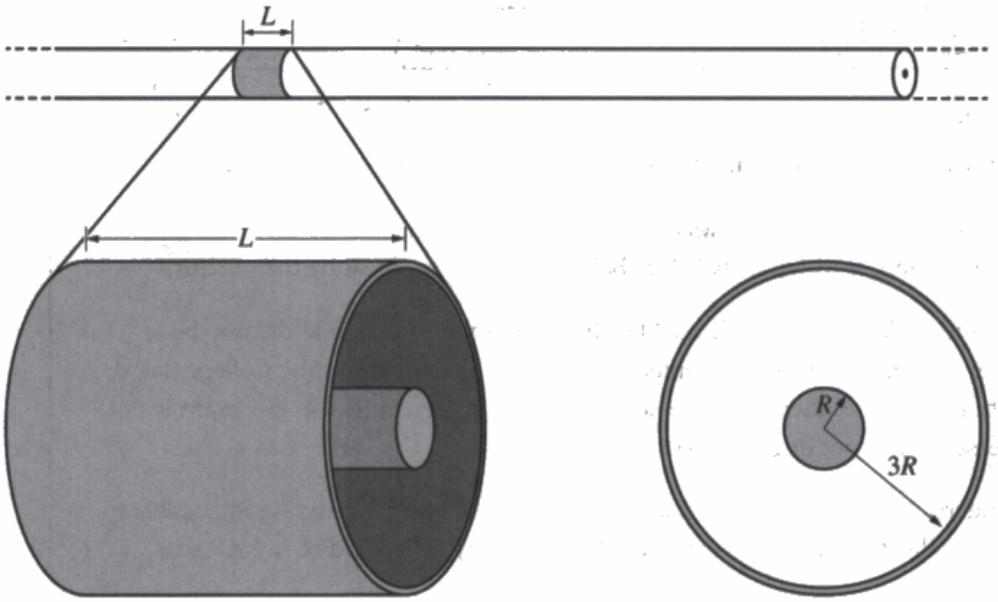

Cutout View

Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1

Continue your response to **QUESTION 1** on this page.

(a) Determine the charge on the outer surface of the cylindrical shell within length L.

$+4Q - (-Q) = +5Q$

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q, R, r, L, and physical constants, as appropriate.

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \, d\vec{A} = E 2\pi r$$

$$E = \frac{q_{enc}}{\varepsilon_0 2\pi r} = \frac{Q\left(\frac{\pi r^2 L}{\pi R^2 L}\right)}{\varepsilon_0 2\pi r} = \frac{Q r^2}{\varepsilon_0 2\pi r R^2} = \frac{Q r}{\varepsilon_0 2\pi R^2}$$

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

2000710

Q5133/3

Question 1

Continue your response to **QUESTION 1** on this page.

(c) The magnitude of the electric field at $r = R$ is 12 N / C. Calculate the value of the electric field at $r = 2R$.

$r=R$: $E = \frac{q_{enc}}{2\pi R \varepsilon_0} = 12$

$r=2R$: $E = \frac{q_{enc}}{2\pi 2R\varepsilon_0} = \frac{q_{enc}}{2\pi R\varepsilon_0}/2 = 12/2 = 6\ N/C$

(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

$\Delta V = -\int_R^{3R} \vec{E} \cdot d\vec{r} = -\int_R^{3R} \frac{Q}{2\pi r \varepsilon_0} dr$

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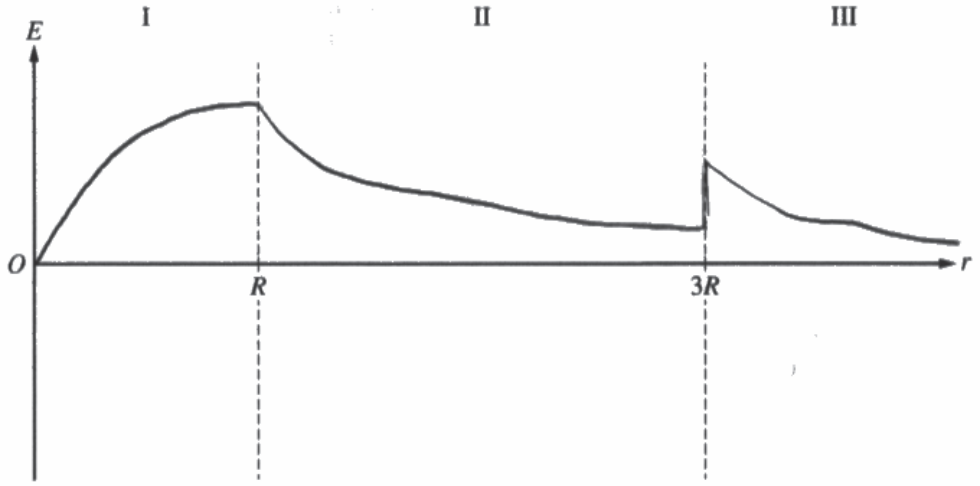
Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1

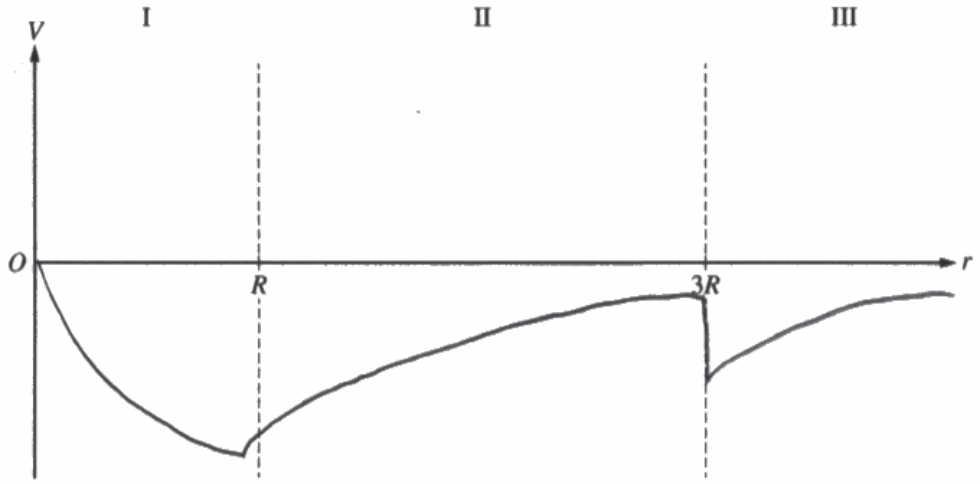
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.



ii. On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.



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2000710

Q5183/5

Question 1

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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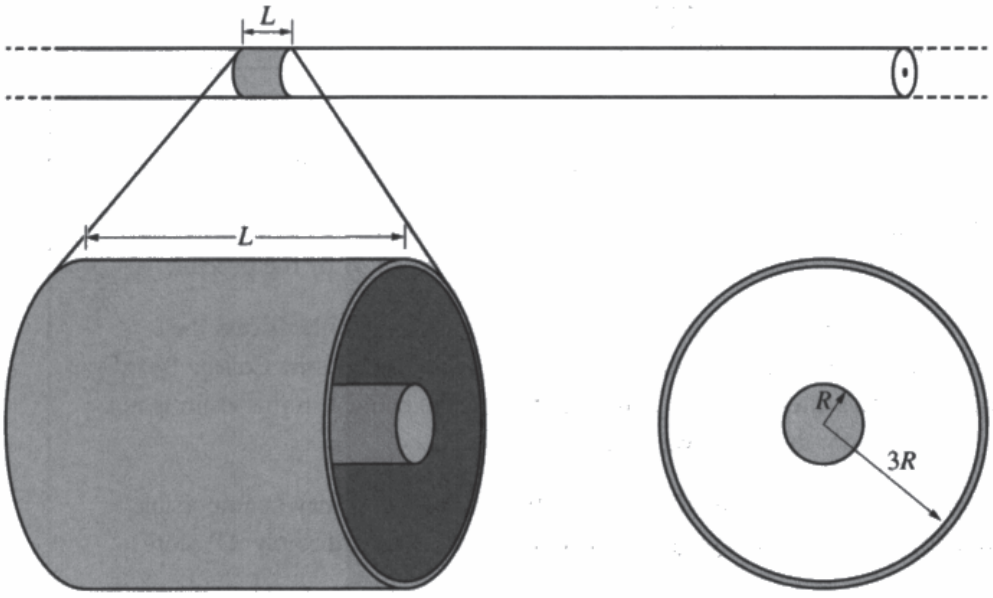

Cutout View

Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

GO ON TO THE NEXT PAGE.

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Question 1

Continue your response to **QUESTION 1** on this page.

(a) Determine the charge on the outer surface of the cylindrical shell within length L.

$Q_{net} = Q_{outer} - Q_{inner} = +4Q - Q = 3Q$

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q, R, r, L, and physical constants, as appropriate.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

due to cylindrical symmetry, $\vec{E}$ is constant

$$E \oint d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$A = \pi r^2$

$Q_{enc} = 3Q$

$$EA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E \pi r^2 = \frac{Q_{enc}}{\varepsilon_0}$$

$$E = \frac{3Q}{\pi r^2 \varepsilon_0}$$

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2001424

Question 1

Continue your response to **QUESTION 1** on this page.

(c) The magnitude of the electric field at $r = R$ is 12 N / C. Calculate the value of the electric field at $r = 2R$.

$12\frac{N}{C} = \frac{3Q}{\pi R^2 \varepsilon_0}$ $\qquad \frac{3Q}{\pi(2R)^2\varepsilon_0} = \frac{3Q}{4\pi R^2\varepsilon_0} = \frac{1}{4}\left(12\frac{N}{C}\right) = 3N/C$

(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

$|\Delta V| = \int \vec{E}\cdot d\vec{r}$

$= \int_R^{3R} \frac{3Q}{\pi r^2 \varepsilon_0}\, dr$

$= \frac{3Q}{\pi\varepsilon_0}\int_R^{3R} \frac{1}{r^2}\, dr =$

$= \frac{3Q}{\pi\varepsilon_0}\left[2\ln r\right]_R^{3R}$

$= \frac{3Q}{\pi\varepsilon_0}\left[2\ln 3R - 2\ln R\right]$

$u = r^2$ $\quad du = 2r\,dr$ $\quad dr = \frac{du}{2r}$

$\int \frac{1}{u}\frac{du}{2r} = \ln u$

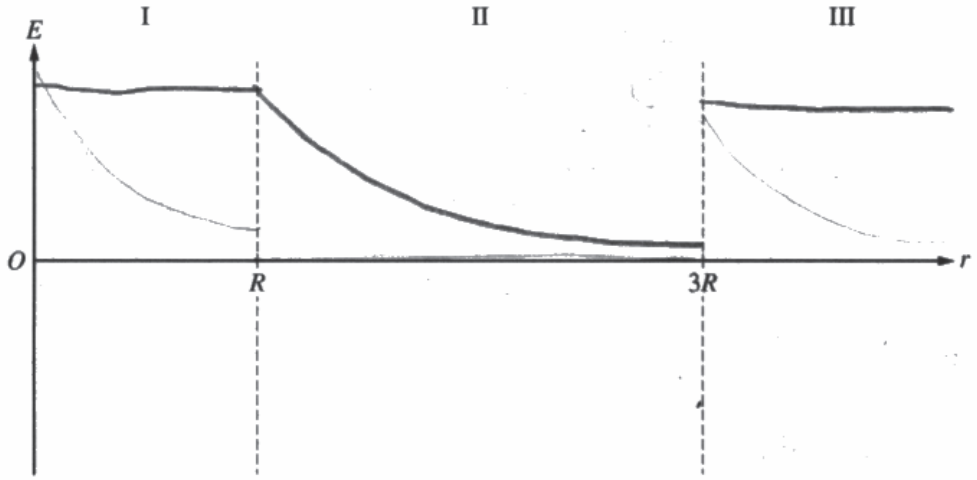
Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1

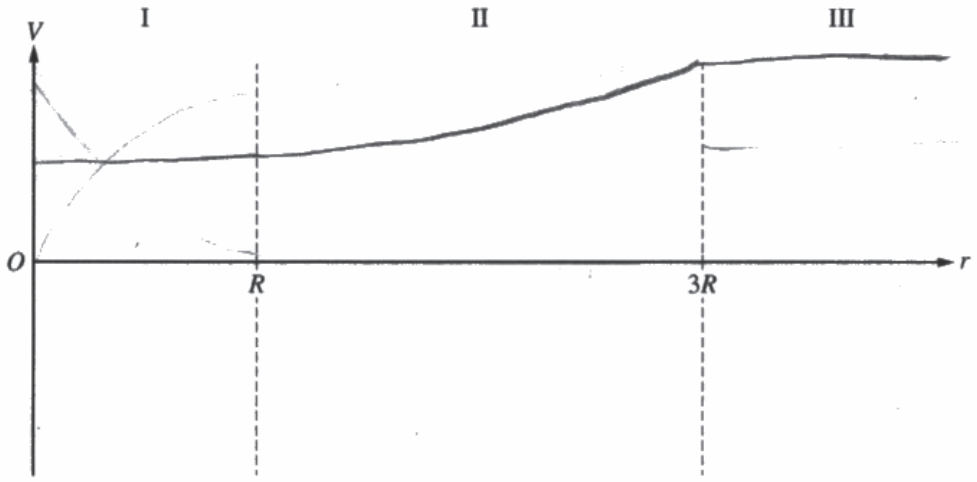
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.



ii. On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.



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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

2001424

Q5133/5

Question 1

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Determine the induced charge on the surface of a conductor in the presence of other cylindrical charge distributions.
- Use symbolic and numerical reasoning to derive and calculate electric field strength and electric potential difference using Gauss's law and the definition of electric potential.
- Calculate the potential difference between two points by integrating the electric field over a distance.
- Sketch graphs showing the relationships between electric field and electric potential with respect to radial position for static cylindrical charge distributions.

Sample: 1A
Score: 15

Part (a) earned 1 point because the response correctly indicates a charge of $+5Q$ on the outer surface of the shell. Part (b) earned 3 points. The first point was earned because the response correctly uses Gauss's law, including an attempt at a substitution for both area and enclosed charge. The second point was earned because the response indicates and uses the correct enclosed charge. The third point was earned because the response indicates and uses the correct area. Part (c) earned 2 points. The first point was earned because the response correctly indicates that the electric field has a reciprocal dependence on r by writing an expression for the electric field and dividing the given E value by 2. The second point was earned because the response indicates the correct answer of 6 N/C. Part (d) earned 3 points. The first point was earned because the response correctly uses the equation relating potential difference to electric field, including an attempt at integration limits and evaluating the integral. The second point was earned because the response substitutes the correct expression for the electric field or the expression for electric field from part (c). The third point was earned because the response integrates the expression for electric field that was substituted in the previous point using correct integration limits. Part (e)(i) earned 3 points. The first point was earned because the response correctly indicates that E is positive with a magnitude that increases linearly in the region $0 < r < R$. The second point was earned because the response correctly indicates a positive, decreasing, concave-up graph for $r > R$. The third point was earned because the response correctly indicates a discontinuous increase in magnitude at $r = 3R$. Part (e)(ii) earned 3 points. The first point was earned because the response correctly indicates that the potential is concave down and always decreasing as r increases within Region I. The second point was earned because the response correctly indicates that the potential is concave up and always decreasing as r increases within Region II and concave up and always decreasing as r increases within Region III. The third point was earned because the response correctly indicates that the potential graph is continuous across all three regions.

Question 1 (continued)

Sample: 1B
Score: 9

Part (a) earned 1 point because the response correctly indicates a charge of $+5Q$ on the outer surface of the shell. Part (b) earned 2 points. The first point was earned because the response correctly uses Gauss's law, including an attempt at a substitution for both area and enclosed charge. The second point was earned because the response indicates and uses the correct enclosed charge. The third point was not earned because the response uses the incorrect area for the surface area for a cylinder. Part (c) earned 2 points. The first point was earned because the response correctly indicates that the electric field has a reciprocal dependence on r by writing an expression for the electric field and dividing the given E value by 2. The second point was earned because the response indicates the correct answer of 6 N/C. Part (d) earned 2 points. The first point was earned because the response correctly uses the equation relating potential difference to electric field, including an attempt at integration limits and evaluating the integral. The second point was earned because the response substitutes the expression for electric field from part (c). The expression is not correct as it is missing the L in the denominator; however, the point is awarded for substitution and not accuracy to avoid double jeopardy with the third point in part (d). The third point was not earned because the response does not integrate correctly the expression for electric field that was substituted in the previous point. Part (e)(i) earned 2 points. The first point was not earned because the response incorrectly indicates that E is concave down in the region $0 < r < R$. The second point was earned because the response correctly indicates a positive, decreasing, concave-up graph for $r > R$. The third point was earned because the response correctly indicates a discontinuous increase in magnitude at $r = 3R$. Part (e)(ii) earned 0 points. The first point was not earned because the response incorrectly indicates that the potential is concave up and always increasing as r increases within Region I. The second point was not earned because the response incorrectly indicates that the potential is concave down in Regions II and III. The third point was not earned because the response incorrectly indicates that the potential graph is discontinuous at $3R$.

Sample: 1C
Score: 5

Part (a) earned 0 points because the response does not indicate a charge of $+5Q$ on the outer surface of the shell. Part (b) earned 1 point. The first point was earned because the response correctly uses Gauss's law, including an attempt at a substitution for both area and enclosed charge. The second point was not earned because the response uses the incorrect enclosed charge. The third point was not earned because the response uses the incorrect area for the surface area for a cylinder. Part (c) earned 0 points. The first point was not earned because the response does not correctly indicate that the electric field has a reciprocal dependence on r. The second point was not earned because the response indicates the incorrect answer of 3 N/C. Part (d) earned 2 points. The first point was earned because the response correctly uses the equation relating potential difference to electric field, including an attempt at integration limits and evaluating the integral. The second point was earned because the response substitutes the expression for electric field from part (c). The expression is not correct, but the point is awarded for substitution and not accuracy to avoid double jeopardy with the third point in part (d). The third point was not earned because the response does not integrate correctly the expression for electric field that was substituted in the previous point. Part (e)(i) earned 1 point. The first point was not earned because the response incorrectly indicates that E is concave down in the region $0 < r < R$. The second point was not earned because the response incorrectly indicates that E is constant in Region III. The third point was earned because the response correctly indicates a discontinuous increase in magnitude of the electric field at $r = 3R$. Part (e)(ii) earned 1 point. The first point was not earned because the response does not indicate that the potential is concave down and always decreasing as r increases within Region I. The second point was not earned because the response does not indicate that the potential is concave up and always decreasing as r increases within Region II and concave up and always decreasing as r increases within Region III. The third point was earned because the response correctly indicates that the potential graph is continuous across all three regions.