2022

AP® | CollegeBoard

AP® Physics C: Electricity and Magnetism

Sample Student Responses and Scoring Commentary Set 1

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Free-Response Question 2

☑ Scoring Guidelines

☑ Student Samples

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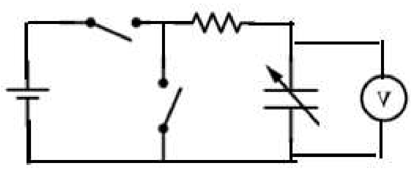
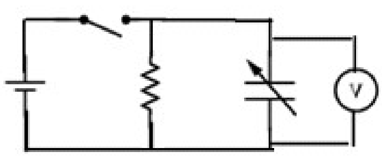
Question 2: Free-Response Question — 15 points

(a)	For a schematic diagram with the capacitor in series with the resistor **OR** For a schematic diagram with the resistor on the parallel path	**1 point**
	For a schematic diagram with the voltmeter in parallel with the capacitor	**1 point**
	For a schematic diagram that uses a switch to connect the battery to the capacitor	**1 point**
	For a schematic diagram that uses a switch that allows the capacitor to discharge through the resistor	**1 point**

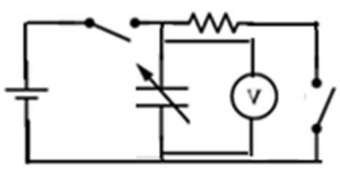
Example Responses



OR



OR



Total for part (a) 4 points

(b) For using an appropriate loop equation — **1 point**

Example Response

$V_C - V_R = 0$

For correctly substituting $\frac{q}{C}$ and IR in a loop equation consistent with the first point — **1 point**

Example Response

$\frac{q}{C} = IR$

	For using a differential expression consistent with the loop rule in the first point that includes a substitution of $I = -\frac{dq}{dt}$ for the current	**1 point**

Example Response

$$\frac{q}{C} = -\frac{dq}{dt}R$$

$$-\frac{1}{RC}dt = \frac{1}{q}dq$$

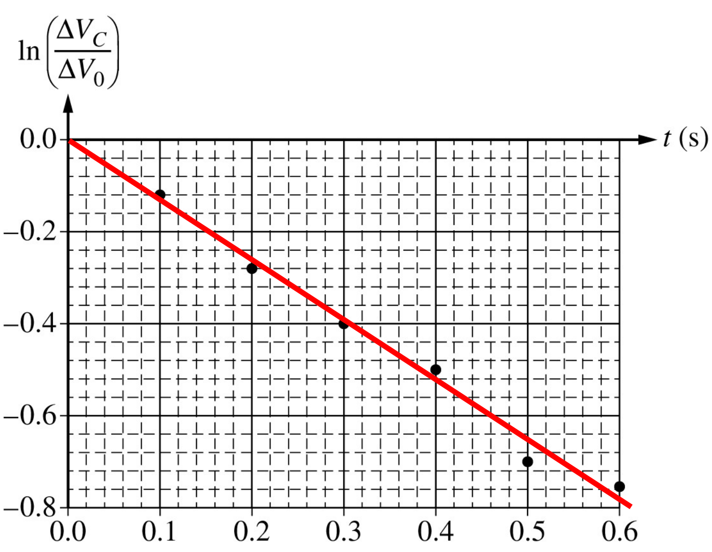
$$-\frac{1}{RC}\int_{t=0}^{t=t} dt = \int_{q=q_0}^{q=q} \frac{1}{q}dq$$

$$\frac{-t}{RC} = \ln\left(\frac{q(t)}{q_0}\right) \therefore \frac{q(t)}{q_0} = e^{-t/RC} \therefore \frac{q(t)}{C} = \frac{q_0}{C}e^{-t/RC} \therefore V(t) = V_0 e^{-t/RC}$$

Scoring Note: The point can be earned regardless of the sign used in the substitution of $\frac{dq}{dt}$ for the current.

	Total for part (b)	**3 points**
(c)(i)	For drawing an appropriate best-fit line	**1 point**

Example Response



(c)(ii) For using an appropriate equation to relate the unknown capacitance to the data **1 point**

Example Response

$$V(t) = V_0 e^{-t/RC} \therefore \frac{V(t)}{V_0} = e^{-t/RC} \therefore \ln\left(\frac{V(t)}{V_0}\right) = -\frac{t}{RC}$$

For correctly calculating the slope using two points from the best-fit line **1 point**

Example Response

$$\text{slope} = \frac{\Delta\left(\ln\left(\frac{V}{V_0}\right)\right)}{\Delta t} = \frac{(-0.70 - (-0.40))}{(0.54\ \text{s} - 0.30\ \text{s})} = -1.25\ \text{s}^{-1}$$

Scoring Note: A response may earn this point regardless of the associated unit.

For correctly relating the slope to the unknown capacitance **1 point**

Example Response

$$\text{slope} = -\frac{1}{RC} \therefore C = \frac{-1}{\text{slope} \times (R)} = \frac{-1}{\left(-1.25\ \text{s}^{-1}\right) \times (150\ \text{k}\Omega)} = 5\mu\text{F}$$

Total for part (c) 4 points

(d) For selecting "Less steep" and an attempt at a relevant justification **1 point**

For correctly relating the correct change in capacitance to the slope of the graph **1 point**

Example Response

Increasing the area of the plates increases the capacitance of the capacitor; thus, the magnitude of the slope will decrease.

Scoring Note: Part (d) is scored consistently with part (c).

Total for part (d) 2 points

(e)(i) Correct answer: "No"

For selecting "No" AND correctly describing the effects of the internal resistance of the battery on the slope of the graph at $t = 0$ **1 point**

Example Response

No. The capacitor still discharges only though resistor R, so the slope is the same.

(e)(ii)	For selecting "No" AND correctly describing the effects of the internal resistance of the battery on the initial value of the graph	**1 point**

Example Response

No. The best-fit line does not change, because the internal resistance of the battery does not affect the final potential difference across the charging capacitor.

Scoring Note: This point is scored with consistency with the circuit drawn in part (b).

Total for part (e) 2 points

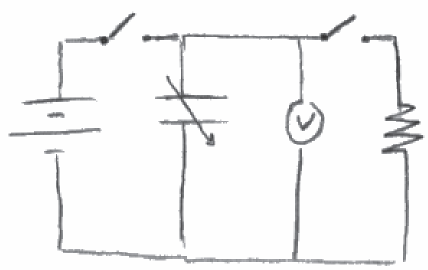
Total for question 2 15 points

Question 2

Begin your response to QUESTION 2 on this page.

2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

$V - IR = 0$ $\quad$ $Q(t) = Q_0 e^{-t/RC}$

$\frac{Q}{C} = IR$ $\quad$ $\frac{d}{dt} Q(t) \cdot R = -I_0 R e^{-t/RC}$

$\frac{dQ}{dt} R = \frac{Q}{C}$ $\quad$ $\Delta V_c(t) = V_0 e^{-t/RC}$

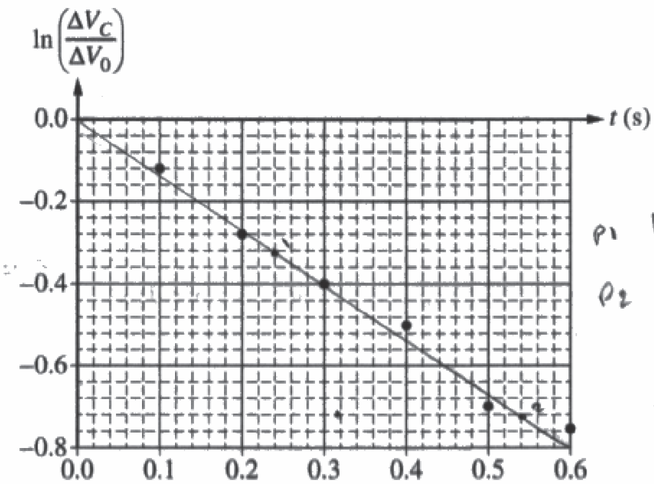
$\int_{Q_0}^{Q} \frac{dQ}{Q} = \int_0^t \frac{dt}{RC} \Rightarrow -\ln \frac{Q}{Q_0} = \frac{t}{RC} \Rightarrow Q = Q_0 e^{-t/RC}$

GO ON TO THE NEXT PAGE.

Question 2

Continue your response to QUESTION 2 on this page.

(c) The experiment is performed using a resistor of $R = 150\ \text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



P1 (0.24, −0.32)
P2 (0.54, −0.72)

i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the unknown capacitance C.

$$\frac{\ln\left(\frac{\Delta V_c}{\Delta V_0}\right)}{t} = m = \frac{\Delta y}{\Delta x} = \frac{-0.72 + 0.32}{0.54 - 0.24} = -1.33$$

$$\frac{-\ln\left(\frac{\Delta V_c}{\Delta V_0}\right)}{t} = \frac{1}{RC}$$

$$C = \frac{1}{(-m)R} = \frac{1}{150 \cdot 10^3 \cdot 1.33} \approx 5.01\ \text{F}$$

GO ON TO THE NEXT PAGE.

Question 2

Continue your response to **QUESTION 2** on this page.

(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

___ More steep   ✓ Less steep   ___ Unchanged

Briefly justify your answer.

$C = \frac{k\varepsilon_0 A}{d}$ A↑ : C↑ : $C \propto \frac{1}{k}$

the slope is inversely proportional to capacitance such that if capacitance were to increase due to an increase in Area, the slope would have to be less steep.

(e) The ideal battery is then replaced with a non-ideal battery with internal resistance *r*, and the experiment is repeated.

i. Would the slope of the graph in this final experiment change compared to the graph in part (c)?

✓ Yes   ___ No

Briefly justify your answer.

Although y would remain unchanged as it refers to change in voltages, the rate at which the capacitor charges and discharges would be increased by the additional internal resistance as per $\tau = RC$

ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

___ Yes   ✓ No

Briefly justify your answer.

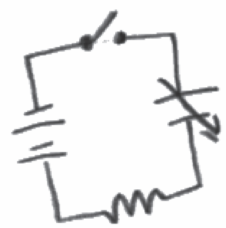
The initial voltage would still be immediately equal when the capacitor begins to discharge meaning it would remain at ln (1) = 0

Question 2

Begin your response to QUESTION 2 on this page.

2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

$I = -\frac{dQ}{dt} = \frac{\Delta V_C}{R}$ $\qquad -\frac{dQ}{dt} = \frac{Q}{RC}$

$\Delta V_C = \frac{Q}{C}$ $\qquad$ if $Q = e^{-\frac{t}{RC}}$

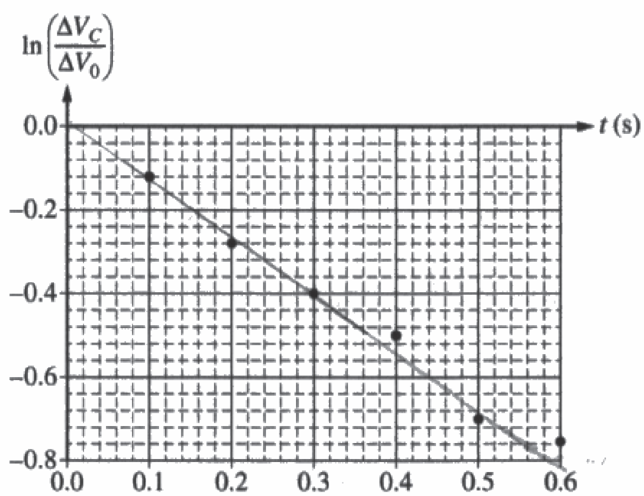
$\Delta V_0 = \frac{Q_0}{C}$ $\qquad \frac{dQ}{dt} = -\frac{Q}{RC}$ $\quad Q = e^{-\frac{t}{RC}}$

$\Delta V_C = \Delta V_0 e^{-\frac{t}{RC}}$

Question 2

Continue your response to **QUESTION 2** on this page.

(c) The experiment is performed using a resistor of $R = 150\ \text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the unknown capacitance C.

Slope ~ −0.75 = $-\frac{1}{RC}$

$C = \frac{1}{R(0.75)}$

$= 8.88 \cdot 10^{-6}$ F

Question 2

Continue your response to **QUESTION 2** on this page.

(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

___ More steep ✓ Less steep ___ Unchanged

Briefly justify your answer.

increasing the surface area would increase the capacitance which would reduce $1/RC$ which is the slope's magnitude

(e) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.

i. Would the slope of the graph in this final experiment change compared to the graph in part (c)?

✓ Yes ___ No

Briefly justify your answer.

We are changing the resistance so we are changing $-\frac{1}{RC}$ which is the slope

ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ✓ No

Briefly justify your answer.

The vertical intercept is when $V_C = V_0$ at $t=0$ so $\ln(1)=0$ the vertical intercept will always be $(0,0)$

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Q5136/8

Question 2

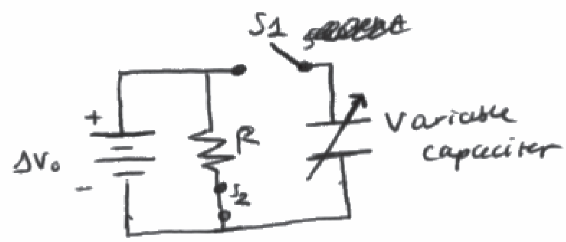
Begin your response to QUESTION 2 on this page.

2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

Battery	Voltmeter	Resistor	Variable Capacitor	Switch

(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.

- ideal battery (ΔV_0)
- a single resistor (R)
- an uncharged capacitor (C)
- 1 or more switches



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

$\Delta V = IR$

$\Delta V = RC$

$\Delta V = \frac{Q}{c}$

$\Delta V_c(t) = \Delta V_0 e^{-\frac{t}{RC}}$

$(IR)(c) = RC\frac{dQ}{dt} \cdot \frac{Q}{C}$

RC^{Q}

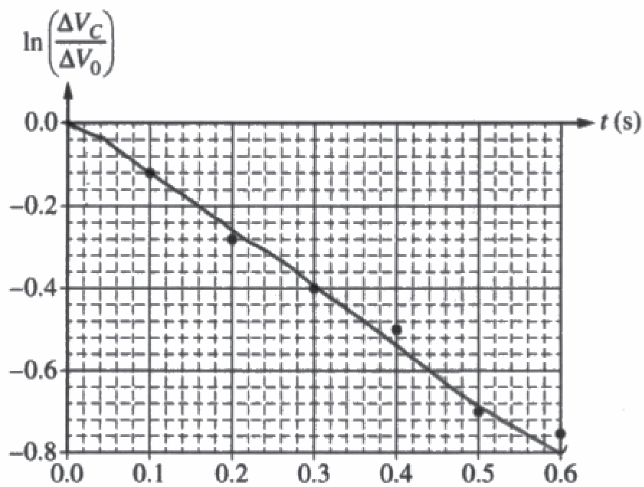
$\ln e \Delta V_c = \ln e \Delta V_0 + -\frac{t}{RC}$

$\Delta V_c(t) = \Delta V_0 e^{-\frac{t}{RC}}$

Question 2

Continue your response to **QUESTION 2** on this page.

(c) The experiment is performed using a resistor of $R = 150\ \text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the unknown capacitance C.

$\ln \frac{\Delta V_c}{\Delta V_0} \cdot s$

$(0.1, -0.12)$ and $(0.3, -0.4)$

$\frac{\Delta y}{\Delta x} = m \Rightarrow \frac{-0.4 - (-0.12)}{0.3 - 0.1} = |-1.4|$

$\ln | 1.4 | = 0.336$

$C = 0.336\ F$

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

0005670

Q5136/7

Question 2

Continue your response to **QUESTION 2** on this page.

(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

✓ More steep ___ Less steep ___ Unchanged

Briefly justify your answer.

~~[illegible]~~ $\frac{\mu_0 A}{d}$ If A is increased the slope will be more steep.

(e) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.

i. Would the slope of the graph in this final experiment change compared to the graph in part (c)?

✓ Yes ___ No

Briefly justify your answer.

The internal resistance would cause the voltage drop to decrease therefore the graph/slope will change.

ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ✓ No

Briefly justify your answer.

This because if there is no voltage there is no voltage drop so it always starts at the origin.

Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Identify the behavior of capacitors in circuits, specifically the properties of charging and discharging RC circuits, including their time dependence.
- Draw a circuit diagram that allows a capacitor to be charged and then discharged through a resistor and ammeter using given circuit elements.
- Use Kirchoff's and Ohm's laws to write a differential equation for a discharging RC circuit that can be integrated to determine the voltage across the capacitor as a function of time.
- Associate the parameters in an equation for an RC circuit with the characteristics of a corresponding graph.
- Use a graph to determine the capacitance of a capacitor using the slope of the line and an equation for the voltage across a discharging capacitor.
- Provide reasoning to justify a claim concerning the changes of the slope and intercept of the graph based on a model for the capacitor circuit.

Sample: 2A
Score: 14

Part (a) earned 4 points. The first point was earned because a schematic diagram with the resistor on a parallel path with the capacitor is shown. The second point was earned because the diagram shows the voltmeter in parallel with the capacitor. The third point was earned because the schematic diagram uses a switch to connect the battery to the capacitor. The fourth point was earned because the schematic diagram uses a switch that allows the capacitor to discharge through the resistor. Part (b) earned 3 points. The first point was earned because an appropriate loop equation is used. The second point was earned because both $\frac{q}{C}$ and IR are substituted into the loop equation. The third point was earned because the differential expression $I = \frac{dq}{dt}$ is substituted into the loop equation to make an integrable equation. Part (c)(i) earned 1 point because an appropriate best-fit line is shown on the graph. Part (c)(ii) earned 3 points. The first point was earned because the response correctly relates the unknown capacitance to the data. The second point was earned because the response correctly calculates the slope using two points from the best-fit line. The third point was earned because the calculated slope is correctly related to the unknown capacitance. Part (d) earned 2 points. The first point was earned because "Less steep" is selected and a justification is attempted. The second point was earned because the justification for the change in capacitance and its relation to the slope is correct. Part (e)(i) earned 0 points because the correct answer "No" is not checked. Part (e)(ii) earned 1 point because the correct answer "No" is selected, and a correct justification is given.

Question 2 (continued)

Sample: 2B
Score: 10

Part (a) earned 2 points. The first point was earned because a schematic diagram with the capacitor in series with the resistor is shown. The second point was not earned because the diagram does not show the voltmeter in parallel with the capacitor. The third point was earned because the schematic diagram uses a switch to connect the battery to the capacitor. The fourth point was not earned because the schematic diagram does not contain a switch that allows the capacitor to discharge through the resistor. Part (b) earned 3 points. The first point was earned because an appropriate loop equation is used. The second point was earned because both $\frac{q}{C}$ and IR are substituted into the loop equation. The third point was earned because the differential expression $I = \frac{dq}{dt}$ is substituted into the loop equation to make an integrable equation. Part (c)(i) earned 1 point because an appropriate best-fit line is shown on the graph. Part (c)(ii) earned 1 point. The first point was not earned because the response does not relate the unknown capacitance to the data. The second point was not earned because the response does not show a correct calculation of the slope using two points from the best-fit line. The third point was earned because the calculated slope is correctly related to the unknown capacitance. Part (d) earned 2 points. The first point was earned because "Less steep" is selected and a justification is attempted. The second point was earned because the justification for the change in capacitance and its relation to the slope is correct. Part (e)(i) earned 0 points because the correct answer "No" is not checked. Part (e)(ii) earned 1 point because the correct answer "No" is selected, and a correct justification is given.

Sample: 2C
Score: 5

Part (a) earned 2 points. The first point was earned because a schematic diagram with the resistor on a parallel path with the capacitor is shown. The second point was not earned because the diagram does not show the voltmeter in parallel with the capacitor. The third point was earned because the schematic diagram uses a switch to connect the battery to the capacitor. The fourth point was not earned because the schematic diagram does not contain a switch that allows the capacitor to discharge through the resistor. Part (b) earned 0 points. The first point was not earned because an appropriate loop equation is not given. The second point was not earned because both $\frac{q}{C}$ and IR are shown but not substituted into a loop equation. The third point was not earned because the differential expression $I = \frac{dq}{dt}$ is not substituted into a loop equation. Part (c)(i) earned 1 point because an appropriate best-fit line is shown on the graph. Part (c)(ii) earned 1 point. The first point was not earned because the response does not relate the unknown capacitance to the data. The second point was earned because the response correctly calculates the slope using two points from the best-fit line. The third point was not earned because the calculated slope is not correctly related to the unknown capacitance. Part (d) earned 1 point. The first point was earned because the choice of more steep is commensurate with the statement in part (c) that the capacitance is equal to the slope. The second point was not earned because the justification for the change in capacitance and its relation to the slope is not correct; the capacitance is not related to the area of the plates. Part (e)(i) earned 0 points because the correct answer "No" is not checked. Part (e)(ii) earned 0 points because the correct answer "No" is not selected.