2022

AP® CollegeBoard

# AP® Physics C: Electricity and Magnetism

## Sample Student Responses and Scoring Commentary Set 2

**Inside:**

Free-Response Question 2

☑ Scoring Guidelines

☑ Student Samples

☑ Scoring Commentary

## Question 2: Free-Response Question — 15 points

**(a)** For a schematic diagram with the capacitor in series with the resistor — **1 point**

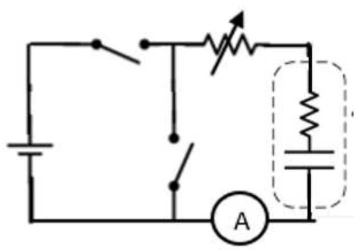
**Scoring Note:** The response may earn this point even if the variable resistor is not included in the circuit diagram.

For a schematic diagram with the ammeter in series with the capacitor and resistor — **1 point**

For a schematic diagram that uses a switch to connect the battery to the capacitor — **1 point**

For a schematic diagram that uses a switch that allows the capacitor to discharge through the resistor — **1 point**

**Example Response**




**Total for part (a) — 4 points**

**(b)** For using an appropriate loop equation by substituting a correct expression for the potential difference across the capacitor in terms of $I$, $C$, and $r_C$ and $IR$ for the potential difference across the variable resistor, if included — **1 point**

**Example Response**

$$V_C - V_R = 0$$

$$\frac{q}{C} - Ir_C = IR \therefore \frac{q}{C} = I(R + r_C)$$

For substituting $R + r_C$ as the total resistance of the circuit — **1 point**

**Example Response**

$$R_{\text{total}} = R + r_C$$

**Scoring Notes:**

- This point is earned if the above substitution is made anywhere in part (b).
- If the variable resistor is not included in the expression, accept expressions without $R$ throughout.

For a correct differential equation consistent with the first point that could be used to determine the current $I$ through the capacitor as a function of time $t$ — **1 point**

**Example Response**

$$\frac{dq}{dt}\frac{1}{C} = (R + r_C)\frac{dI}{dt}$$
$$-I\frac{1}{C} = (R + r_C)\frac{dI}{dt}; \quad I = -\frac{dq}{dt}$$

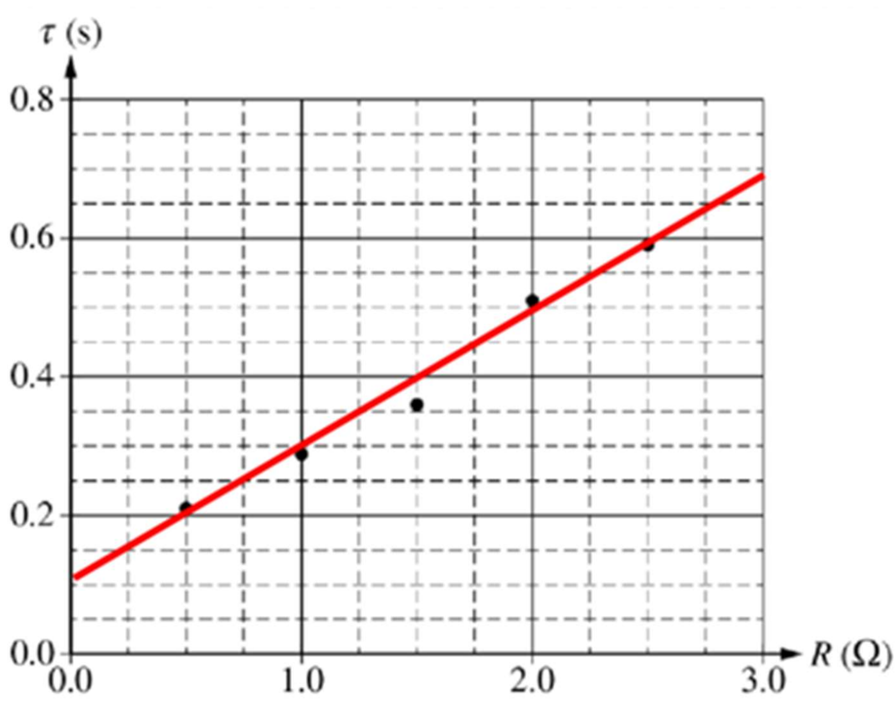
Where $q$ is the charge on one plate of the capacitor that decreases over time

$$\frac{1}{I}\frac{dI}{dt} = \frac{-1}{C(R + r_C)}$$
$$\ln\frac{I}{I_0} = \frac{-t}{C(R + r_C)}$$
$$I = I_0 e^{\frac{-t}{C(R+r_C)}}$$

**Total for part (b) 3 points**

**(c)(i)** For drawing an appropriate best-fit line — **1 point**

**Example Response**



| | | |
|---|---|---|
| **(c)(ii)** | For correctly determining the slope of the line<br><br>**Example Response**<br><br>$\text{slope} = \dfrac{(0.24 - 0.08)\ \text{s}}{(1.0 - 0.2)\ \Omega} = 0.2\ \text{F}$ | **1 point** |
| | For using the equation $\tau = RC$ to determine that the slope must be $C$<br><br>**Example Response**<br><br>$\tau = RC$<br>$\tau = (R + r_C)C$<br>$\tau = CR + r_C C$ | **1 point** |
| | For correctly relating the vertical intercept to the internal resistance of the capacitor<br><br>**Example Response**<br>Vertical intercept $= \tau_0 = r_C C$<br><br>Vertical intercept $= 0.08$ s<br><br>$r_C = \dfrac{\text{vertical intercept}}{C}$<br>$= \dfrac{0.08\ \text{s}}{0.2\ \text{F}}$<br><br>$r_C = 0.4\ \Omega$ | **1 point** |
| | **Total for part (c)** | **4 points** |
| **(d)** | For selecting "Less than" and an attempt at a relevant justification | **1 point** |
| | For a correct justification that the internal resistance would be less due to the unknown resistance that is measured being the equivalent resistance of the ammeter and capacitor | **1 point** |
| | **Example Response**<br>*The internal resistance of the ammeter would add to the internal resistance of the capacitor due to the fact the circuit elements are in series; this would result in an equivalent resistance that is measured in this experiment. Thus, the internal resistance of the capacitor is smaller than this equivalent resistance measured.* | |
| | **Total for part (d)** | **2 points** |
| **(e)** | For selecting "Remain unchanged" and an attempt at a relevant justification | **1 point** |
| | For a correct justification that the slope of the line is capacitance, which is independent of resistance | **1 point** |

**Example Response**

*The relationship between the time constant and the resistance, the slope, is the capacitance, which does not change regardless of how large the value of the resistance is.*

**Scoring Note:** This point is scored with consistency with the circuit drawn in part (a).

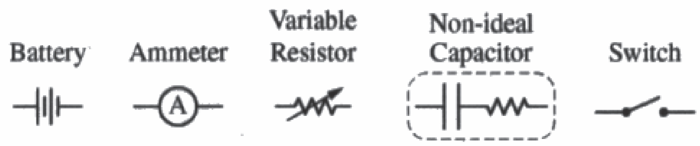
**Total for part (e) 2 points**

**Total for question 2 15 points**

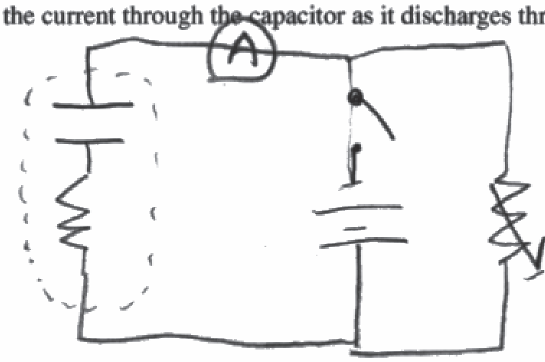
## Question 2

**Begin your response to QUESTION 2 on this page.**

2. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance $r_C$. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference $\Delta V_0$, a single ammeter, a single variable resistor of resistance $R$, a single uncharged non-ideal capacitor of capacitance $C$, and one or more switches as needed.




(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the current through the capacitor as it discharges through the resistor.




The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the current $I$ through the capacitor as a function of time $t$ is $I(t) = I_0 e^{\frac{-t}{(R+r_C)C}}$ as the capacitor discharges.

$$\frac{Q}{C} - IR - Ir_c = 0 \qquad I = \frac{dq}{dt}$$

$$Q = \frac{dq}{dt}(R+R_c)C$$

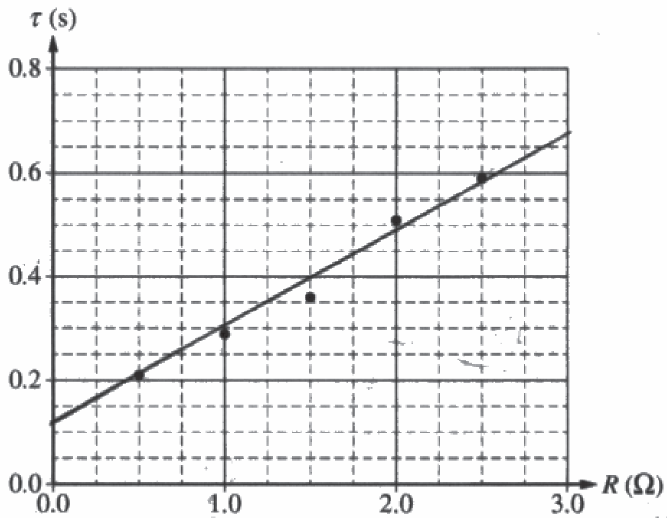
$$\frac{dq}{Q} = \frac{dt}{(R+R_c)C}$$

$$\ln(Q) = \frac{t}{(R+R_c)C}$$

## Question 2

Continue your response to **QUESTION 2** on this page.

(c) The students determine the time constant $\tau$ for the circuit as a function of the resistance $R$. The students' data are shown in the following graph.




i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the internal resistance $r_C$ of the capacitor.

$C = \text{slope}$

$\tau = (R + r_c)C \qquad C = \frac{0.5 - 0.3}{1} = 0.2$

$\tau = RC + r_c C$

$r_c C = y_{int} = 0.125$

$r_c = \frac{0.125}{0.2} = \boxed{.625\,\Omega}$

## Question 2

Continue your response to **QUESTION 2** on this page.

(d) The ammeter is found to be nonideal. Is the actual value for the internal resistance $r_C$ for the capacitor greater than, less than, or equal to the experimental internal resistance of the capacitor calculated in part (c)?

___ Greater than  ✓ Less than  ___ Equal to

Briefly justify your answer using features of the graph in part (c).

If the ammeter is nonideal, then it has resistance. This means the expirimental value of the IR includes both the true value and the resistance of the ammeter. Thus the actual value will be less than the experimental value

(e) The values of the variable resistor in the original experiment ranged from 0.5 Ω to 2.5 Ω. The experiment is repeated with values ranging from 3.0 Ω to 6.0 Ω. Would the slope of the best-fit line be more steep, be less steep, or remain unchanged compared to the graph in part (c)?

~~✓~~ More steep  ___ Less steep  ✓ Remain unchanged

Briefly justify your answer.

The slope of the line is equal to C, aka the capacitance. Thus resistance has nothing to do w/ the value of the slope

## Question 2

Continue your response to **QUESTION 2** on this page.



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## Question 2

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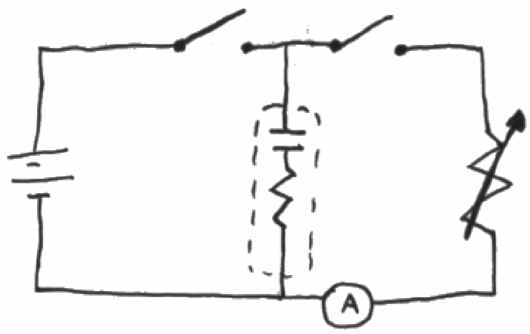
2. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance $r_C$. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference $\Delta V_0$, a single ammeter, a single variable resistor of resistance $R$, a single uncharged non-ideal capacitor of capacitance $C$, and one or more switches as needed.


(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the current through the capacitor as it discharges through the resistor.




The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the current $I$ through the capacitor as a function of time $t$ is $I(t) = I_0 e^{\frac{-t}{(R+r_C)C}}$ as the capacitor discharges.

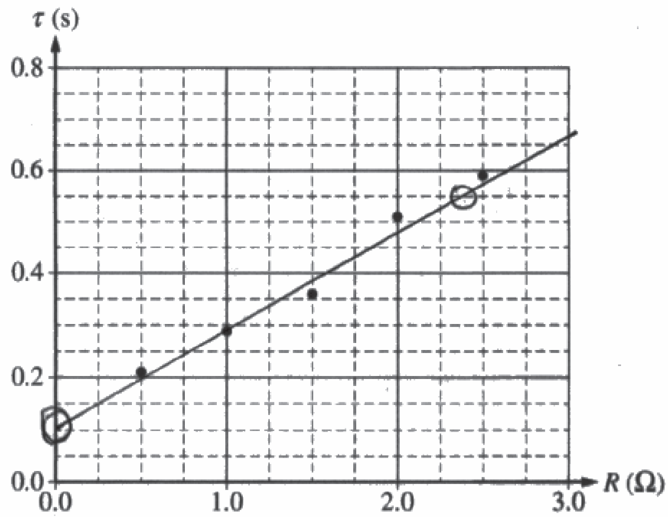
$I = I_0 \left(e^{-t/RC}\right)$

$R = R_{tot} = (R + r_c)$

## Question 2

Continue your response to **QUESTION 2** on this page.

(c) The students determine the time constant $\tau$ for the circuit as a function of the resistance $R$. The students' data are shown in the following graph.




i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the internal resistance $r_C$ of the capacitor.

(0, 0.1)
(2.375, 0.55)

Slope = $\frac{0.55 - 0.1}{2.375 - 0}$ = 0.189 S/Ω

(Slope$^{-1}$)t = Ω
= ~~5.278Ω~~
= 5.278t

@ t = 0

~~Ω~~
Ω = 0

ℰ − IR − IC = 0
ℰ − I$r_c$ = 0





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0001564

Q5133/7

## Question 2

Continue your response to **QUESTION 2** on this page.

(d) The ammeter is found to be nonideal. Is the actual value for the internal resistance $r_C$ for the capacitor greater than, less than, or equal to the experimental internal resistance of the capacitor calculated in part (c)?

___ Greater than  X Less than  ___ Equal to

Briefly justify your answer using features of the graph in part (c).

resistors in series create a larger req than an individual resistor, therefore

$R_c = r_{eq} - r_{ammeter}$

(e) The values of the variable resistor in the original experiment ranged from 0.5 Ω to 2.5 Ω. The experiment is repeated with values ranging from 3.0 Ω to 6.0 Ω. Would the slope of the best-fit line be more steep, be less steep, or remain unchanged compared to the graph in part (c)?

___ More steep  X Less steep  ___ Remain unchanged

Briefly justify your answer.

$\text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{\Delta \$}{\Delta \Omega}$

with greater $\Delta \Omega$, the slope will be less steep.



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## Question 2

Continue your response to **QUESTION 2** on this page.





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## Question 2

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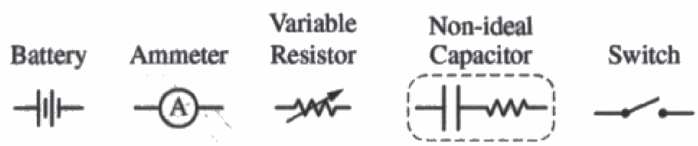
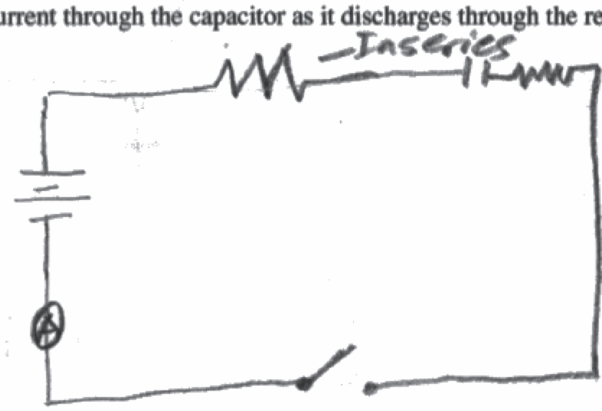
2. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance $r_C$. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference $\Delta V_0$, a single ammeter, a single variable resistor of resistance $R$, a single uncharged non-ideal capacitor of capacitance $C$, and one or more switches as needed.




(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the current through the capacitor as it discharges through the resistor.




The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the current $I$ through the capacitor as a function of time $t$ is $I(t) = I_0 e^{\frac{-t}{(R+r_C)C}}$ as the capacitor discharges.

As time increases discharge accumulates, hence ROC current is negative and inversely proportional to $(R + r_c)C$



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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5133/6

## Question 2

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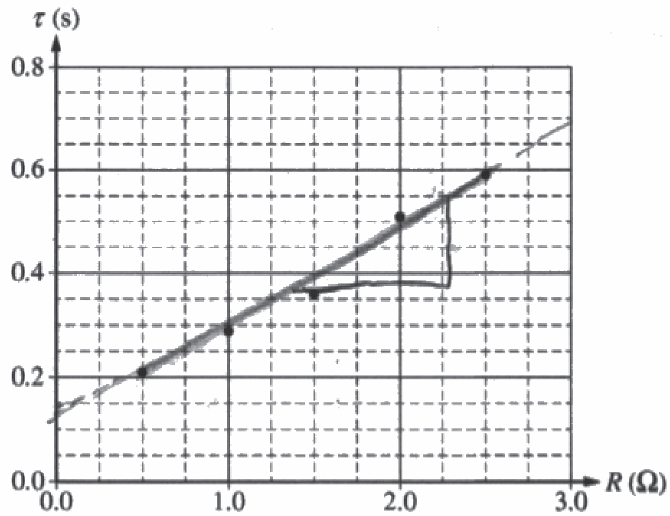
(c) The students determine the time constant $\tau$ for the circuit as a function of the resistance $R$. The students' data are shown in the following graph.




i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the internal resistance $r_C$ of the capacitor.

Resistance = Pointcharge·lenth / Arena

$R = \frac{\rho l}{A}$ $\quad R_2 = 2 = \frac{0.51L}{L^2} = 0.51L$

$L(0.51-0.29)$ $\quad R_1 = 1 = \frac{0.29L}{L^2} = 0.29L$

$L = 0.22 \cdot \mathcal{E}^2 = 2.17\Omega$

## Question 2

Continue your response to **QUESTION 2** on this page.

(d) The ammeter is found to be nonideal. Is the actual value for the internal resistance $r_C$ for the capacitor greater than, less than, or equal to the experimental internal resistance of the capacitor calculated in part (c)?

✓ Greater than    ___ Less than    ___ Equal to

Briefly justify your answer using features of the graph in part (c).

Some Charge is lost in non-ideal ammeter, hence internal resistance of capacitor is lower than real.

(e) The values of the variable resistor in the original experiment ranged from 0.5 Ω to 2.5 Ω. The experiment is repeated with values ranging from 3.0 Ω to 6.0 Ω. Would the slope of the best-fit line be more steep, be less steep, or remain unchanged compared to the graph in part (c)?

___ More steep    ✓ Less steep    ___ Remain unchanged

Briefly justify your answer.

2.5 − 0.5 > 6 − 3

2Ω > 3Ω

Since Ω is the indep variable (x axis) of graph, increasing Ω values while keeping τ constant would flatten the curve more.

## Question 2

Continue your response to **QUESTION 2** on this page.





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## Question 2

**Note:** Student samples are quoted verbatim and may contain spelling and grammatical errors.

### Overview

The responses were expected to demonstrate the ability to:

- Identify the behavior of capacitors in circuits, specifically the properties of charging and discharging RC circuits, including their time dependence.
- Draw a circuit diagram that allows a capacitor to be charged and then discharged through a resistor and ammeter using given circuit elements.
- Use Kirchoff's and Ohm's laws to write a differential equation for a discharging RC circuit that can be integrated to determine the current through the capacitor circuit as a function of time.
- Associate the parameters in an equation for an RC circuit with the characteristics of a corresponding graph.
- Use a graph to determine the internal resistance of a capacitor using the slope of the line and an equation for the current in a circuit with a discharging capacitor.
- Provide reasoning to justify a claim concerning the changes of the slope and intercept of the graph.

**Sample: 2A**
**Score: 15**

Part (a) earned 4 points. The first point was earned because the diagram drawn correctly shows that the capacitor is in series with the resistor. The second point was earned because the diagram drawn correctly shows that the ammeter is in series with the capacitor and resistor. The third point was earned because the diagram drawn correctly shows a switch that connects the battery to the capacitor. The fourth point was earned because the diagram drawn correctly shows a switch that allows the capacitor to be discharged through the resistor. Part (b) earned 3 points. The first point was earned because the response uses an appropriate loop equation with the correct substitutions for $V_C$ and $V_R$. The second point was earned because the response indicates that the total resistance of the circuit is $R + r_C$. The third point was earned because the response uses a correct differential equation. Part (c)(i) earned 1 point because an appropriate best fit line is drawn on the graph. Part (c)(ii) earned 3 points. The first point was earned because an acceptable value for the slope of the line is determined. The second point was earned because the slope of the graph is correctly determined to be capacitance. The third point was earned because the vertical intercept of the graph is correctly related to the internal resistance of the capacitor, resulting in an acceptable value of the internal resistance of the capacitor. Part (d) earned 2 points. The first point was earned because "Less than" is selected, followed by a relevant justification. The second point was earned because the response includes a correct justification. Part (e) earned 2 points. The first point was earned because "Remain unchanged" is selected, followed by a relevant justification. The second point was earned because the response includes a correct justification.

## Question 2 (continued)

**Sample: 2B**
**Score: 9**

Part (a) earned 4 points. The first point was earned because the diagram drawn correctly shows that the capacitor is in series with the resistor. The second point was earned because the diagram drawn correctly shows that the ammeter is in series with the capacitor and resistor. The third point was earned because the diagram drawn correctly shows a switch that connects the battery to the capacitor. The fourth point was earned because the diagram drawn correctly shows a switch that allows the capacitor to be discharged through the resistor. Part (b) earned 1 point. The first point was not earned because the response does not use a loop equation. The second point was earned because the response indicates that the total resistance of the circuit is $R + r_C$. The third point was not earned because the response does not use a correct differential equation. Part (c)(i) earned 1 point because an appropriate best-fit line is drawn on the graph. Part (c)(ii) earned 1 point. The first point was earned because an acceptable value for the slope of the line is determined. The second point was not earned because the slope of the graph is not determined to be capacitance. The third point was not earned because the vertical intercept of the graph was not related to the internal resistance of the capacitor. Part (d) earned 2 points. The first point was earned because "Less than" is selected, followed by a relevant justification. The second point was earned because the response includes a correct justification. Part (e) earned 0 points. The first point was not earned because "Remain unchanged" is not selected. The second point was not earned because the justification is incorrect.

**Sample: 2C**
**Score: 4**

Part (a) earned 3 points. The first point was earned because the diagram drawn correctly shows that the capacitor is in series with the resistor. The second point was earned because the diagram drawn correctly shows that the ammeter is in series with the capacitor and resistor. The third point was earned because the diagram drawn correctly shows a switch that connects the battery to the capacitor. The fourth point was not earned because the diagram drawn does not show a switch that allows the capacitor to be discharged through the resistor. Part (b) earned 0 points. The first point was not earned because the response does not use a loop equation. The second point was not earned because the response does not indicate that the total resistance of the circuit is $R + r_C$. The third point was not earned because the response does not use a differential equation. Part (c)(i) earned 1 point because an appropriate best-fit line is drawn on the graph. Part (c)(ii) earned 0 points. The first point was not earned because a value for the slope is not determined. The second point was not earned because the slope of the graph is not determined to be capacitance. The third point was not earned because the vertical intercept of the graph was not related to the internal resistance of the capacitor. Part (d) earned 0 points. The first point was not earned because "Less than" is not selected. The second point was not earned because the justification is incorrect. Part (e) earned 0 points. The first point was not earned because "Remain unchanged" is not selected. The second point was not earned because the justification is incorrect.