2022

AP® Physics C: Electricity and Magnetism

Scoring Guidelines
Set 1

Question 1: Free-Response Question — 15 points

(a) For correctly determining the charge on the outer surface of the shell — **1 point**

Example Response

$q_{net} = q_{inner} + q_{outer}$
$q_{outer} = q_{net} - q_{inner}$
$q_{outer} = +3Q - (+Q)$
$q_{outer} = +2Q$

Scoring Note: A correct response may earn a point even if no work is shown.

Total for part (a) 1 points

(b) For using Gauss's law by substituting for either the area or the enclosed charge — **1 point**

Example Response

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \cdot dA = E\left(4\pi r^2\right)$$

For using a correct expression for q_{enc} as a function of r — **1 point**

Example Response

$$q_{enc} = \rho V = \left(\frac{-Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = -Q\frac{r^3}{R^3}$$

Scoring Note: The response may earn this point regardless of the sign of q_{enc}.

For using the correct area — **1 point**

Example Response

$$E = q_{enc}\frac{1}{\varepsilon_0\left(4\pi r^2\right)} = \left(-Q\frac{r^3}{R^3}\right)\frac{1}{\varepsilon_0\left(4\pi r^2\right)}$$
$$E = -Q\frac{r}{4\pi\varepsilon_0 R^3}$$

Scoring Note: The response may earn this point regardless of the sign of E.

Total for part (b) 3 points

(c) For recognizing the electric field varies as an inverse square of the separation distance from the center of the nonconducting sphere **1 point**

Example Response

$$E \propto \frac{1}{r^2}$$

For the correct magnitude, including units, of the electric field at $2R$ **1 point**

Example Response

$$E_{new} = \frac{E_{old}}{4} = \frac{8 \text{ N/C}}{4}$$
$$E_{new} = 2 \text{ N/C}$$

Total for part (c) **2 points**

(d) For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral **1 point**

Example Response

$$V_f - V_i = -\int_{s_i}^{s_f} E dr$$
$$\Delta V = -\int_{r=R}^{r=4R} E dr$$

For substituting the correct expression for the electric field or an expression for E that is consistent with the explicit functional dependence of $E(r)$ from part (c) **1 point**

Example Response

$$\Delta V = -\int_{r=R}^{r=4R} -\frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\int_{r=R}^{r=4R} \frac{1}{r^2} dr$$

	For correctly integrating the electric field expression that was substituted in the previous point with correct limits of integration	**1 point**

Example Response

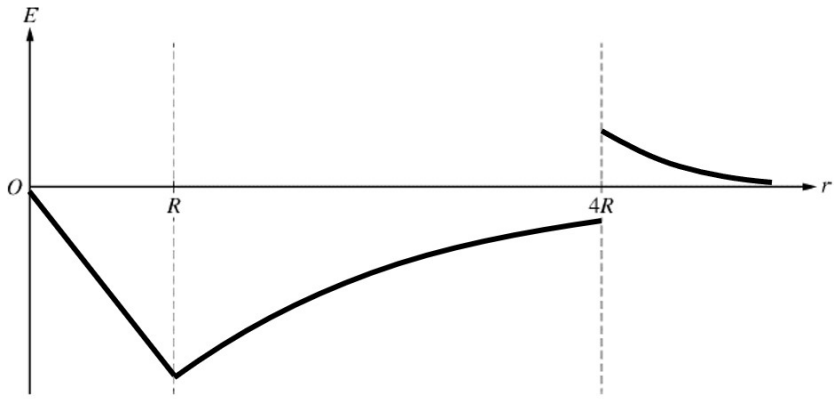
$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r=R}^{r=4R} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r=4R}^{r=R} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{4R}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{4-1}{4R}\right)$$

$$\Delta V = \frac{3Q}{16\pi\varepsilon_0 R}$$

Alternate Solution

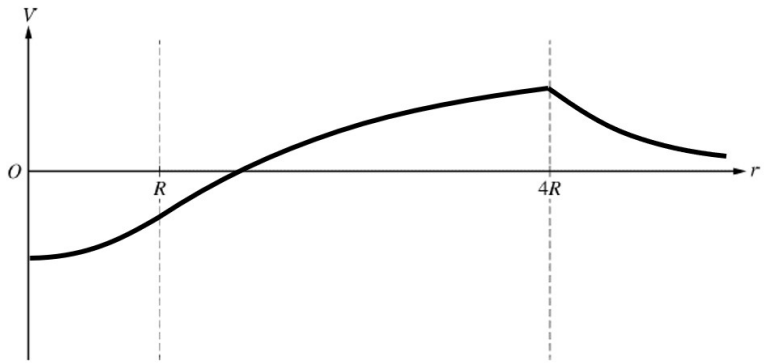
	For using a difference in potentials treating the inner sphere as a point charge by symmetry $\Delta V = \frac{Q}{4\pi\varepsilon_0 r_f} - \frac{Q}{4\pi\varepsilon_0 r_i}$	**1 Point**
	For using the correct charge on both of the terms; Q *not* Q *and* $3Q$	**1 Point**
	For using the correct values for r $\Delta V = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 4R}$ $\Delta V = \frac{3Q}{16\pi\varepsilon_0 R}$	**1 Point**
		Total for part (d) 3 points
(e)(i)	For indicating that E is negative and the magnitude of E increases linearly from 0 to R	**1 point**
	For a curve that is continuous at R and asymptotically approaches zero in the region $R < r < 4R$	**1 point**
	For a positive, concave-up, and decreasing curve for $r > 4R$	**1 point**

Example Response



(e)(ii)	For a potential curve that is always increasing from 0 to $4R$	**1 point**
	For a potential curve that is decreasing for $r > 4R$	**1 point**
	For a continuous graph across all three regions	**1 point**

Example Response



Scoring Note: The intercept of the curve on the vertical axis is irrelevant. The intercept of the curve on the horizontal axis is irrelevant. The curve can, hence, cross the horizontal axis at any location or even be entirely on the negative side of the horizontal axis as long as the other criteria are met.

Total for part (e) 6 points

Total for question 1 15 points

Question 2: Free-Response Question — 15 points

(a)

For a schematic diagram with the capacitor in series with the resistor — **1 point**

OR

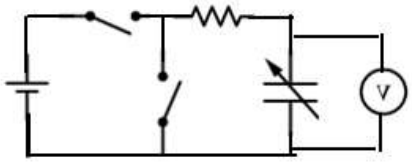
For a schematic diagram with the resistor on the parallel path

For a schematic diagram with the voltmeter in parallel with the capacitor — **1 point**

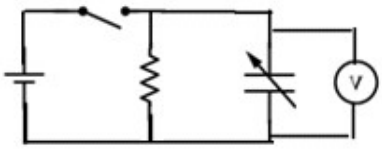
For a schematic diagram that uses a switch to connect the battery to the capacitor — **1 point**

For a schematic diagram that uses a switch that allows the capacitor to discharge through the resistor — **1 point**

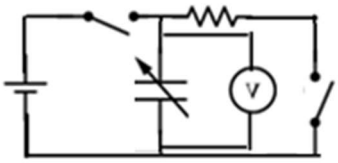
Example Responses



OR



OR



Total for part (a) — **4 points**

(b)

For using an appropriate loop equation — **1 point**

Example Response

$V_C - V_R = 0$

For correctly substituting $\frac{q}{C}$ and IR in a loop equation consistent with the first point — **1 point**

Example Response

$\frac{q}{C} = IR$

For using a differential expression consistent with the loop rule in the first point that includes a substitution of $I = -\frac{dq}{dt}$ for the current **1 point**

Example Response

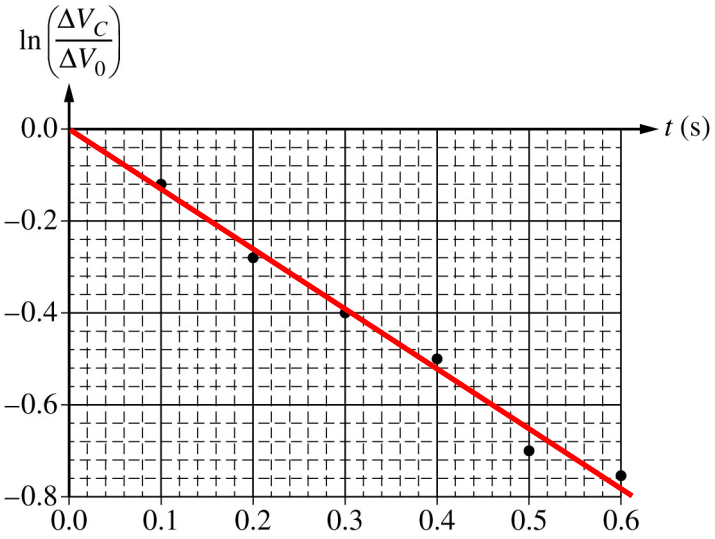
$$\frac{q}{C} = -\frac{dq}{dt}R$$
$$-\frac{1}{RC}dt = \frac{1}{q}dq$$
$$-\frac{1}{RC}\int_{t=0}^{t=t} dt = \int_{q=q_0}^{q=q} \frac{1}{q}dq$$
$$\frac{-t}{RC} = \ln\left(\frac{q(t)}{q_0}\right) \therefore \frac{q(t)}{q_0} = e^{-t/RC} \therefore \frac{q(t)}{C} = \frac{q_0}{C}e^{-t/RC} \therefore V(t) = V_0 e^{-t/RC}$$

Scoring Note: The point can be earned regardless of the sign used in the substitution of $\frac{dq}{dt}$ for the current.

Total for part (b) 3 points

(c)(i) For drawing an appropriate best-fit line **1 point**

Example Response



(c)(ii) For using an appropriate equation to relate the unknown capacitance to the data **1 point**

Example Response

$$V(t) = V_0 e^{-t/RC} \;\therefore\; \frac{V(t)}{V_0} = e^{-t/RC} \;\therefore\; \ln\left(\frac{V(t)}{V_0}\right) = -\frac{t}{RC}$$

For correctly calculating the slope using two points from the best-fit line **1 point**

Example Response

$$\text{slope} = \frac{\Delta\left(\ln\left(\frac{V}{V_0}\right)\right)}{\Delta t} = \frac{(-0.70 - (-0.40))}{(0.54\text{ s} - 0.30\text{ s})} = -1.25\text{ s}^{-1}$$

Scoring Note: A response may earn this point regardless of the associated unit.

For correctly relating the slope to the unknown capacitance **1 point**

Example Response

$$\text{slope} = -\frac{1}{RC} \;\therefore\; C = \frac{-1}{\text{slope} \times (R)} = \frac{-1}{\left(-1.25\text{ s}^{-1}\right) \times (150\text{ k}\Omega)} = 5\mu\text{F}$$

Total for part (c) **4 points**

(d) For selecting "Less steep" and an attempt at a relevant justification **1 point**

For correctly relating the correct change in capacitance to the slope of the graph **1 point**

Example Response

Increasing the area of the plates increases the capacitance of the capacitor; thus, the magnitude of the slope will decrease.

Scoring Note: Part (d) is scored consistently with part (c).

Total for part (d) **2 points**

(e)(i) Correct answer: "No"

For selecting "No" AND correctly describing the effects of the internal resistance of the battery on the slope of the graph at $t = 0$ **1 point**

Example Response

No. The capacitor still discharges only though resistor R, so the slope is the same.

(e)(ii)	For selecting "No" AND correctly describing the effects of the internal resistance of the battery on the initial value of the graph	**1 point**

Example Response

No. The best-fit line does not change, because the internal resistance of the battery does not affect the final potential difference across the charging capacitor.

Scoring Note: This point is scored with consistency with the circuit drawn in part (b).

Total for part (e) 2 points

Total for question 2 15 points

Question 3: Free-Response Question — 15 points

(a)

For selecting "Out of the page" and an attempt at a relevant justification — **1 point**

For a justification that correctly relates how the changing current in the long wire changes the flux through the loop with respect to time — **1 point**

For indicating the induced current will oppose the change in magnetic flux — **1 point**

Example Response

Because the current in the straight wire is decreasing, the magnetic field, which is originally pointing out of the page, is decreasing. Hence, the induced current produces a field that is directed out of the page to compensate for the decreasing flux.

Total for part (a) 3 points

(b)

For using an appropriate integral equation, with a substitution of an expression for magnetic field, to calculate magnetic flux — **1 point**

Example Response

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

For writing a correct equation for the magnetic field as a function of distance from the wire — **1 point**

Example Response

$$B = \frac{\mu_0 I}{2\pi r}$$

For substituting the value of $t = 3$ s to find the electric current — **1 point**

Example Response

$$I(t) = C - Dt$$
$$I(3\text{ s}) = 10\text{ A} - (2\text{ A/s})(3\text{ s})$$
$$I(3\text{ s}) = 4\text{ A}$$

For integrating B with correct limits and a correct substitution for dA, to determine the total flux through the loop — **1 point**

Example Response

$$\Phi_B = \int_d^{d+W} \frac{\mu_0 I L}{2\pi r} dr$$
$$\Phi_B = \frac{\mu_0 I L}{2\pi} \ln[r]_d^{d+W}$$
$$\Phi_B = \frac{\mu_0 I L}{2\pi} \ln\left[\frac{d+W}{d}\right]$$
$$\Phi_B = \frac{(4\pi \times 10^{-7}\ \text{T•m/A})(4\text{ A})(0.04\text{ m})}{2\pi} \ln\left[\frac{0.03\text{ m}}{0.01\text{ m}}\right] = 3.52 \times 10^{-8}\ \text{T•m}^2$$

Total for part (b) 4 points

(c) For using Faraday's law to determine the emf across the light bulb **1 point**

Example Response

$$\varepsilon = \left|\frac{d\Phi}{dt}\right|$$
$$\varepsilon = \frac{d}{dt}\left(\frac{\mu_0 (C - Dt) L}{2\pi} \ln\left[\frac{d + W}{d}\right]\right)$$
$$\varepsilon = \left(\frac{\mu_0 DL}{2\pi} \ln\left[\frac{d + W}{d}\right]\right)$$

For using Ohm's law to find the current in the light bulb consistent with the emf determined from the previous point **1 point**

Example Response

$$I = \frac{\varepsilon}{R}$$
$$I = \left(\frac{\mu_0 DL}{2\pi R} \ln\left[\frac{d + W}{d}\right]\right)$$

For correct substitutions of the values of $\frac{dI}{dt}$ and R **1 point**

Example Response

$$I = \frac{(4\pi \times 10^{-7}\ \text{T•m/A})(2.0\ \text{A/s})(0.04\ \text{m})}{(2\pi \text{•} 10\Omega)} \ln\left[\frac{0.03\ \text{m}}{0.01\ \text{m}}\right]$$
$$I = 1.8 \times 10^{-9}\ \text{A}$$

Total for part (c) 3 points

(d) For selecting: "The current in the long wire changes at a faster rate than expected." **1 point**

For correctly justifying the selection **1 point**

Scoring Note: A response cannot earn this point if the incorrect selection is chosen.

Example Response

If the current in the wire changes at a faster rate, there will be a greater rate of change of magnetic flux. So the induced emf and current will be higher.

Total for part (d) 2 points

(e)	For selecting "$I_2 < I_1$" with an attempt at a relevant justification	**1 point**
	For indicating the total flux in the loop is less in the new orientation	**1 point**
	For correctly relating the rate of change of the flux to the total flux inside the loop	**1 point**
	Example Response *With the new orientation, some parts of the rectangle are further away from the straight wire, which means that the magnetic flux through the rectangle will be less. The rate of change of the flux has the same dependence on distance and will also decrease, resulting in a smaller current.*	
	Total for part (e)	**3 points**
	Total for question 3	**15 points**