Chief Reader Report on Student Responses:

2021 AP® Physics C: Mechanics Free-Response Questions

• Number of Students Scored	48,803		
• Number of Readers	461 (for all Physics exams)		
• Score Distribution	Exam Score	N	%At
	5	11,468	23.5
	4	13,980	28.6
	3	10,412	21.3
	2	7,279	14.9
	1	5,664	11.6
• Global Mean	3.38		

The following comments on the 2021 free-response questions for AP® Physics C: Mechanics were written by the Chief Reader, Shannon Willoughby, Montana State University. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student preparation in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question #1 **Task:** Newton's laws **Topic:** Atwood Machine

Max. Points: 15 **Mean Score:** 9.07

What were the responses to this question expected to demonstrate?

The responses to this question were expected to demonstrate the following knowledge and skills:

- Derive a complete set of equations for a modified Atwood's machine both with and without friction.
- Describe an object's motion and the forces on the object both in equilibrium and while accelerating. Derive an expression for the net force in a system for these cases.
- Identify relationships within a system of connected objects.
- Determine the coefficient of friction between surfaces.
- Read and interpret data on a graph.
- Correctly draw and use a free body diagram.
- Defend or refute a student's contention.
- Calculate the slope of a best-fit line and use an appropriate equation to determine the acceleration due to gravity.

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

Some students struggled with drawing correct force diagrams in part B. This is a key skill for applying Newton's second law, and if this step is not done correctly then students will not typically calculate the correct solution. Students typically need a fair bit of practice with this skill, especially with systems that contain more than one object, and hence more than one free body diagram.

An associated skill is to correctly solve Newton's second law separately for each object (and associated force diagram).

In part D, many students did not adequately justify their reasoning for the difference between the measured and calculated quantities.

In part F, many students struggled with correctly justifying their reasoning for how the acceleration of the system would change if the surface was inclined at an angle. The skill required here is to understand how the force diagrams would change as a result of tipping the experiment.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Some students were unable to determine the coefficient of static friction for a two-block system at rest.	• Correctly calculated the coefficient of static friction for a two-block system at rest.
• Some students were unable to draw appropriate free-body force diagrams and/or label forces appropriately.	• Correctly drew free-body force diagrams with appropriate forces and correct vector labels.
• Some students were unable to evaluate Newton's second law on individual blocks or a system of connected blocks to determine a specific quantity.	• Correctly wrote out Newton's second law expressions based on the forces from each free-body diagram and combine them algebraically to derive an equation that

	could be used to calculate the coefficient of kinetic friction.
• Many students could not provide a sufficient justification to account for an observed discrepancy between a calculated and measured quantity.	• Correctly justified a potential explanation to account for the observed difference in a quantity.
• Many students were unable to relate the slope of a line to the calculation of a particular quantity.	• Wrote out the equation for the slope and indicated its relationship to the acceleration due to gravity.
• Some students did not correctly describe the relationship between forces acting on the objects of a system and the resulting acceleration.	• Correctly identified at least one reasonable force and explained specifically how it would alter the experimentally found value for *g*.
• Many students could not provide a sufficient justification to account for a change in the acceleration due to an alteration of the experimental setup.	• Correctly justified the change in acceleration due to a change in the experimental setup.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

Practice algebra skills like manipulating equations with variables, drawing a best fit line, and calculating slope from a line.

Practice drawing and using free body diagrams. Common mistakes can be avoided by reminding students that: (1) vectors need to originate from *and touch* the dot, (2) vectors need to be drawn in the direction of the forces rather than along curved lines and, (3) labels must be consistent with the particular object represented by the dot. For instance, instead of treating each dot in the system independently, students frequently tried to use the corresponding force on the *other* dot as the label, such as identifying tension on block 1 as m_2g (weight of block 2) instead of F_T.

Include example problems with both static and accelerating systems. Provide practice on identifying a positive direction and writing equations with consistent signs. On the Atwood's machine, students had some trouble identifying the "positive" force, for example, writing $f - T = m_1a$ rather than $T - f = m_1a$.

Question #2	**Task:** Rotational Inertia, Center of Mass, Rotational Dynamics	**Topic:** Rotation of a rigid object
	Max. Points: 15	**Mean Score:** 3.33

What were the responses to this question expected to demonstrate?

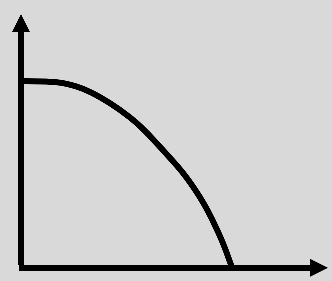
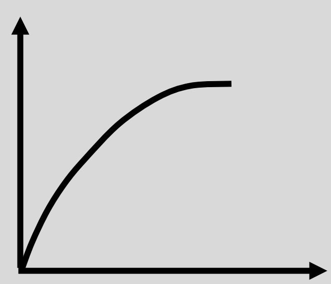
The responses to this question were expected to demonstrate the following:

- Use integral calculus to derive the rotational inertia of a long, thin rod. This required that the response correctly determine the density of the object, M/2L, and to set up a correct integral with correct limits.
- Determine the center of mass of a compound object made from two rods attached at a right angle, using the center of mass equation for discrete objects.
- Compare the rotational inertia of two objects, from part (a) and part (b), and justify the choice using conceptual understanding of rotational inertia.
- Sketch graphs to represent the rotational acceleration and velocity as a function of time for the object from part (b) as it swings down. Answering correctly required applying understanding of torque OR recognizing that the motion is analogous to a physical pendulum and that α and ω are related by calculus.
- Determine whether the angular acceleration would increase, decrease, or remain constant as the object rotates downwards, pulled by gravity. The response required understanding that angular acceleration is caused by torque and that the torque depends on the angle of rotation, and therefore would decrease as the object rotates downward.
- Derive an expression for the angular velocity of the object after it rotates 90° from its initial position. This required conservation of energy, equating the change in gravitational energy to the change in rotational kinetic energy.

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Few students were able to apply calculus correctly to this task.
- Many responses correctly stated the values for the center of mass without showing work. Some were able to write the appropriate equation but could not apply it to the situation.
- Responses were generally able to identify which object had the smaller rotational inertia, but very few were able to correctly apply the definition of rotational inertia to justify the choice.
- Few responses sketched the correct angular acceleration and angular velocity graphs, but a much larger number correctly drew an α graph that was the derivative of the ω graph.
- Responses were generally able to identify that the angular acceleration would decrease, but fewer were able to adequately explain the reasoning using the definition of torque.
- Few responses were able to completely solve for the angular velocity, but many were able to partially answer by applying conservation of energy to connect gravitational potential energy to rotational kinetic energy.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Students had difficulty determining the density of the object and relating it to the differential dm in the integral. Many responses used L, rather than the full length of the object, $2L$.	• The density depends on the total length of the object, $2L$. • $\lambda = \frac{M}{2L} \quad dm = \lambda\, dr = \frac{M}{2L} dr$
• Students had difficulty in correctly applying the limits of integration to a valid integral for rotational inertia. Frequently, the r^2 was simply replaced with $(2L)^2$ in the integral. Another common error was integrating r^2dm to mr^2 and then applying the limits of integration.	• $\int_0^{2L} r^2 dm = \int_0^{2L} r^2\, \lambda dr = \lambda \left[\frac{r^3}{3}\right]_0^{2L}$
• Students frequently assumed that the center of mass was in the center of the object ($L/2$, $L/2$), or at the vertex of the shape (L,L).	• $\frac{\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right) L}{M} = \frac{3L}{4}$
• Students attempted to justify the comparison of the rotational inertias of objects A and B by referring only to length, or only to center of mass, and not to distribution of mass relative to the pivot point.	• Object B has a smaller rotational inertia than Object A because half of the mass of object A is at a larger distance from the pivot point than for the corresponding parts of B.
• Students drew angular acceleration graphs that did not begin with nearly zero slope or were not concave down. Linear graphs, exponential decay curves, and horizontal lines were common. • Students drew α graphs that were not related to the derivative of the ω graph, showing a lack of understanding that $d\omega/dt = \alpha$ regardless of the shape of the ω graph.	 •
• Students drew angular velocity graphs that did not end with zero slope or were not concave down. Parabolic graphs and linear graphs were common.	 •

• Students did not completely justify why the angular acceleration would decrease as the object B swings downward, for instance by not identifying which angle the response was referring to or by not including a complete reasoning.	• The angle ϕ between the radius vector from the pivot to the center of mass and the weight force is decreasing, and that leads to a decrease in torque because $\tau = \vec{r} \times \vec{F_g} = rF_g \sin\phi$ • The component of the gravity force that is perpendicular to the line between the pivot and the center of mass is decreasing as the object rotates. • The horizontal distance from the pivot point to the center of mass is decreasing as the object rotates, and that decreases the torque caused by the gravity force acting on the center of mass.
• Students attempted to use torque or conservation of angular momentum to derive an expression for the angular velocity ω of the object when it had rotated down 90°. • Students did not begin the derivation with a fundamental relationship or equation, jumping into the derivation midway, or just stating an answer. • Students included linear kinetic energy of the object in the energy equation. • Students did not use the allowed symbols in the derivation (e.g., using I or I_A rather than the intended I_B).	$U_{gi} = K_{rf}$ $mg\,\Delta h = \frac{1}{2}I\omega^2$ $mg\left(\frac{L}{2}\right) = \frac{1}{2}I_B\omega^2$ • $\sqrt{\frac{mgL}{I_B}} = \omega$

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

Teachers should model for students the process of deriving equations and have students practice derivations in non-test situations. Emphasize the importance of beginning with a statement of a fundamental principle or of fundamental equations and using only the allowed symbols in the final result. Having students work in small groups to carry out a derivation can help students to build confidence in their ability to derive equations. Requiring students to verbally explain their derivation can help to build a deeper understanding of the physics relationships and the process of derivation.

Students need experience explaining or justifying relationships between quantities using physics principles. Allowing students to do this verbally can build confidence and experience, but requiring written explanations is critical to success on the AP Exam. Asking students to critique example justifications (both good and bad) can help them see how far the reasoning needs to go in order to make sense and completely support their point. Require students to state a principle or equation as part of the explanation and to clearly explain the relationship between the variables that are relevant to the justification. For example, "$A = 1/B$, so as B increases, A will decrease." Teachers should encourage students to attempt an explanation for any check-box question, applying relevant physics concepts.

Students should practice sketching graphs by hand, as predictions or descriptions of phenomena. Teachers should emphasize that "sketch" means that numerical values are not important, but concavity, slope (constant or changing, steep, shallow, zero), intercept (positive, negative, or zero), or asymptote are critical information to represent accurately. The sketches in (d) were often close to the correct shapes, but not close enough to earn the points, because of the beginning (α graph) or ending (ω graph) slope.

Because student responses on paper forms are expected to continue to be digitally scanned prior to being scored, legible handwriting is even more important. Neatness, size, and weight of the writing contribute to readability on the scanned documents.

Question #3 **Task:** Force diagram, conservation of energy, projectile motion, and graph analysis **Topic :** Block launched from spring into projectile motion

Max. Points: 15 **Mean Score:** 4.44

What were the responses to this question expected to demonstrate?

The responses to this question were expected to demonstrate an understanding of the following concepts:

- Newton's laws, including how to identify and explain the forces acting on an object in circular motion.
- Recognition of the minimum requirement for an object to continue around a loop.
- Conservation of energy and how spring potential energy is transformed into gravitational potential and kinetic energy.
- The relationship of initial projectile velocity to relevant quantities of motion, such as time of flight, height, and distance traveled.
- Analysis and interpretation of sections of a given graph.

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Student responses often added extraneous forces on the block in circular motion and did not justify the forces on the block.
- Student responses generally used the conservation of energy correctly.
- Many student responses indicated that the normal force was equal to the centripetal force and added the weight of the block in solving for the net force on the block.
- Many student responses derived an expression for the compression of the spring assuming the block had zero velocity at the top of the track.
- Students generally connected the total height of the system to the time and correctly used the velocity in part (b)(i) to find the distance traveled.
- Many student responses correctly explained why there was not any data before Δx_{MIN}.
- Many students did not correctly explain the condition that lead to D_{MIN} or how Δx related to D.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Assumed the block was not touching the track and/or included friction and other extraneous forces in the diagram.	• Included the correct interpretation of the problem by not drawing in any extraneous forces and having the block touch the track.
• Did not draw in two different arrows: one for each vector and connect them to the dot.	• Included two separate labeled arrows and correctly connected them to the dot.
• Did not justify the two correct forces drawn.	• Justified what these two forces were caused by.

• Used the wrong value for height for potential energy: 4R or (3R +Δx).	• Substituted the correct height of $3R$ into the gravitational potential energy equation.
• Wrote the Net Force as $\frac{mv^2}{r} + mg$.	• $F_{\text{NET}} = \frac{mv^2}{r}$
• Misinterpreted the question in part (c) and solved for the minimum spring compression required to launch the block to a height of 3R with no velocity.	• Correctly identified that the minimum compression would be when the normal force is zero and set the answer to part (b)(ii) equal to *mg*.
• Solved for time of flight in part (d) using $h = 3R$.	• Solved for time of flight using $h = 4R$.
• Misinterpreted missing data to mean that the block did not leave the spring.	• The block does not make it to point B on the track.
• Stated that D_{MIN} occurs when Δx_{MIN} occurs.	• Δx_{MIN} occurs at the point when the block has the minimum speed needed to reach point B, which then leads to the D_{MIN}.
• Stated that the graph is linear so as Δx increases so does D.	• From part (d), as the spring is compressed the distance the block travels goes up.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

Having students start with the large dot and making sure each individual vector is attached and labeled is a great way to enhance their understanding of force diagrams. In addition to writing out the sum of the forces, ask the students to explain why each force is on the force diagram. Recognize that centripetal force is the sum of the forces that cause circular motion rather than a force that goes onto a force diagram.

Give students opportunities to describe physical reasons behind different types of graphs and not just the mathematical relationships that the graphs are showing. Doing different demonstrations in class and showing a graph about the demonstration helps them focus on the physical meaning of the graph rather than just the mathematical relationship of the graph.

Create multi-concept problems that involve three or four different areas of physics and have the students analyze each part. In addition, ask groups of students to create their own problems along with solutions to the problems, followed by each group assigning their problems to other groups to solve. This technique could help students learn how to connect different topics of the course together as well as help them review the material as they are solving their own problems as well as the problems of other groups.