2021

AP® CollegeBoard

AP® Physics C: Mechanics

Sample Student Responses and Scoring Commentary Set 1

Inside:

Free Response Question 2

- ☑ Scoring Guideline
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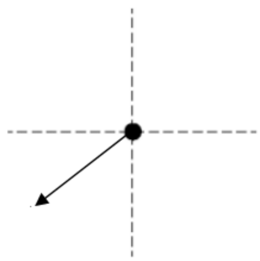
Question 2: Free-Response Question — 15 points

(a)		For a single acceleration vector pointing down and to the left and attempting a justification	**1 point**
		For a correct justification	**1 point**

Example response for part (a)

The block is changing direction with a centripetal component of the acceleration toward the center of the circle and a gravitational acceleration downward. Therefore, the acceleration of the block will be down and to the left.



Total for part (a) — 2 points

(b) i. For correctly using conservation of energy for the block at point B — **1 point**

$$K_A + U_{gA} = K_B + U_{gB}$$

$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$$

For correctly substituting into the above equation — **1 point**

$$0 + mgh = \frac{1}{2}mv_B^2 + mgR$$

$$v_B = \sqrt{2g(h - R)}$$

ii. For correctly substituting the expression for v_B from part (b)(i) into an expression for centripetal force — **1 point**

$$F_c = \frac{mv^2}{r} = \frac{mv_B^2}{R} = \frac{m\left(\sqrt{2g(h-R)}\right)^2}{R}$$

For correctly substituting into an equation for vector addition to derive an expression for the net force at point B — **1 point**

$$F_{net} = \sqrt{F_c^2 + (mg)^2}$$

$$F_{net} = \sqrt{\left(\frac{mv_B^2}{R}\right)^2 + (mg)^2} = \sqrt{\left(\frac{2mg}{R}(h-R)\right)^2 + (mg)^2}$$

Total for part (b) — 4 points

(c)	For correctly using conservation of energy for the speed of the block at point C	**1 point**

$$K_A + U_{gA} = K_C + U_{gC}$$

$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_C^2 + mgh_C$$

$$0 + mgh = \frac{1}{2}mv_C^2 + mg(2R)$$

$$v_C = \sqrt{2g(h-2R)}$$

For correctly applying Newton's second law at point C — **1 point**

$$F_c = \frac{mv_C^2}{r}$$

$$F_N + mg = \frac{mv_C^2}{r}$$

Set the normal force equal to zero

$$0 + mg = \frac{mv_C^2}{R} \therefore v_C = \sqrt{Rg}$$

For combining the two equations above — **1 point**

$$\sqrt{Rg} = \sqrt{2g(h-2R)} \therefore R = 2(h-2R) \therefore R/2 = h - 2R$$

$$h = 2.5R$$

Total for part (c) 3 points

(d)	For correctly using conservation of energy for the block compressing the spring	**1 point**
	For correctly substituting the gravitational and elastic potential energies into an equation for conservation of energy	**1 point**

$$mgh = \frac{1}{2}kx_{MAX}^2$$

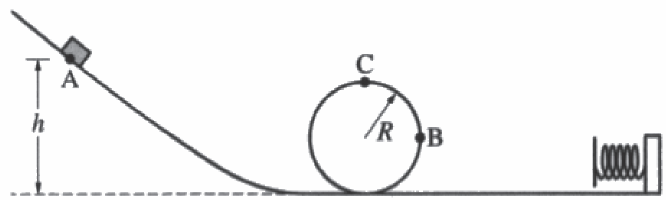
For correctly substituting for the spring constant k into the equation above — **1 point**

$$x_{MAX} = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2mgh}{mg/(2R)}} = \sqrt{4hR} = \sqrt{(4)(0.30\text{ m})(0.10\text{ m})} = 0.35\text{ m}$$

Total for part (d) 3 points

(e) i.	For a correct justification	**1 point**
	Example response for part (e)(i) *Because the block does not make it through the loop at this height, it will not compress the spring.*	
ii.	For indicating that as the height increases, the compression of the spring increases	**1 point**
	For indicating that the height is proportional to the square of the compression of the spring	**1 point**
	Example response for part (e)(ii) *From the equation in part (c), the compression of the spring is directly proportional to the square root of the height that the block is released. Thus, the graph would be an x-axis parabola, as shown.*	
	Total for part (e)	**3 points**
	Total for question 2	**15 points**

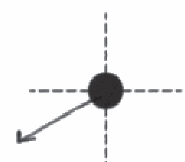
Begin your response to **QUESTION 2** on this page.



Note: Figure not drawn to scale.

2. A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.



Justify your answer.

the block is under the influence of two forces: gravity, which yields a downwards acceleration, & the normal force in the loop, yielding an acceleration to the left. the resultand vector will be what is shown above.

(b)

i. Derive an expression for the speed v of the block at point B.

$E_1 = E_2$

$mgh = mgR + \frac{1}{2}mv^2 \longrightarrow v^2 = 2(gh - gR) \rightarrow v = \boxed{\sqrt{2g(h-R)} \text{ m/s.}}$

ii. Derive an expression for the magnitude of the net force F on the block at point B.

$F_{net} = F_{gravity} + F_{centripetal} = mg + \frac{mv^2}{R} = mg + \frac{m}{R}(2g(h-R))$

$= F_{net} = \boxed{\frac{2mgh}{R} - mg}$ N.

$C = mg + \frac{2mgh}{R} - \frac{2mg\cancel{R}}{\cancel{R}}$

Continue your response to **QUESTION 2** on this page.

(c) In terms of R, derive an expression for the minimum height h_{min} necessary for the block to maintain contact with the track through point C.

$a_c = g = \frac{v^2}{R} \longrightarrow v_{min} = \sqrt{gR}$

$E_i = E_f$

$\cancel{m}gh = \cancel{m}g(2R) + \frac{1}{2}\cancel{m}v^2$

$v = \sqrt{2(gh - 2gR)}$

$(\sqrt{gR})^2 = (\sqrt{2(gh-2gR)})^2$

$gR = 2gh - 4gR$

$\hookrightarrow 2gh = \frac{5gR}{2g}$

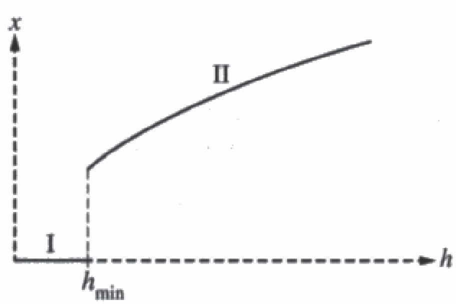
$\hookrightarrow h = \boxed{\frac{5}{2}R}$ m.

(d) It is determined that $h = 0.30\text{ m}$ and $R = 0.10\text{ m}$. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

E_2

$mgh = \frac{1}{2}kx^2 \rightarrow x_{max} = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2mgh}{(\frac{mg}{2R})}} = \sqrt{4hR} = \sqrt{4(0.3)(0.1)} = \boxed{0.3464 \text{ m.}}$

(e) A graph of the maximum compression of the spring as a function of height is shown below. The height h_{min} is the height calculated in part (c).



i. Explain why section I appears as a horizontal line segment on the horizontal axis.

Since $a_c < g$ @ point C, the cart will fall off the track, and will not reach the spring. It follows there can be no spring compression.

ii. Explain the reason for the shape of section II on the graph.

as calculated in part (d), $x_{max} = \sqrt{4hr}$, $\therefore x_{max} \propto \sqrt{h}$. The graph of section II follows the expected shape of a square-root function.

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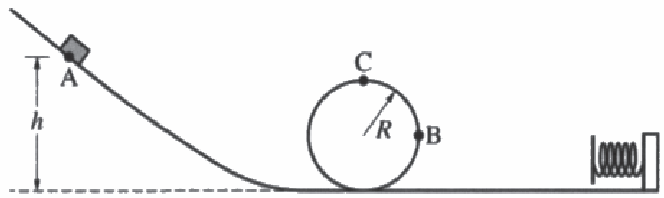
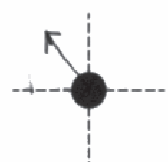
Begin your response to **QUESTION 2** on this page.



Note: Figure not drawn to scale.

2. A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.



Justify your answer. Acceleration points in the direction of motion (up) and also toward the center of the loop.

(b)

i. Derive an expression for the speed v of the block at point B.

$U_i = K_f$

$mgh = \frac{1}{2}mv^2 \rightarrow \boxed{v = \sqrt{2gh}}$

ii. Derive an expression for the magnitude of the net force F on the block at point B.

$F = \frac{mv^2}{r} = \frac{m(\sqrt{2gh})^2}{R} = \boxed{\frac{2ghm}{R}}$

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) In terms of R, derive an expression for the minimum height h_{min} necessary for the block to maintain contact with the track through point C.

$F_g = F_c$ $\qquad$ $U_i = K_F$

$\cancel{m}g = \frac{\cancel{m}v^2}{r}$ $\qquad$ $mgh = \frac{1}{2}mv^2$

$V = \sqrt{2gh_{min}}$

$g = \frac{(\sqrt{2gh_{min}})^2}{R} = \frac{2gh_{min}}{R}$

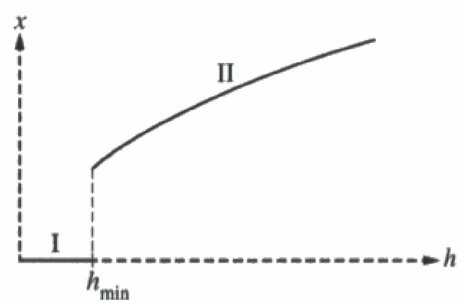
$\boxed{h_{min} = R/2}$

(d) It is determined that $h = 0.30\text{ m}$ and $R = 0.10\text{ m}$. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

$U_i = U_F$

$mgh = \frac{1}{2}k\Delta x^2 \rightarrow \Delta x = \sqrt{\frac{2mgh}{\frac{mg}{2R}}} = \sqrt{4hR} = \sqrt{4(.3)(.1)} = \boxed{.35m}$

(e) A graph of the maximum compression of the spring as a function of height is shown below. The height h_{min} is the height calculated in part (c).



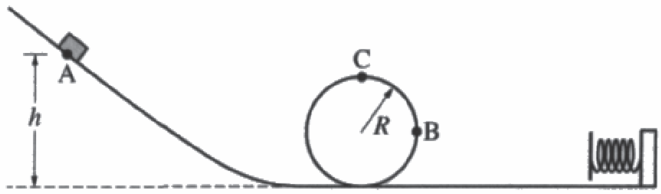
i. Explain why section I appears as a horizontal line segment on the horizontal axis.

The block will never reach the spring for $h < h_{min}$, as h_{min} is the minimum height needed to complete the loop. Thus, no compression is shown.

ii. Explain the reason for the shape of section II on the graph.

With an increasing height, the maximum compression also increases, but exponentially.

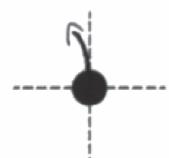
Begin your response to **QUESTION 2** on this page.



<u>Note</u>: Figure not drawn to scale.

2. A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.



Justify your answer.

The cart will be following the track and will continue from point B to C.

(b)

i. Derive an expression for the speed v of the block at point B.

$V = V_0 + at \qquad V = 0 + \frac{F_{net}}{m}(t)$

ii. Derive an expression for the magnitude of the net force F on the block at point B.

$F = ma$

Continue your response to **QUESTION 2** on this page.

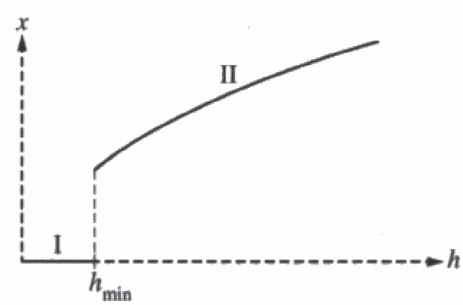
(c) In terms of R, derive an expression for the minimum height h_{min} necessary for the block to maintain contact with the track through point C.

$h_{min} = 2h_c = 4R$

(d) It is determined that $h = 0.30\,\text{m}$ and $R = 0.10\,\text{m}$. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

$K = \frac{mg}{2R}$ $K = \frac{m(9.8)}{2(.10)} = \frac{9.8m}{.2}$ $K = 49m$

(e) A graph of the maximum compression of the spring as a function of height is shown below. The height h_{min} is the height calculated in part (c).



i. Explain why section I appears as a horizontal line segment on the horizontal axis.

B/c I or the line segment is showing how there must be a hmin, inorder for the block to maintain contact with the track through point C.

ii. Explain the reason for the shape of section II on the graph.

B/c as the h increases, so does the speed of the block. As the speed of the block increases so does the compression of the spring as it makes impact

Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses to this question were expected to demonstrate the following knowledge/skills:

- A block traveling through a vertical loop with negligible friction can be described using two distinct forces: the centripetal force associated with the acceleration of the block along a circular loop and the gravitational force associated with the downward acceleration as the upward-moving block slows.
    - These centripetal and gravitational accelerations/forces must be added vectorially to get the net values.
- For a block traveling down a ramp and through a loop with negligible friction, conservation of energy can be used to find various unknown quantities (such as speed) at several points along the track.
- The centripetal force at any point for the block traveling through a loop with negligible friction will be: $F_c = \frac{mv^2}{r}$.
- To find the minimum speed for the block traveling through a loop with negligible friction, the normal force for the block at the top of the loop will be equal to zero.
- Conservation of energy can be used to find the compression of the spring for a block traveling with negligible friction down a ramp and through a loop onto a horizontal surface, where the block then compresses the spring.
- Identifying functional relationships for and demonstrating understanding of key parts of a graph that describes the physical situation of the block making contact with the spring.
- Substitution of given constants and physical constants, as appropriate.

Sample: M Q2 A
Score: 14

Part (a) earned 2 points because the vector drawn is pointing down and to the left and the justification is correct. Part (b)(i) earned 2 points for the correct use of conservation of energy for the block at point B and for correctly substituting into this equation. Part (b)(ii) earned 1 point. This point was earned for substituting the expression found in (b)(i) into the expression for centripetal force. The net force is incorrect, it does not use vector addition. Part (c) earned 3 points. These points were earned for correctly using conservation of energy at point C, applying Newton's second law at point C, and for combining the two derived equations. Part (d) earned 3 points. These points were earned for using conservation of energy for the block compressing the spring and for all correct substitutions. Part (e)(i) earned 1 point for a correct justification. If the block does not make it through the loop, it will not reach the spring, and it will not compress. Part (e)(ii) earned 2 points for correctly stating that the compression is proportional to the square root of the height, and 1 point for implying that as the height increases, the compression of the spring increases.

Question 2 (continued)

Sample: M Q2 B
Score: 8

Part (a) earned no points. An acceleration vector pointing up and to the left is incorrect. The justification describes the vector drawn but does not give a reason as to why the vector would be in that direction. Part (b)(i) earned no points because the conservation of energy equation given is incorrect at point B. Part (b)(ii) earned 1 point for substituting the expression found in part (b)(i) into the expression for centripetal force. The net force is not equal to the centripetal force. Part (c) earned 2 points. These points were earned for correctly applying Newton's second law at point C and for combining the two derived equations. The conservation of energy equation used is incorrect at point C. Part (d) earned 3 points. These points were earned for using conservation of energy for the block compressing the spring and for all correct substitutions. Part (e)(i) earned 1 point for a correct justification. If the block does not reach the spring, it will not compress. Part (e)(ii) earned 1 point for indicating that as the height increases, the compression of the spring increases. The function is not exponential.

Sample: M Q2 C
Score: 2

Part (a) earned no points. An acceleration vector pointing up and to the left is incorrect. The justification is incorrect. Part (b)(i) earned no points because there is no correct or relevant work. Kinematics cannot be used to derive an expression of the speed of the block at point B. Part (b)(ii) earned no points because there is no correct or relevant work. Part (c) earned no points because there is no correct or relevant work. Part (d) earned no points. There is no correct or relevant work. Although the student used the equation for the spring constant, they did not use this expression in a relevant conservation of energy statement. Part (e)(i) earned 1 point for a correct justification. If the block does not make it through the loop, it will not reach the spring, and it will not compress. Part (e)(ii) earned 1 point for indicating that as the height increases, the compression of the spring increases. The shape of the curve is not addressed in the justification.