2021

AP® CollegeBoard

AP® Physics C: Mechanics

Sample Student Responses and Scoring Commentary Set 2

Inside:

Free Response Question 2

☑ Scoring Guideline

☑ Student Samples

☑ Scoring Commentary

© 2021 College Board. College Board, Advanced Placement, AP, AP Central, and the acorn logo are registered trademarks of College Board. Visit College Board on the web: collegeboard.org.

AP Central is the official online home for the AP Program: apcentral.collegeboard.org.

Question 2: Free-Response Question — 15 points

(a)	For integrating using the correct limits or constant of integration $I = \int_{r=0}^{r=2L} \lambda r^2 dr = \lambda \left[\frac{r^3}{3} \right]_{r=0}^{r=2L} = \frac{\lambda}{3}\left((2L)^3 - 0\right)$	**1 point**
	For correctly relating λ to M and L $\lambda = \frac{m}{\ell} = \frac{M}{2L} \therefore I = \left(\frac{1}{3}\right)\left(\frac{M}{2L}\right)\left(8L^3\right) = \frac{4}{3}ML^2$	**1 point**
	Total for part (a)	**2 points**
(b) i.	For correctly substituting into an equation for the center of mass of an object in the horizontal direction $X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\left[\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right)(L)\right]}{\left(\frac{M}{2} + \frac{M}{2}\right)} = \frac{\left(\frac{ML}{4} + \frac{ML}{2}\right)}{M} = \frac{3}{4}L$	**1 point**
ii.	For correctly substituting into an equation for the center of mass of an object in the vertical direction $Y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\left[\left(\frac{M}{2}\right)(L) + \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)\right]}{\left(\frac{M}{2} + \frac{M}{2}\right)} = \frac{\left(\frac{ML}{2} + \frac{ML}{4}\right)}{M} = \frac{3}{4}L$	**1 point**
	Total for part (b)	**2 points**
(c)	For selecting “Less than” and attempting a relevant justification	**1 point**
	For a correct justification	**1 point**
	Example response for part (c) *Because object B has more of its mass closer to the pivot than object A, the rotational inertia of object B must be less than that of object A.*	
	Total for part (c)	**2 points**

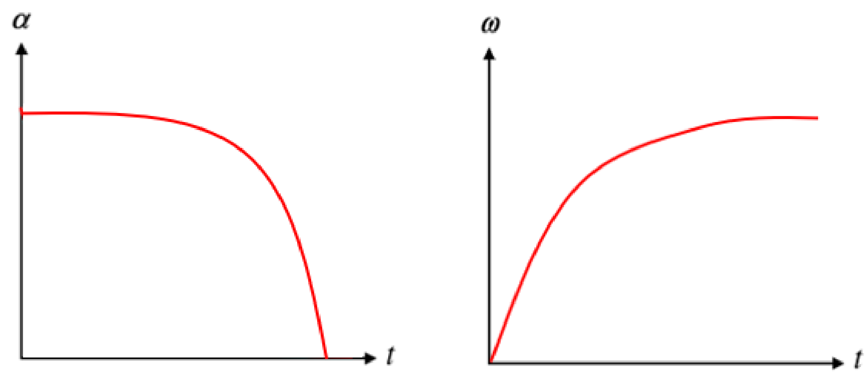
© 2021 College Board

(d) For an acceleration graph that is concave down and begins horizontally **1 point**

For an angular speed graph that is concave down and ends horizontally **1 point**

For consistency between the angular acceleration and angular speed graphs **1 point**

Example responses for part (d)



Total for part (d) 3 points

(e) For selecting "Decreasing" and attempting a relevant justification **1 point**

For a justification that indicates the lever arm for the torque is decreasing **1 point**

Example response for part (e)

Because the horizontal position of the center of mass for the object is moving closer to the pivot, the lever arm for the force of gravity is decreasing so the angular acceleration decreases.

Total for part (e) 2 points

(f) For using conservation of energy **1 point**

$U_{g1} = K_2$

For correctly relating the change in rotational kinetic energy to the change in gravitational potential energy **1 point**

$mgh = \frac{1}{2}I\omega^2$

For correctly substituting for h into the equation above **1 point**

$mg\left(\frac{L}{2}\right) = \frac{1}{2}I\omega^2$

For an expression for ω that uses only the allowed symbols and is algebraically consistent with the previous steps **1 point**

$\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2Mg(L/2)}{I_B}}$

$\omega = \sqrt{\frac{MgL}{I_B}}$

Total for part (f) 4 points

Total for question 2 15 points

Begin your response to **QUESTION 2** on this page.

Object A

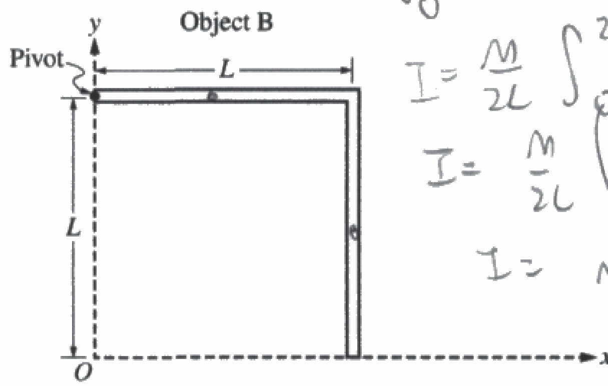
2. Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.

$I = mR^2$ $\quad dm = \frac{M}{2L}dr$

$dI = dm\,r^2$. $\quad I = \int_0^{2L} \frac{M}{2L} r^2\,dr$

Object B



$I = \frac{M}{2L}\int_0^{2L} r^2\,dr$

$I = \frac{M}{2L}\left(\frac{1}{3}r^3\Big|_0^{2L}\right)$

$I = \frac{M\cancel{8}^{4}L^{\cancel{3}2}}{\cancel{6}_{3}\cancel{L}} = \boxed{\frac{4}{3}ML^2}$

Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above. Express your answers in part (b) in terms of L.

(b) Determine the following for the given coordinate system shown in the figure.

i. The x-coordinate of the center of mass of object B Each branch mass $\frac{M}{2}$

$X_{cm} = \frac{\Sigma x_i m_i}{m_{total}} = \frac{\left(\frac{L}{2}\right)\left(\frac{M}{2}\right) + L\left(\frac{M}{2}\right)}{M} = \frac{\frac{3ML}{4}}{M} = \boxed{\frac{3}{4}L}$

ii. The y-coordinate of the center of mass of object B

$Y_{cm} = \frac{\Sigma y_i m_i}{m_{total}} = \frac{\left(\frac{L}{2}\right)\left(\frac{M}{2}\right) + L\left(\frac{M}{2}\right)}{M} = \frac{\frac{3ML}{4}}{M} = \boxed{\frac{3}{4}L}$

Unauthorized copying or reuse of this page is illegal.

Continue your response to **QUESTION 2** on this page.

Object B has a rotational inertia of I_B about its pivot.

(c) Is the value of I_B greater than, less than, or equal to I_A ?

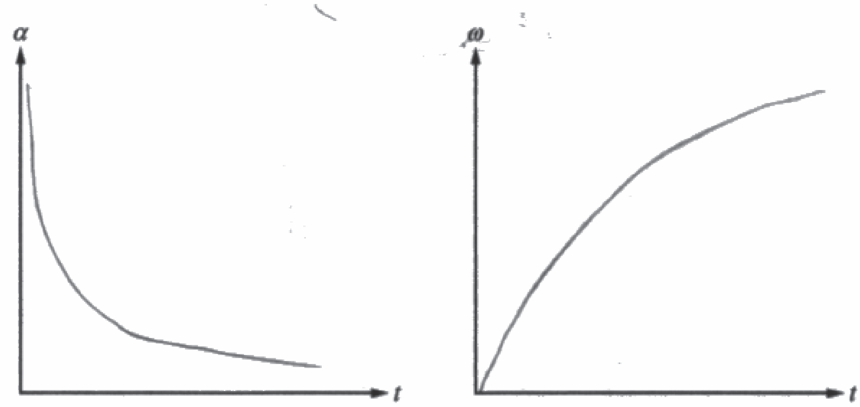
____ Greater than   ✓ Less than   ____ Equal to

Justify your answer.

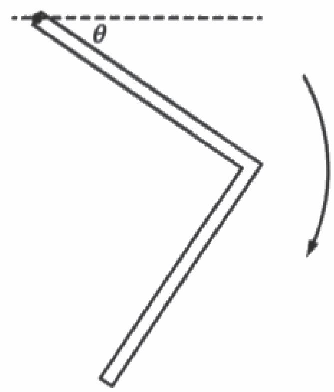
Object A's mass is distributed further away from the pivot up to distance 2L, while Object B mass is distributed closer to the pivot.

Object B is released from rest and begins to rotate about its pivot.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.



Continue your response to **QUESTION 2** on this page.

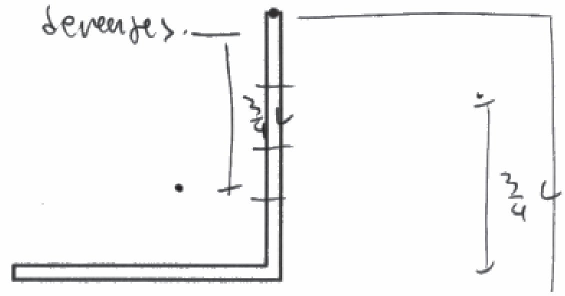


(e) While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

_____ Increasing __✓__ Decreasing _____ Not changing

Justify your answer.

As it rotates towards equilibrium, the component of gravity tangent to the motion of its center of mass decreases, so its acceleration also decreases.



Object B rotates through the position shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M, L, I_B, and physical constants, as appropriate.

PE lost: $mgh = \left(\frac{1}{2}L\right)(10)(M) = 5ML$

converted to rotational KE:

$\frac{1}{2}I\omega^2 = 5ML$

$I\omega^2 = 10ML$

$\boxed{\omega = \sqrt{\frac{10ML}{I_B}}}$

Unauthorized copying or reuse of this page is illegal.

GO ON TO THE NEXT PAGE.

Begin your response to QUESTION 2 on this page.

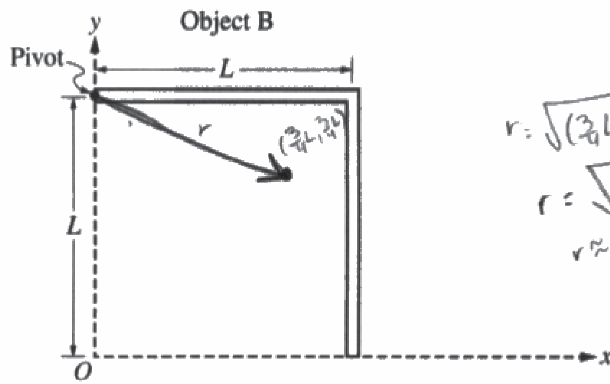
Object A

2. Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.

$I = \int_0^{2L} r^2\,dm$

Object B



$r = \sqrt{(\frac{3}{4}L)^2 + (\frac{1}{4}L)^2}$

$r = \sqrt{0.625 L^2}$

$r \approx 0.791 L$

Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above. Express your answers in part (b) in terms of L.

(b) Determine the following for the given coordinate system shown in the figure.

i. The x-coordinate of the center of mass of object B

$\frac{\frac{1}{2}LM + LM}{2M} = \frac{\frac{3}{2}L\cancel{M}}{2\cancel{M}} = \boxed{\frac{3}{4}L}$

ii. The y-coordinate of the center of mass of object B

$\frac{LM + \frac{1}{2}LM}{2M} = \frac{\frac{3}{2}L\cancel{M}}{2\cancel{M}} = \boxed{\frac{3}{4}L}$

Continue your response to QUESTION 2 on this page.

Object B has a rotational inertia of I_B about its pivot.

(c) Is the value of I_B greater than, less than, or equal to I_A ?

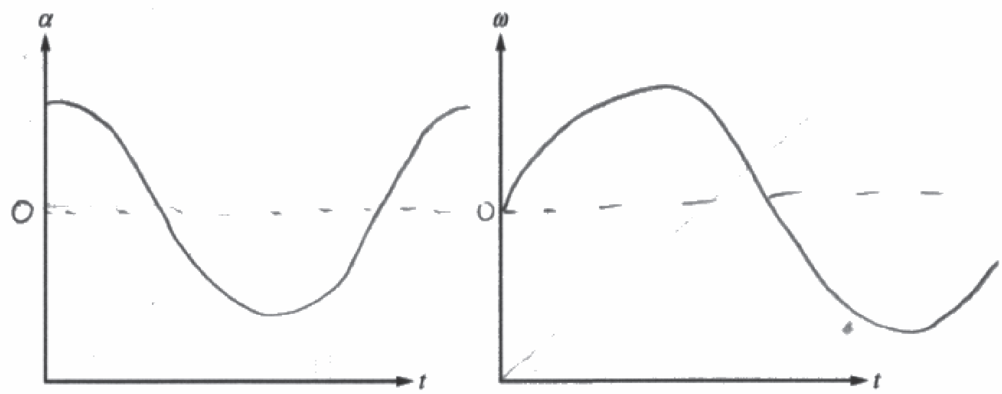
____ Greater than   X Less than   ____ Equal to

Justify your answer.

The distance from the center of mess of object B to the pivot of object B is less than the distance from the center of mess of object A to the pivot of object A, ~~but~~ both have identical messes, therefore, object B has a lesser rotational inertia than object A.

Object B is released from rest and begins to rotate about its pivot.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.

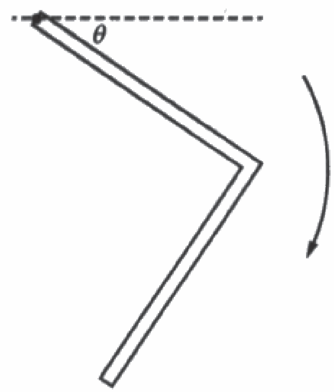


$$\alpha = \frac{\tau_{net}}{I}$$

Unauthorized copying or reuse of this page is illegal.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

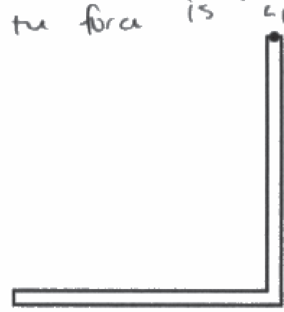


(e) While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

____ Increasing   __X__ Decreasing   ____ Not changing

Justify your answer.

The torque is caused by the weight force in this case and as the object rotates the weight force is applied at angles moving away from 90°. Because the maximum torque — and thus the maximum angular acceleration — occur where the force is applied at 90°, the angular acceleration is decreasing.



Object B rotates through the position shown above.

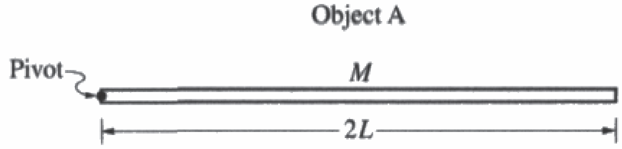
(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M, L, I_B, and physical constants, as appropriate.

$$\omega = \int \alpha \, d\theta$$

$$\omega = \int \frac{\tau_{net}}{I} d\theta$$

$$\omega = \int_0^{90} \frac{Mg(0.791)L\sin\theta}{I_B} d\theta$$

Begin your response to **QUESTION 2** on this page.

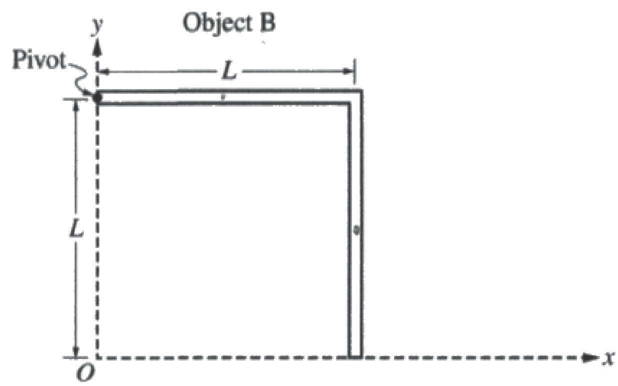


2. Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.

$I = \int r^2\,dm = \int ((2L)^2)\,dm = \int 4L^2\,dm = 4\int L^2\,dm$

$4\int L^2\,dm = 4 \cdot \frac{1}{3}mL^2 = \frac{4}{3}mL^2$



Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above. Express your answers in part (b) in terms of L.

(b) Determine the following for the given coordinate system shown in the figure.

i. The x-coordinate of the center of mass of object B

$$x_{com} = \frac{(.5M \cdot .5L) + (.5m \cdot L)}{m} = \frac{.25mL + .5mL}{m} = \frac{.75mL}{m} = \boxed{.75L}$$

ii. The y-coordinate of the center of mass of object B

$$y_{com} = \frac{(.5M \cdot .5L) + (.5m \cdot L)}{m} = \frac{.75mL}{m} = \boxed{.75L}$$

Continue your response to **QUESTION 2** on this page.

Object B has a rotational inertia of I_B about its pivot.

$I_B = \frac{1}{12}\cancel{ML}^2 + \frac{1}{2}L^2 = \frac{13}{24}L^2$

$I_A = \frac{1}{12}M(2L)^2 =$

(c) Is the value of I_B greater than, less than, or equal to I_A ?

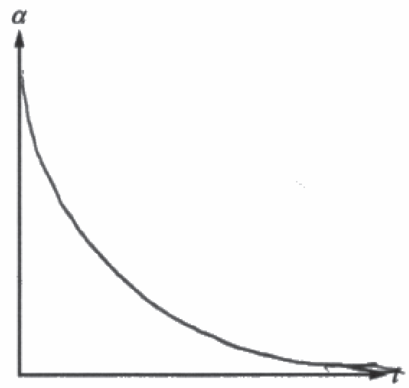
__✓__ Greater than   ~~✓~~ Less than   _____ Equal to

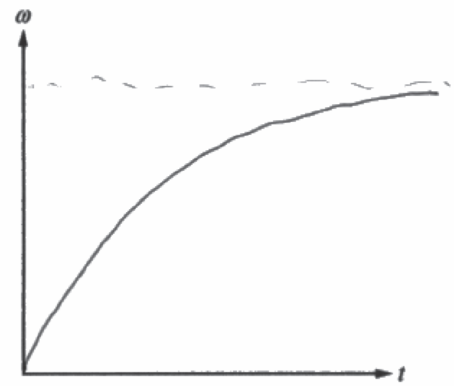
Justify your answer.

~~I is determined by CMR² where C is a coefficient. Since both objects have the same mass,~~ Greater because one rod of B essentially acts as a point mass which would have a higher I.

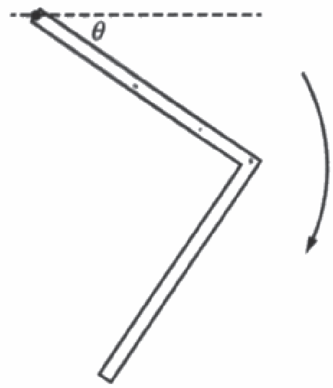
Object B is released from rest and begins to rotate about its pivot.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.





Continue your response to **QUESTION 2** on this page.

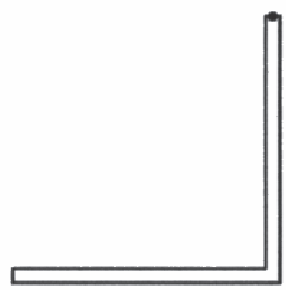


(e) While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

__✓__ Increasing   _____ Decreasing   _____ Not changing

Justify your answer.

The com of the object produces a torque still $\therefore$



Object B rotates through the position shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M, L, I_B, and physical constants, as appropriate.

Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses to this question were expected to demonstrate the following:

- Use integral calculus to derive the rotational inertia of a long, thin rod. This required that the response correctly determine the density of the object, M/2L, and to set up a correct integral with correct limits.
- Determine the center of mass of a compound object made from two rods attached at a right angle, using the center of mass equation for discrete objects.
- Compare the rotational inertia of two objects, from part (a) and part (b), and justify the choice using conceptual understanding of rotational inertia.
- Sketch graphs to represent the rotational acceleration and velocity as a function of time for the object from part (b) as it swings down. Answering correctly required applying understanding of torque OR recognizing that the motion is analogous to a physical pendulum and that a and ω are related by calculus.
- Determine whether the angular acceleration would increase, decrease, or remain constant as the object rotates downwards, pulled by gravity. The response required understanding that angular acceleration is caused by torque and that the torque depends on the angle of rotation, and therefore would decrease as the object rotates downward.
- Derive an expression for the angular velocity of the object after it rotates 90° from its initial position. This required conservation of energy, equating the change in gravitational energy to the change in rotational kinetic energy.

Sample: M Q2 A
Score: 13

Part (a) earned 2 points for proper dm substitution and proper limits with a good integrand. Part (b) earned 2 points. Both points were earned with good algebra and correct answers. Part (c) earned 2 points. Both points were earned with a correct choice of "Less than" and a statement of mass being distributed closer to the pivot point. Part (d) earned 1 point as the decreasing value of alpha matches a decreasing slope of omega. No points were earned as this graph does not start horizontal and is not concave down, and it is not clear that omega ends horizontal. Part (e) earned 2 points. Both points were earned with the choice of "decreasing" and with a good discussion of the component of gravity causing torque decreases. Part (f) earned 4 points: 2 points were earned with correct use of conservation of energy relating gravitational potential energy change to change in rotational kinetic energy. One point was earned as the substitution of 10 as a value of g is acceptable, and 1 point was earned because the final answer is consistent with their algebra, containing I_B and only containing allowable variables.

© 2021 College Board.
Visit College Board on the web: collegeboard.org.

Question 2 (continued)

Sample: M Q2 B
Score: 9

Part (a) earned 1 point for applying the correct limits of integration to a correct integral. It did not earn the second point because M/2L is not present. Part (b) earned 2 points for stating the correct values. Part (c) earned 1 point for a correct choice and attempt at relevant justification. The second point was not earned for the justification because the distance from the center of mass to the pivot is not, by itself, sufficient to justify that I_B is less than I_A. Part (d) earned 3 points. Two points were earned because the alpha and omega graphs are the correct shapes, even though they go beyond the lowest point of the object's motion. One point was earned because the graphs are self-consistent; the derivative of omega would look like alpha. The response shows a strong understanding of the physics, even though it exceeds the time frame of the prompt. Part (e) earned 2 points for a correct choice and relevant justification and for a justification that connects the change of torque to the changing angle. Part (f) earned no points.

Sample: M Q2 C
Score: 4

Part (a) earned no points. The response did not earn the first point because there are not the correct limits applied and did not earn the second point because the density M/2L is not written or used. Part (b) earned 2 points for arriving at the correct CoM values. Part (c) earned no points because the wrong choice is selected; therefore, the justification cannot earn the second point. Part (d) earned 2 points. One point was earned because the omega graph is concave down and ends nearly horizontal. The dashed line indicates that response is intended to be approaching horizontal. One point was earned because the derivative of the omega graph could look like the alpha graph: the first derivative is always positive, and the second derivative is negative. In the alpha graph, alpha is always positive and always decreasing. The response did not earn the first point because the alpha graph is the wrong shape. Part (e) earned no points because the wrong choice is marked. Part (f) earned no points because there is no response.

© 2021 College Board.
Visit College Board on the web: collegeboard.org.