2021

AP® CollegeBoard

# AP® Physics C: Mechanics

## Sample Student Responses and Scoring Commentary Set 1

**Inside:**

Free Response Question 3

☑ Scoring Guideline

☑ Student Samples

☑ Scoring Commentary

## Question 3: Free-Response Question — 15 points

**(a)** For using integral calculus to calculate the rotational inertia of the rod — **1 point**

$$I = \int r^2 dm$$

$$dm = \lambda dr = \gamma x^2 dx$$

For correctly substituting $\gamma x^2$ into the above equation — **1 point**

$$I = \int x^2 \left(\gamma x^2 dx\right) = \int_{x=0}^{x=L} \gamma x^4 dx = \gamma \left[\frac{x^5}{5}\right]_{x=0}^{x=L} = \left(\frac{3M}{L^3}\right)\left(\frac{L^5}{5} - 0\right) = \frac{3}{5}ML^2$$

**Total for part (a) — 2 points**

**(b)** For using integral calculus to determine the center of mass of the rod — **1 point**

$$X_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\int x dm}{\int dm}$$

For correctly substituting $\gamma x^2$ into the numerator of the above equation — **1 point**

For correctly substituting $M$ into the denominator of the above equation OR evaluating the integral $\int dm$ to find the mass of the rod — **1 point**

$$X_{CM} = \frac{\int x\lambda dx}{M} = \frac{\int x\left(\gamma x^2\right)dx}{M} = \frac{\int_{x=0}^{x=L} \gamma x^3 dx}{M} = \frac{\left[\frac{\gamma x^4}{4}\right]_{x=0}^{x=L}}{M} = \frac{\left(\frac{3M}{L^3}\right)\frac{L^4}{4}}{M} = \frac{3}{4}L$$

OR

$$X_{CM} = \frac{\int x\lambda dx}{\int \lambda dx} = \frac{\int x\left(\gamma x^2\right)dx}{\int_{x=0}^{x=L} \gamma x^2 dx} = \frac{\int_{x=0}^{x=L} \gamma x^3 dx}{\left[\frac{\gamma x^2}{3}\right]_{x=0}^{x=L}} = \frac{\left[\frac{\gamma x^4}{4}\right]_{x=0}^{x=L}}{\frac{\gamma L^2}{3}} = \frac{\frac{\gamma L^4}{4}}{\frac{\gamma L^2}{3}} = \frac{3}{4}L$$

**Total for part (b) — 3 points**

**(c)** For selecting "Greater than" with an attempted justification **1 point**

For a correct justification **1 point**

**Example responses for part (c)**

*Because more of the mass of the rod is at the end of the rod opposite point P, more mass is concentrated away from the axis of rotation; thus, the rotational inertia of the rod would be greater around point P than around its center of mass.*

OR

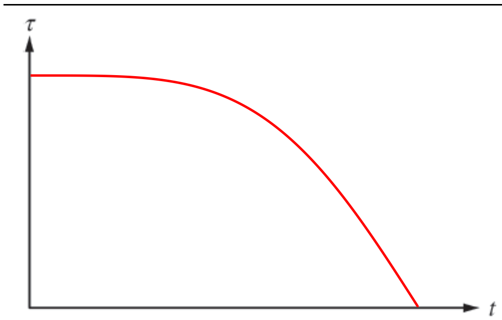
*According to the parallel axis theorem, $I = I_{cm} + md^2$, if the axis is at a position away from the center of mass, the rotational inertia is larger than if the axis were at the center of mass.*

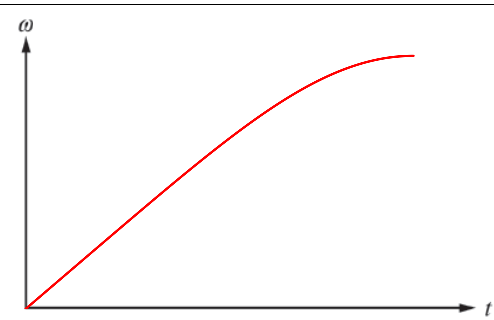
**Total for part (c) 2 points**

**(d)** For a concave down curve that decreases to zero for the graph of $\tau$ as a function of $t$ **1 point**

For a concave down curve that approaches horizontal for the graph of $\omega$ as a function of $t$ **1 point**

For consistency between the two graphs **1 point**




**Total for part (d) 3 points**

**(e)** For selecting "Decreases" with an attempted justification **1 point**

For a correct justification **1 point**

**Example responses for part (e)**

*As the rod rotates downward, the angle $\theta$ in the torque equation $\tau = rF\sin\theta$ decreases. Thus, the torque on the rod decreases.*

OR

*As the rod rotates downward, the lever arm between point P and the rod's center of mass continues to decrease; thus, the torque on the rod decreases.*

**Total for part (e) 2 points**

| | | |
|---|---|---|
| **(f)** | For using conservation of energy to calculate the speed of the rotating rod | **1 point** |

$$U_i + K_i = U_f + K_f$$
$$U_i + 0 = 0 + K_f$$
$$U_i = K_f$$

For correctly substituting into the above equation — **1 point**

$$mgh_i = \frac{1}{2}I\omega_f^2$$

For correctly solving for the linear speed of point S — **1 point**

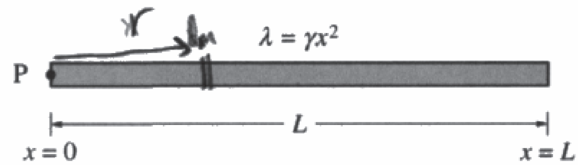
$$Mg\left(\frac{3}{4}L\right) = \frac{1}{2}\left(\frac{3}{5}ML^2\right)\left(\frac{v}{L}\right)^2$$

$$\frac{3}{4}MgL = \frac{3}{10}Mv^2 \therefore v = \sqrt{\tfrac{5}{2}gL} = \sqrt{\tfrac{5}{2}\left(9.8\ \mathrm{m/s^2}\right)(1.0\ \mathrm{m})} = 4.9\ \mathrm{m/s}$$

**Total for part (f)** — **3 points**

**Total for question 3** — **15 points**

Begin your response to **QUESTION 3** on this page.




3. A triangular rod of length $L$ and mass $M$ has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \frac{3M}{L^3}$ and $x$ is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia $I$ of the rod about an axis perpendicular to the page and through point P is $\frac{3}{5}ML^2$.

$I = \int r^2 dm \Rightarrow I = \int_0^L \gamma x^4 dx = \frac{\gamma x^5}{5}\Big|_0^L = \frac{\gamma L^5}{5} - 0$

$dm = \lambda dx$
$= \gamma r^2 dr$

$\Rightarrow I = \frac{3M}{L^3}\left(\frac{L^5}{5}\right) \Rightarrow I = \frac{3ML^2}{5}$

$\boxed{I = \frac{3ML^2}{5}}$

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of $L$. $X_{cm} = \frac{\int r dm}{M} \Rightarrow X_{cm} = \frac{\int_0^L \gamma r^3 dr}{M} \Rightarrow X_{cm} = \frac{\gamma r^4}{4M}\Big|_0^L \Rightarrow X_{cm} = \frac{\gamma L^4}{4M} - 0$

$\Rightarrow X_{cm} = \frac{3\cancel{M}}{\cancel{L^3}}\left(\frac{L^4}{4\cancel{M}}\right) \Rightarrow \boxed{X_{cm} = \frac{3}{4}L}$

(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

__✓__ Greater than ____ Less than ____ Equal to

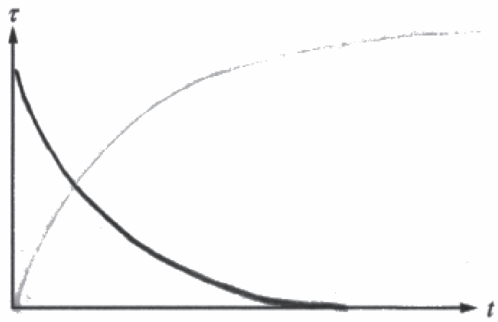
Justify your answer.

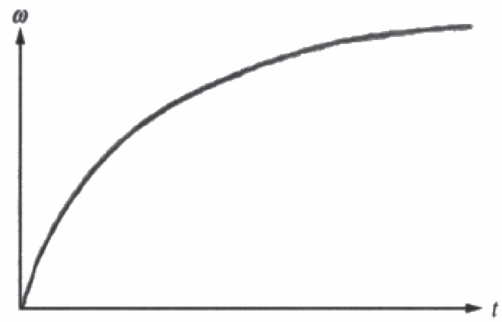
This is because if the rod was rotating @ its center of mass, then the mass would be more centered around the axis than if it were rotating @ point P.

Continue your response to **QUESTION 3** on this page.

The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

(d) On the axes below, sketch graphs of the magnitude of the net torque $\tau$ on the rod and the angular speed $\omega$ of the rod as functions of time $t$ from the time the rod is released until the time its center of mass reaches its lowest point.




(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

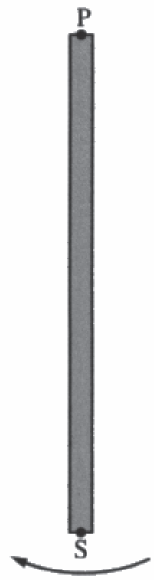
_____ Increasing    __✓__ Decreasing    _____ Not changing

Justify your answer.

As the rod rotates, it follows $\Sigma \tau = I\alpha$

$\Sigma \tau = \tau_g = \vec{r} \times \vec{F_g}$, which has a decreasing magnitude as when the bar rotates, the component of gravity perpendicular to the radius decreases. Thus the magnitude of angular acceleration decreases.

Continue your response to **QUESTION 3** on this page.




(f) The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed $v$ of point S as the rod swings through the vertical position shown.

$$E_g = E_k \rightarrow mgh = \frac{1}{2} I \omega^2$$

$$mg\frac{3}{4}L = \frac{1}{2}\left(\frac{3mL^2}{5}\right)\omega^2 \rightarrow \frac{3mg}{4}L = \frac{3mL^2}{10}$$





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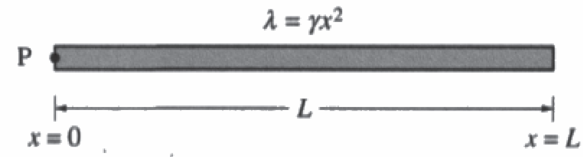
Begin your response to **QUESTION 3** on this page.




3. A triangular rod of length $L$ and mass $M$ has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and $x$ is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia $I$ of the rod about an axis perpendicular to the page and through point P is $\frac{3}{5}ML^2$.

$x^3 = \dfrac{\lambda}{\gamma} = \dfrac{\lambda L^3}{3M}$ $\quad \lambda = \dfrac{dM}{dL} \quad dM = \lambda\, dx$

$dM = \dfrac{3M}{L^3} x^2\, dx$

$I = \int x^2\, dm =$

$I = \int_0^L x^2 \cdot \dfrac{3M}{L^3} x^2\, dx = \int_0^L x^4 \cdot \dfrac{3M}{L^3}\, dx = \dfrac{3M}{L^3} \cdot \dfrac{L^5}{5} = \dfrac{3ML^2}{5}$

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of $L$.

M

L

$\int_0^x M\, dL = \int_0^L M\, dL$

(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

✓ Greater than _____ Less than _____ Equal to

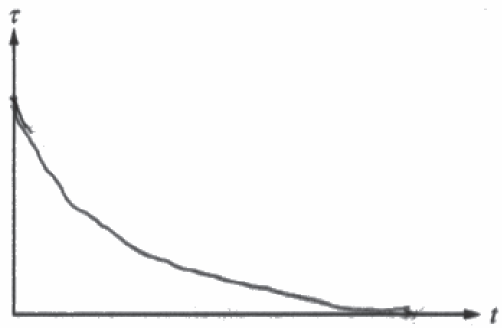
Justify your answer.

There is more mass at a greater radius when the axis is at point P.

Continue your response to **QUESTION 3** on this page.

The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P. $\tau = mg \cos\theta \cdot l$

(d) On the axes below, sketch graphs of the magnitude of the net torque $\tau$ on the rod and the angular speed $\omega$ of the rod as functions of time $t$ from the time the rod is released until the time its center of mass reaches its lowest point.





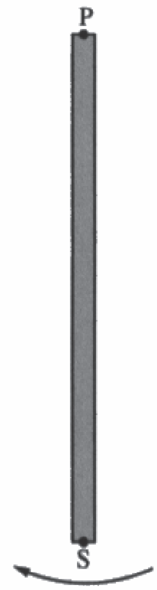
(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

____ Increasing  ✓ Decreasing  ____ Not changing

Justify your answer.

The rod is decreasing in angular acceleration as the torque decrease with a constant rotational inertia.

Continue your response to **QUESTION 3** on this page.




(f) The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed $v$ of point S as the rod swings through the vertical position shown.

$$mgh = \frac{1}{2}mv^2$$

$$V = \sqrt{2gh}$$





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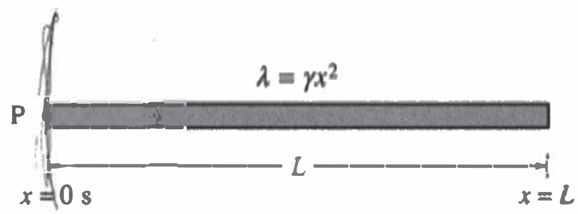
**Begin your response t QUESTION 3 this page.**




3. A triangular rod of length $L$ and mass $M$ has a nonuniform mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \frac{3M}{L^3}$ and $x$ is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia $I$ of the rod about an axis perpendicular to the page and through point P is $\frac{3}{5}ML^2$.

$$\int_0^L r^2 dm = \int_0^L \lambda^2 \, 1 \, dx = \int_0^L \left(\frac{3M}{L^3}\cdot x^2\right)^2 dx =$$

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of $L$.

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{M\cdot(0) + m(\lambda)}{M} = \frac{M\cdot \frac{3M}{L^3}\cdot(L)^2}{M} = \frac{3M}{L}$$

(c) For an axis perpendicular to the page, is the value of the rotational inertia of ~~the rod~~ around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

✓ Greater than ____ Less than ____ Equal to

Justify your answer.

Continue your response to **QUESTION 3** on this page.

The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

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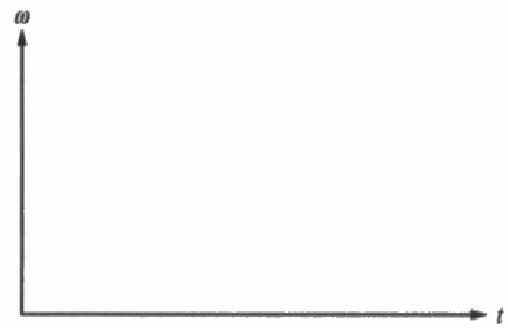


(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_____ Increasing     _____ Decreasing     __✓__ Not changing

Justify your answer.



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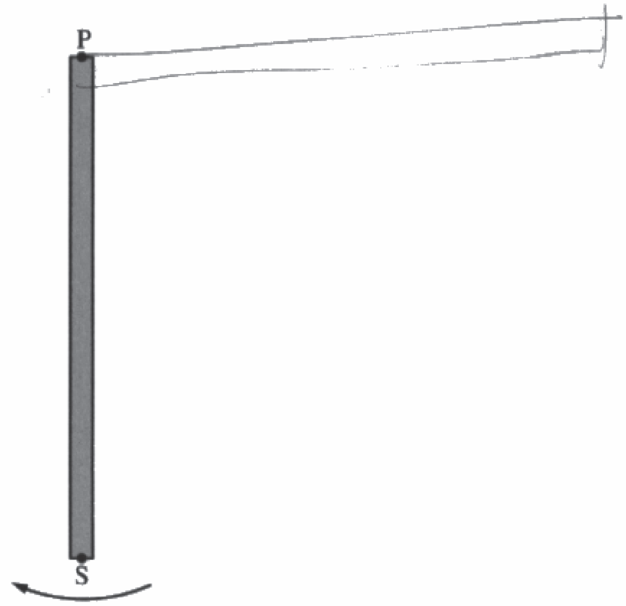
Continue your response to **QUESTION 3** on this page.




(f) The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed $v$ of point S as the rod swings through the vertical position shown.

$m = 3.0 \quad L = 1.0\,m$

$I = \frac{3}{5} M L^2$

$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$

$v = \sqrt{2gh}$

$= 4.427$

$= 4.4\ m/s$

## Question 3

**Note:** Student samples are quoted verbatim and may contain spelling and grammatical errors.

**Overview**

The responses to this question were expected to demonstrate the following:

- Use of calculus to derive equations for both moment of inertia and center of mass for a nonuniform rod with a given linear mass density function.
- Conceptually describe the relationship between mass distribution and the moment of inertia of an object rotating around different axes.
- Address, both conceptually and mathematically, what would happen if the rod were released from a horizontal position and allowed to swing downward, including:
  - drawing graphs for both torque and angular speed.
  - using conservation of energy to calculate the linear speed of the endpoint of the rod when the rod reached a vertical orientation.

**Sample: M Q1 A**
**Score: 13**

Part (a) earned 2 points. One point was earned for using a correct integral Calculus equation to determine moment of inertia, and 1 point was earned for substituting λdx into the correct equation. Part (b) earned 3 points. These points were earned for using integral Calculus to find the center of mass, for correctly evaluating the Calculus in the numerator, and for dividing by M. Part (c) earned 2 points. One point was earned for the correct answer with an attempt at justification, and the second point was earned for a correct justification. Part (d) earned 2 points. No point was earned for a correct torque graph, but 1 point was earned for a correct angular speed graph, and 1 point was earned for consistency between the two graphs (decreasing slope of angular speed graph -> decreasing torque). Part (e) earned two points for the correct answer with an attempt at justification and a correct justification. Part (f) earned two points. One point was earned for using conservation of energy, and the second point was earned for setting $U_g = K_{rot}$ . No point was earned for an incorrect answer.

**Sample: M Q1 B**
**Score: 8**

Part (a) earned two points. One point was earned for using a correct integral Calculus equation to determine moment of inertia, and 1 point was earned for substituting λdx into the correct equation. Part (b) earned no points for not using integral Calculus to find the center of mass, for not correctly evaluating the Calculus in the numerator, and for not dividing by M. Part (c) earned 1 point for the correct answer with an attempt at justification, and 1 point for a correct justification. Part (d) earned 2 points. No point was earned for an incorrect torque graph, but 1 point was earned for a correct angular speed graph, and 1 point for consistency between the two graphs (decreasing slope of angular speed graph -> decreasing torque). Part (e) earned 1 point for the correct answer with an attempt at justification, but no points for an incorrect justification (no explanation for torque decrease). Part (f) earned 1 point for using conservation of energy, but no point for not setting $U_g = K_{rot}$ , and no point for an incorrect answer.

## Question 3 (continued)

**Sample: M Q1 C**
**Score: 3**

Part (a) earned 1 point for using a correct integral Calculus equation to determine moment of inertia, but no point for substituting $\lambda dx$ into the correct equation. Part (b) earned 1 point for dividing by M. No points were earned for using integral Calculus to find the center of mass or for correctly evaluating the Calculus in the numerator.
Part (c) earned no points. There is no correct answer with an attempt at justification and no justification. Part (d) earned no points because there is no correct torque graph, no correct angular speed graph, and no consistency between the two graphs. Part (e) earned no points because there is not a correct answer with an attempt at justification, and no correct justification. Part (f) earned 1 point for using conservation of energy, but no point for setting $U_g = K_{rot}$, and no point for an incorrect answer.