2021

AP CollegeBoard

# AP® Physics C: Mechanics

## Sample Student Responses and Scoring Commentary Set 2

**Inside:**

Free Response Question 3

☑ Scoring Guideline

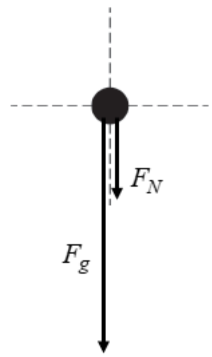
☑ Student Samples

☑ Scoring Commentary

## Question 3: Free-Response Question — 15 points

| | | | |
|---|---|---|---|
| **(a)** | | For correctly drawing and labeling the weight of the block | **1 point** |
| | | For correctly drawing and labeling the force exerted by the track on the block | **1 point** |
| | | For a correct justification consistent with the diagram | **1 point** |

**Example response for part (a)**




*The force $F_g$ represents the weight of the block and always points downward. The force $F_N$ represents the force the track exerts on the block to keep it moving in a circular path and points perpendicular to the surface of the track.*

**Scoring Note**: Examples of appropriate labels for the force due to gravity include: $F_G$, $F_g$, $F_{grav}$, $W$, $mg$, $Mg$, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$, $F_{\text{Block,E}}$ .The labels G or g are not appropriate labels for the force due to gravity. $F_n$, $F_N$, $N$, "normal force," "ground force," or similar labels may be used for the normal force.

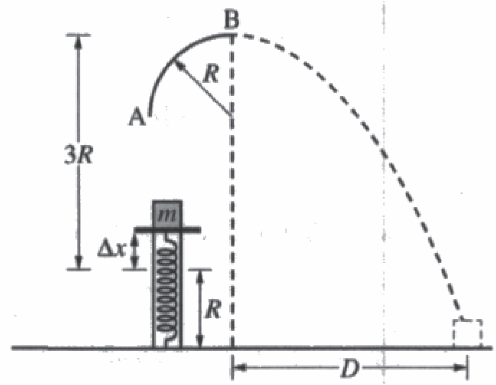
**Scoring Note**: If extraneous forces are present, a maximum of 2 points can be earned.

**Total for part (a) — 3 points**

| | | | |
|---|---|---|---|
| **(b)** | **i.** | For using conservation of energy<br>$U_1 + K_1 = U_2 + K_2 \therefore U_1 + 0 = U_2 + K_2$ | **1 point** |
| | | For correctly relating the elastic potential energy at maximum spring compression to the gravitational potential energy at point B<br>$U_{s1} + 0 = U_{g2} + K_2 \therefore K_2 = U_{s1} - U_{g2}$ | **1 point** |
| | | For a correct substitution into the equation above<br>$\frac{1}{2}mv_B^2 = \frac{1}{2}k(\Delta x)^2 - mgh_2$<br>$v_B^2 = \frac{k}{m}(\Delta x)^2 - 2g(3R) \therefore v_B = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}$ | **1 point** |

| | | | |
|---|---|---|---|
| | **ii.** | For correctly relating the centripetal force to speed from part (b)(i)<br>$F_C = \dfrac{mv^2}{r} = \dfrac{mv_B^2}{R}$ | **1 point** |
| | | For an answer consistent with part (b)(i)<br>$F_C = \dfrac{m}{R}\left(\sqrt{\dfrac{k}{m}(\Delta x)^2 - 6gR}\right)^2 = \dfrac{k(\Delta x)^2}{R} - 6mg$ | **1 point** |
| | | **Total for part (b)** | **5 points** |
| **(c)** | | For correctly relating the net force to the diagram from part (a) and setting the normal force equal to zero | **1 point** |
| | | For correctly substituting into the equation above<br>$\dfrac{k(\Delta x)^2}{R} - 6mg = mg \therefore \dfrac{k(\Delta x)^2}{R} = 7mg \therefore \Delta x = \sqrt{\dfrac{7mgR}{k}}$ | **1 point** |
| | | **Total for part (c)** | **2 points** |
| **(d)** | | For correctly relating the height of fall to the time of fall<br>$y = y_0 + v_{oy}t + \dfrac{1}{2}a_y t^2 \therefore H = 0 + 0 + \dfrac{1}{2}gt^2 \therefore t = \sqrt{\dfrac{(2)(4R)}{g}} = \sqrt{\dfrac{8R}{g}}$ | **1 point** |
| | | For correctly substituting into the equation for constant velocity consistent with part (b)(i)<br>$D = v_x t = \left(\sqrt{\dfrac{k}{m}(\Delta x)^2 - 6gR}\right)\left(\sqrt{\dfrac{8R}{g}}\right)$ | **1 point** |
| | | **Total for part (d)** | **2 points** |
| **(e)** | **i.** | For a correct justification | **1 point** |
| | **ii.** | For indicating that as the maximum compression of the spring increases, the distance D increases | **1 point** |
| | | For indicating that the minimum value is due to the minimum speed needed to get through the track. | **1 point** |
| | | **Example responses for part (e)**<br>*The block needs a minimum speed to make it through point B on the track; thus, the horizontal line segment represents compressions of the spring for which the block does not make it to point B.*<br>OR<br>*From the equation in part (d), the compression of the spring is directly proportional to the horizontal distance traveled by the block; thus, the graph would be a straight line. The minimum value is the distance traveled when the compression of the spring generates the minimum speed needed to reach point B on the track.* | |
| | | **Total for part (e)** | **3 points** |
| | | **Total for question 3** | **15 points** |

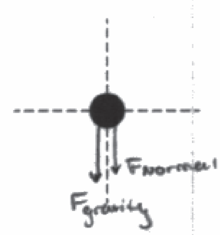
Begin your response to **QUESTION 3** on this page.




Note: Figure not drawn to scale.

3. A block of mass $m$ is placed on top of an ideal spring of spring constant $k$. The block is pushed against the spring, compressing the spring a distance $\Delta x$. The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature $R$ that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance $D$ from the end of the track. Express all algebraic answers in terms of $m$, $k$, $\Delta x$, $R$, and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

(a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.




Justify your choice of vectors.

The force of gravity is included because it always pulls objects toward the earth (down). Depending on how fast the block is moving, it might also experience a normal force from the ramp. The normal force is always perpendicular to the surface, so it too points down



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b)

i. Derive an expression for the speed $v$ of the block at point B.

$E_f = E_i$

$K_f + U_{gf} = U_{s,i}$

$\frac{1}{2}mv^2 + mgh = \frac{1}{2}kx^2$

$mv^2 + 2mg(3R) = k(\Delta x)^2$

$\boxed{v = \sqrt{\frac{k(\Delta x^2) - 6mgR}{m}}}$

ii. Derive an expression for the magnitude of the net force $F$ on the block at point B.

$F_{net} = \frac{mv^2}{R}$

$= \frac{m\left(\frac{k(\Delta x)^2 - 6mgR}{m}\right)}{R}$

$\boxed{F = \frac{k(\Delta x)^2}{R} - 6mg}$

(c) Derive an expression for the minimum value of $\Delta x_{min}$ required in order for the block to maintain contact with the track through point B.

$F_{grav} = \frac{mv^2}{r}$

$\cancel{m}g = \frac{\cancel{m}v^2}{R}$

$v = \sqrt{gR}$

$\sqrt{gr} = \sqrt{\frac{k(\Delta x^2) - 6mgR}{m}}$

$mgR = k(\Delta x)^2 - 6mgR$

$7mgR = k(\Delta x)^2$

$\boxed{\Delta x_{min} = \sqrt{\frac{7mgR}{k}}}$

The procedure is repeated several times with the distance $\Delta x > \Delta x_{min}$.

(d) Calculate the distance $D$ that the block travels.

$y_f = y_i + v_i t + \frac{1}{2}at^2$

$0 = 3R - \frac{(g)}{2}t^2$

$\sqrt{\frac{6R}{g}} = t$

$x_f = x_i + vt$

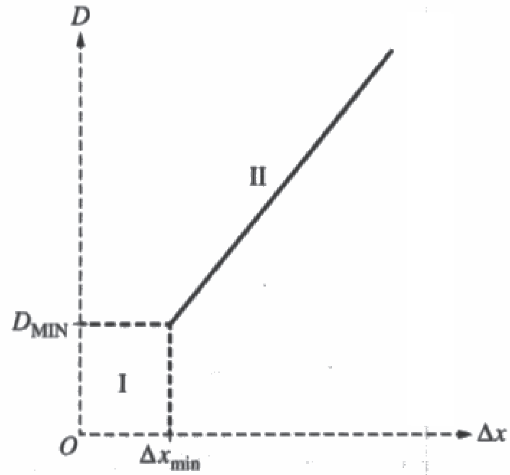
$D = v(t)$

$D = \sqrt{\frac{k(\Delta x)^2 - 6mgR}{m}} \cdot \sqrt{\frac{6R}{g}}$

$\boxed{D = \sqrt{\frac{6kR(\Delta x)^2 - 6mgR^2}{mg}}}$

Continue your response to **QUESTION 3** on this page.

(e) The graph below shows the best-fit line drawn by the students through their data of $D$ as a function of $\Delta x$.



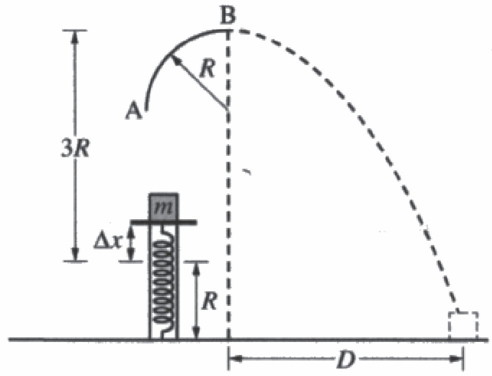

i. Explain why there are no data for section I of the graph.

when $\Delta x$ is less than $\Delta x_{min}$, the block does not maintain contact with the track until point B

ii. Explain the reason for the shape and minimum value of section II on the graph.

D is directly proportional to the square root of $(\Delta x)^2$, which means that D is proportional to $\Delta x$; this linear relationship produces a line

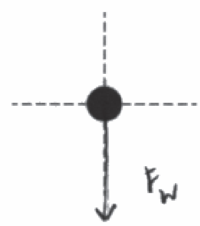
Begin your response to **QUESTION 3** on this page.




<u>Note:</u> Figure not drawn to scale.

3. A block of mass $m$ is placed on top of an ideal spring of spring constant $k$. The block is pushed against the spring, compressing the spring a distance $\Delta x$. The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature $R$ that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance $D$ from the end of the track. Express all algebraic answers in terms of $m$, $k$, $\Delta x$, $R$, and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

(a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.




Justify your choice of vectors.

At point B the only force acting on the block is gravity; because the block is perpendicular to the ground at point B, the track does not exert normal force on it.

Continue your response to **QUESTION 3** on this page.

(b)

i. Derive an expression for the speed $v$ of the block at point B.

$\Delta KE + \Delta U_{g} + \Delta U_s = 0$

$\frac{1}{2}m(v_f^2 - \cancel{v_i^2}) + mgh - \frac{1}{2}k(\Delta x)^2 = 0$

$\frac{1}{2}mv^2 + mg(3R) - \frac{1}{2}k(\Delta x)^2 = 0$

$\rightarrow \frac{1}{2}mv^2 = \frac{1}{2}k(\Delta x)^2 - 3mgR$

$v = \sqrt{\dfrac{\frac{1}{2}k(\Delta x)^2 - 3mgR}{\frac{1}{2}m}}$

ii. Derive an expression for the magnitude of the net force $F$ on the block at point B.

$mg = F_{net}$

(c) Derive an expression for the minimum value of $\Delta x_{min}$ required in order for the block to maintain contact with the track through point B.

$F_s = -k\Delta x$

The procedure is repeated several times with the distance $\Delta x > \Delta x_{min}$.

(d) Calculate the distance $D$ that the block travels.

$v = \sqrt{\dfrac{\frac{1}{2}k(\Delta x)^2 - 3mgR}{\frac{1}{2}m}}$

$0 = 4R + \frac{1}{2}gt^2$

$-4R = -\frac{1}{2}gt^2$

$8R = gt^2$

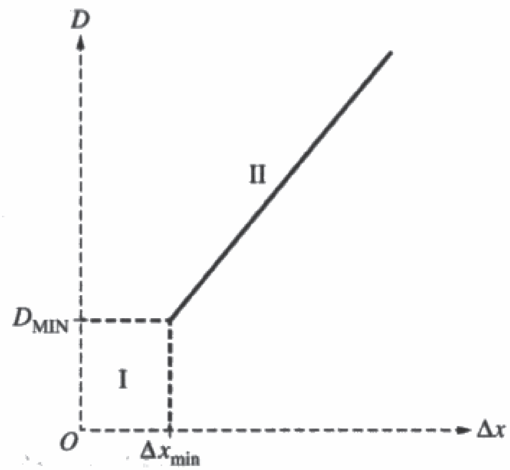
$t = \sqrt{\dfrac{8R}{g}}$

$\boxed{D = \sqrt{\dfrac{\frac{1}{2}k(\Delta x)^2 - 3mgR}{\frac{1}{2}M}}\left(\sqrt{\dfrac{8R}{g}}\right)}$

$=$

Continue your response to **QUESTION 3** on this page.

(e) The graph below shows the best-fit line drawn by the students through their data of $D$ as a function of $\Delta x$.

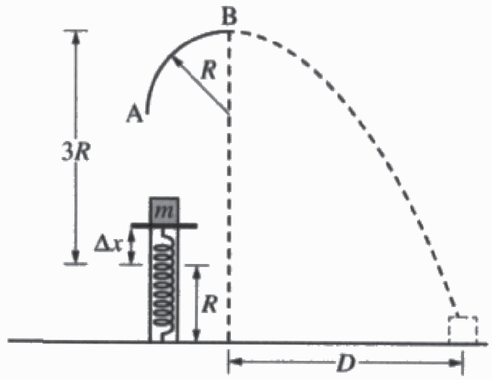



i. Explain why there are no data for section I of the graph.

If $\Delta x < \Delta x_{min}$, the block falls before it exits the track, so it lacks the horizontal velocity to go a distance D beyond point B.

ii. Explain the reason for the shape and minimum value of section II on the graph.

Begin your response to **QUESTION 3** on this page.

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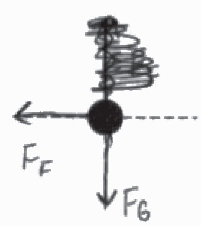
(a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.




Justify your choice of vectors.

Continue your response to **QUESTION 3** on this page.

(b)

i. Derive an expression for the speed $v$ of the block at point B.

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}I(\omega)^2 + \frac{1}{2}mv^2 \rightarrow \quad v = \sqrt{\frac{k\Delta x^2}{2m}}$$

ii. Derive an expression for the magnitude of the net force $F$ on the block at point B.

$$\Sigma F = ma \qquad \Sigma F_x = F_F$$
$$\Sigma F_y = F_g$$

(c) Derive an expression for the minimum value of $\Delta x_{min}$ required in order for the block to maintain contact with the track through point B.

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}I(\omega)^2 + \frac{1}{2}mv^2$$

The procedure is repeated several times with the distance $\Delta x > \Delta x_{min}$.

(d) Calculate the distance $D$ that the block travels.

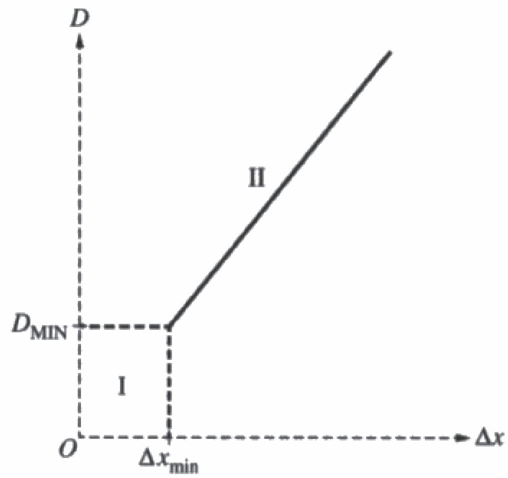
$$v = \sqrt{\frac{k\Delta x^2}{2m}}$$

$$v = \frac{D}{t}$$

$$t\sqrt{\frac{k\Delta x^2}{2m}} = D$$

Continue your response to **QUESTION 3** on this page.

(e) The graph below shows the best-fit line drawn by the students through their data of $D$ as a function of $\Delta x$.




i. Explain why there are no data for section I of the graph.

The block doesn't travel until after $\Delta x$ min.

ii. Explain the reason for the shape and minimum value of section II on the graph.

That is the starting position after $\Delta x_{min}$ has been reached.

# Question 3

**Note:** Student samples are quoted verbatim and may contain spelling and grammatical errors.

**Overview**

The responses to this question were expected to demonstrate an understanding of the following concepts:

- Newton's laws, including how to identify and explain the forces acting on an object in circular motion.
- Recognition of the minimum requirement for an object to continue around a loop.
- Conservation of energy and how spring potential energy is transformed into gravitational potential and kinetic energy.
- The relationship of initial projectile velocity to relevant quantities of motion, such as time of flight, height, and distance traveled.
- Analysis and interpretation of sections of a given graph.

**Sample: M Q3 A**
**Score: 13**

Part (a) earned 3 points for correctly labeling each individual vector and correct justification. Part (b) earned 5 points for correctly using conservation of energy to solve for velocity and relating it to the net force on the block at point B. Part (c) earned 2 points for correctly showing zero normal force and substituting into the correct equation. Part (d) earned 1 point for using the velocity from (b)(i). Incorrect relation given for height of fall and time. Part (e) earned 2 points for correctly explaining the missing data in section I and the relationship between $\Delta x$ and D in section II. No explanation of why the graph started at $\Delta x_{min}$.

**Sample: M Q3 B**
**Score: 7**

Part (a) earned 1 point for correctly labeling the weight. No vector is drawn in for the normal force and the justification of the drawn vector is incorrect. Part (b) earned 3 points for correctly using conservation of energy to solve for velocity. No relationship to centripetal force and its relation to (b)(i). Part c) earned no points as the normal force is not set equal to zero, and the velocity is not used from part (b). Part (d) earned 2 points for correctly relating the total height to time and using the velocity from (b)(i). Part (e) earned 1 point for a correct explanation of missing data in section I. There is no explanation for section II of the graph.

**Sample: M Q3 C**
**Score: 3**

Part (a) earned 1 point for the correctly drawn in weight. Normal force is not drawn in, and friction is not present. Part (b) earned 1 point for the use of conservation of energy. Part (c) earned no points as the normal force is not set equal to zero, and the velocity is not used from part (b). Part (d) earned 1 point for using the velocity from (b)(i). No relationship between height and time of fall is given. Part (e) earned no points for an incorrect explanation of sections I and II of the graph.