2021

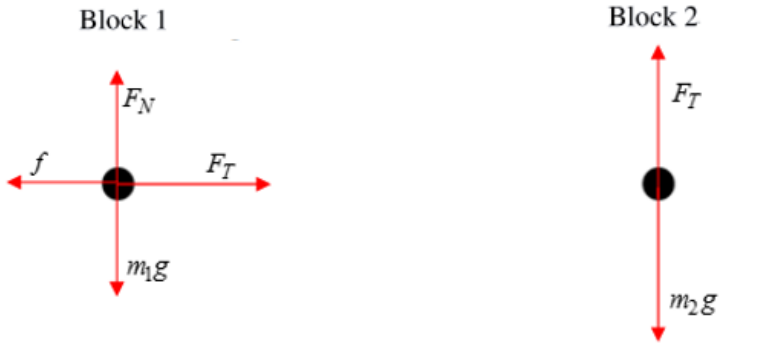
AP® Physics C: Mechanics

Scoring Guidelines
Set 2

Question 1: Free-Response Question — 15 points

(a)	For correctly evaluating Newton's second law equations for the system at rest $m_2 g - f = (m_1 + m_2)a = 0 \therefore m_2 g = f$	**1 point**
	For correctly substituting for static friction into above equation: $m_2 g = f = \mu_s F_N = \mu_s m_1 g$ $\mu_s = \frac{m_2}{m_1} = \frac{(0.20\ \text{kg})}{(0.44\ \text{kg})} = 0.45$	**1 point**
		Total for part (a) 2 points
(b)	For correctly drawing and labeling the horizontal forces of friction and tension on block of mass m_1	**1 point**
	For correctly drawing and labeling the vertical forces of weight and normal force on block of mass m_1	**1 point**
	For correctly drawing and labeling forces of weight and tension on block of mass m_2	**1 point**
	For indicating that the gravitational forces on each block are different	**1 point**

Example responses for part (b)

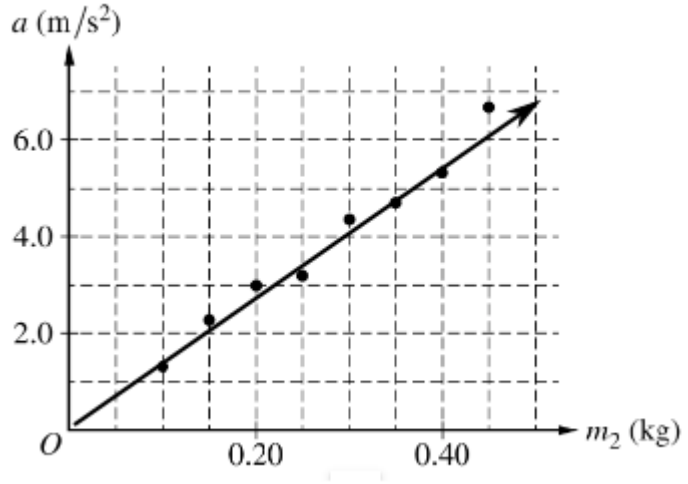


Scoring note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{E,Block}$, $F_{Block,E}$. The labels G or g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force.

Total for part (b) 4 points

(c)	For correctly evaluating Newton's second law equation for block 1: $T - f = m_1 a$	**1 point**
	For correctly evaluating Newton's second law equation for block 2: $m_2 g - T = m_2 a$ Combining the two equations $m_2 g - f = (m_1 + m_2)a \therefore f = m_2 g - (m_1 + m_2)a$ **Scoring note:** Both points are earned for a single correct Newton's second law equation for the two-block system.	**1 point**
	For correctly substituting for kinetic friction into above equation $f = \mu_k F_N = \mu_k m_1 g = m_2 g - (m_1 + m_2)a \therefore \mu_k = \frac{m_2 g - (m_1 + m_2)a}{m_1 g}$ $\mu_k = \frac{(0.20\text{ kg})\left(9.8\text{ m/s}^2\right) - (0.44\text{ kg} + 0.20\text{ kg})\left(2.3\text{ m/s}^2\right)}{(0.44\text{ kg})\left(9.8\text{ m/s}^2\right)} = 0.11$	**1 point**
		Total for part (c) 3 points
(d)	For selecting "Yes" and attempting a relevant justification	**1 point**
	For a correct justification	**1 point**
	Example response for part (d) *If the track is not level, the angle of the track must be incorporated into the equation for acceleration, and this could account for the larger coefficient of kinetic friction.*	
		Total for part (d) 2 points

(e) **i.** For drawing an appropriate best-fit line **1 point**



ii. For calculating slope using two points from the best-fit line **1 point**

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(6-2)\left(\text{m}/\text{s}^2\right)}{(0.45-0.15)(\text{kg})} = 13.3\ \text{m}/\text{kg}\cdot\text{s}^2$$

For correctly using an expression that relates the slope to the acceleration due to gravity **1 point**

From $y = mx + b$

$a =$ (slope)m_2 + (y-intercept)

$$a = \frac{m_2 g}{(m_1 + m_2)} \therefore \text{slope} = \frac{g}{(m_1 + m_2)}$$

$$g = \text{slope} \times (m_1 + m_2) = \left(13.3\ \text{m}/\text{kg}\cdot\text{s}^2\right)(0.44\ \text{kg} + 0.20\ \text{kg}) = 8.5\ \text{m}/\text{s}^2$$

Total for part (e) 3 points

(f) For a correct justification **1 point**

Example response for part (f)

The acceleration would be greater because there would be a component of the gravitational force on block 1 along the surface, which would be in the same direction as the tension force.

Total for question 1 15 points

Question 2: Free-Response Question — 15 points

(a)

For integrating using the correct limits or constant of integration — **1 point**

$$I = \int_{r=0}^{r=2L} \lambda r^2 dr = \lambda \left[\frac{r^3}{3} \right]_{r=0}^{r=2L} = \frac{\lambda}{3}\left((2L)^3 - 0\right)$$

For correctly relating λ to M and L — **1 point**

$$\lambda = \frac{m}{\ell} = \frac{M}{2L} \therefore I = \left(\frac{1}{3}\right)\left(\frac{M}{2L}\right)\left(8L^3\right) = \frac{4}{3}ML^2$$

Total for part (a) 2 points

(b) i.

For correctly substituting into an equation for the center of mass of an object in the horizontal direction — **1 point**

$$X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\left[\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right)(L)\right]}{\left(\frac{M}{2} + \frac{M}{2}\right)} = \frac{\left(\frac{ML}{4} + \frac{ML}{2}\right)}{M} = \frac{3}{4}L$$

ii.

For correctly substituting into an equation for the center of mass of an object in the vertical direction — **1 point**

$$Y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\left[\left(\frac{M}{2}\right)(L) + \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)\right]}{\left(\frac{M}{2} + \frac{M}{2}\right)} = \frac{\left(\frac{ML}{2} + \frac{ML}{4}\right)}{M} = \frac{3}{4}L$$

Total for part (b) 2 points

(c)

For selecting "Less than" and attempting a relevant justification — **1 point**

For a correct justification — **1 point**

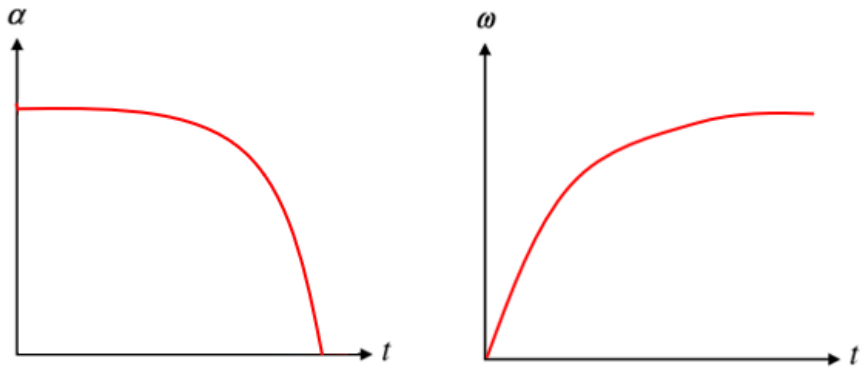
Example response for part (c)

Because object B has more of its mass closer to the pivot than object A, the rotational inertia of object B must be less than that of object A.

Total for part (c) 2 points

(d)	For an acceleration graph that is concave down and begins horizontally	**1 point**
	For an angular speed graph that is concave down and ends horizontally	**1 point**
	For consistency between the angular acceleration and angular speed graphs	**1 point**

Example responses for part (d)



Total for part (d) 3 points

(e)	For selecting "Decreasing" and attempting a relevant justification	**1 point**
	For a justification that indicates the lever arm for the torque is decreasing	**1 point**

Example response for part (e)

Because the horizontal position of the center of mass for the object is moving closer to the pivot, the lever arm for the force of gravity is decreasing so the angular acceleration decreases.

Total for part (e) 2 points

(f)	For using conservation of energy $U_{g1} = K_2$	**1 point**
	For correctly relating the change in rotational kinetic energy to the change in gravitational potential energy $mgh = \frac{1}{2} I\omega^2$	**1 point**
	For correctly substituting for h into the equation above $mg\left(\frac{L}{2}\right) = \frac{1}{2} I\omega^2$	**1 point**
	For an expression for ω that uses only the allowed symbols and is algebraically consistent with the previous steps $\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2Mg(L/2)}{I_B}}$ $\omega = \sqrt{\frac{MgL}{I_B}}$	**1 point**

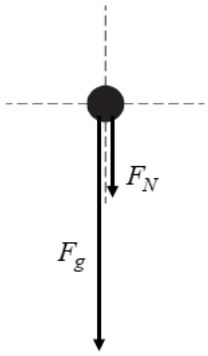
Total for part (f) 4 points

Total for question 2 15 points

Question 3: Free-Response Question — 15 points

(a)		For correctly drawing and labeling the weight of the block	**1 point**
		For correctly drawing and labeling the force exerted by the track on the block	**1 point**
		For a correct justification consistent with the diagram	**1 point**

Example response for part (a)



The force F_g represents the weight of the block and always points downward. The force F_N represents the force the track exerts on the block to keep it moving in a circular path and points perpendicular to the surface of the track.

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Scoring Note: If extraneous forces are present, a maximum of 2 points can be earned.

		Total for part (a)	**3 points**
(b)	**i.**	For using conservation of energy $U_1 + K_1 = U_2 + K_2 \therefore U_1 + 0 = U_2 + K_2$	**1 point**
		For correctly relating the elastic potential energy at maximum spring compression to the gravitational potential energy at point B $U_{s1} + 0 = U_{g2} + K_2 \therefore K_2 = U_{s1} - U_{g2}$	**1 point**
		For a correct substitution into the equation above $\frac{1}{2}mv_B^2 = \frac{1}{2}k(\Delta x)^2 - mgh_2$ $v_B^2 = \frac{k}{m}(\Delta x)^2 - 2g(3R) \therefore v_B = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}$	**1 point**

	ii.	For correctly relating the centripetal force to speed from part (b)(i) $F_C = \frac{mv^2}{r} = \frac{mv_B^2}{R}$	**1 point**
		For an answer consistent with part (b)(i) $F_C = \frac{m}{R}\left(\sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}\right)^2 = \frac{k(\Delta x)^2}{R} - 6mg$	**1 point**
		Total for part (b)	**5 points**
(c)		For correctly relating the net force to the diagram from part (a) and setting the normal force equal to zero	**1 point**
		For correctly substituting into the equation above $\frac{k(\Delta x)^2}{R} - 6mg = mg \therefore \frac{k(\Delta x)^2}{R} = 7mg \therefore \Delta x = \sqrt{\frac{7mgR}{k}}$	**1 point**
		Total for part (c)	**2 points**
(d)		For correctly relating the height of fall to the time of fall $y = y_0 + v_{oy}t + \frac{1}{2}a_y t^2 \therefore H = 0 + 0 + \frac{1}{2}gt^2 \therefore t = \sqrt{\frac{(2)(4R)}{g}} = \sqrt{\frac{8R}{g}}$	**1 point**
		For correctly substituting into the equation for constant velocity consistent with part (b)(i) $D = v_x t = \left(\sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}\right)\left(\sqrt{\frac{8R}{g}}\right)$	**1 point**
		Total for part (d)	**2 points**
(e)	**i.**	For a correct justification	**1 point**
	ii.	For indicating that as the maximum compression of the spring increases, the distance D increases	**1 point**
		For indicating that the minimum value is due to the minimum speed needed to get through the track.	**1 point**
		Example responses for part (e) *The block needs a minimum speed to make it through point B on the track; thus, the horizontal line segment represents compressions of the spring for which the block does not make it to point B.* OR *From the equation in part (d), the compression of the spring is directly proportional to the horizontal distance traveled by the block; thus, the graph would be a straight line. The minimum value is the distance traveled when the compression of the spring generates the minimum speed needed to reach point B on the track.*	
		Total for part (e)	**3 points**
		Total for question 3	**15 points**