2022

AP® CollegeBoard

AP® Physics C: Mechanics

Sample Student Responses and Scoring Commentary Set 1

Inside:

Free-Response Question 3

- ☑ Scoring Guidelines
- ☑ Student Samples
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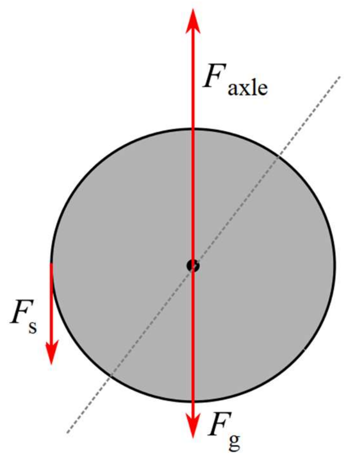
Question 3: Free-Response Question — 15 points

(a)	For indicating that the sum of the torques on the disk equals zero $\Sigma\tau_{\text{on disk}} = 0$ $\tau_g = \tau_s$ **OR** For indicating that the sum of the forces equals zero $\Sigma F = 0$ $F_g = F_s$	**1 point**
	For correctly substituting the expressions for the forces $F_g R = F_s R$ $m_B g R = k\Delta x R$ $m_B g = k\Delta x$ **OR** $F_g = F_s$ $m_B g = k\Delta x$	**1 point**
	For correctly substituting for Δx $m_B g = kR\theta$ $m_B = \dfrac{kR\theta}{g}$	**1 point**
	Total for part (a)	**3 points**
(b)	For drawing and labeling the force of the tension exerted on the disk anywhere between point P and the left edge of the disk, including point P and the left edge of the disk, tangent to the disk	**1 point**
	For correct location and label of the force due to gravity exerted on the disk, directed straight down	**1 point**
	For correct location and label of the force exerted on the disk by the axle, directed such that the disk remains in translational equilibrium (i.e., $\Sigma F = 0$)	**1 point**

Example Response



Scoring Notes:

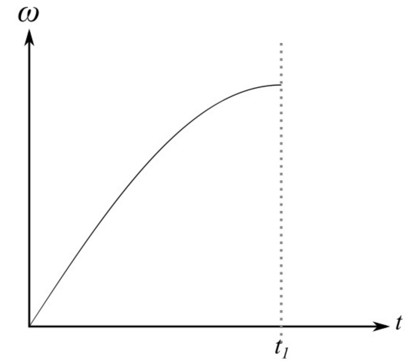
- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force, which can be used instead of F_{axle}. F_{spring}, F_s, T_{spring}, T, "spring force," or similar labels may be used for the tension force exerted by the spring.
- A response with extraneous forces or vectors can earn a maximum of two points.

	Total for part (b)	3 points
(c)	For indicating that the net torque is due only to the force exerted on the disk by the tension in the rotational form of Newton's second law $\tau_s = I_d \alpha$	**1 point**
	For correctly expressing the torque on the disk by the tension in terms of the spring force, which is equal to the tension, and the lever (moment) arm $F_s R = I_d \alpha$	**1 point**
	For correctly substituting for F_s $-k\Delta x R = I_d \alpha$	**1 point**
	For correctly substituting I_d and Δx, or an expression for Δx consistent with part (a) $-k(R\theta)R = \frac{1}{2} M_d R^2 \alpha$ $\alpha = -\frac{2k\theta}{M_d}$ **Scoring Note:** The negative sign is not necessary to earn this point.	**1 point**
	Total for part (c)	**4 points**

(d) For a sketch that starts at zero and monotonically increases until time $t = t_1$ **1 point**

For a sketch that is concave down between time $t = 0$ and $t = t_1$ **1 point**

Example Response



Scoring Note: Any part of the graph beyond t_1 is not considered in scoring.

Total for part (d) 2 points

(e) For indicating that the torque exerted by the force due to gravity on the disk increased **1 point**

Example Response

The force due to gravity on the disk now has a non-zero lever arm and hence it exerts a larger torque on the disk.

For indicating that the torque exerted by the tension caused by the force due to gravity on the block increased **1 point**

Example Response

The force exerted by the right side of the string (from the block) on the disk has a longer lever arm, hence the torque it exerts is larger.

For indicating that the torque exerted by the tension increased **1 point**

Example Response

The counterclockwise torque due to the tension caused by the spring must increase to counteract the increase in clockwise torques due to the force due to gravity of the disk and tension caused by the force of gravity due to block to keep the disk in equilibrium.

Scoring Notes:

- A response that references the torque due to the force at the axle staying the same can earn all 3 points.
- A response that references the torque due to the force on the axle changing, or any additional torques can earn a maximum of 2 points.

Total for part (e) 3 points

Total for question 3 15 points

Question 3

Begin your response to **QUESTION 3** on this page.

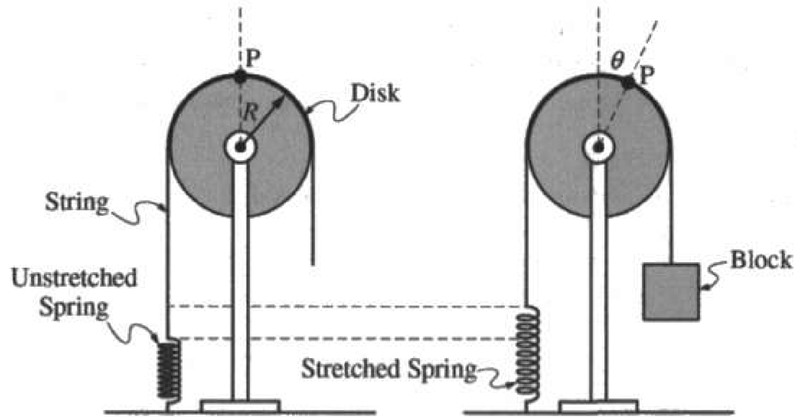


Figure 1 Figure 2

Note: Figures not drawn to scale.

3. A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \frac{1}{2}M_d R^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

Question 3

Continue your response to **QUESTION 3** on this page.

(a) Derive an expression for the mass m_B of the block.

$\Sigma F_{total} = (ma)_{total}$ $\qquad x = r\theta$

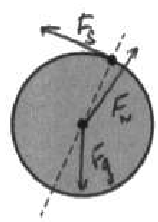
$m_B g - kx = 0 \quad \leftarrow \quad x = R\cdot\theta$

$m_B g - k\cdot R\theta = 0$

$m_B g = kR\theta$

$m_B = \frac{kR\theta}{g}$

(b) At time $t = 0$, the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.



(c) Derive an expression for the angular acceleration α of the disk immediately after the string is cut.

F_s = spring force = $-kx$

$x = R\theta$

$\Sigma \tau = I\alpha$

$R\cdot F_s \sin 90 = \frac{1}{2} m_d R^2 \alpha$

$R\cdot kx = \frac{1}{2} m_d R^2 \alpha$

$R\cdot kR\theta = \frac{1}{2} m_d R^2 \alpha$

$\frac{2}{m_d}\cdot k\theta = \frac{1}{2} m_d \alpha \cdot \frac{2}{m_d}$

$\alpha = \frac{2k\theta}{m_d}$

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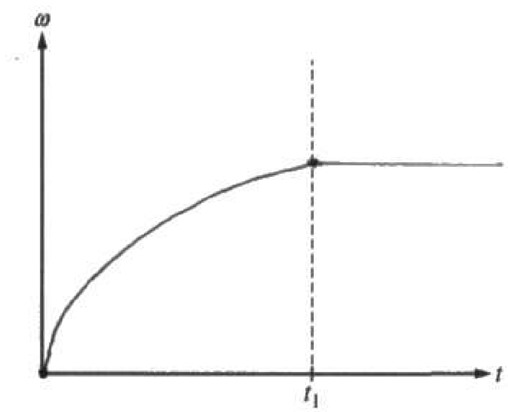
Question 3

Continue your response to **QUESTION 3** on this page.

(d) At $t = t_1$, the disk has rotated and point P is again directly above the axle. Sketch a graph of the magnitude of the angular velocity ω of the disk as a function of time t from $t = 0$ to $t = t_1$.



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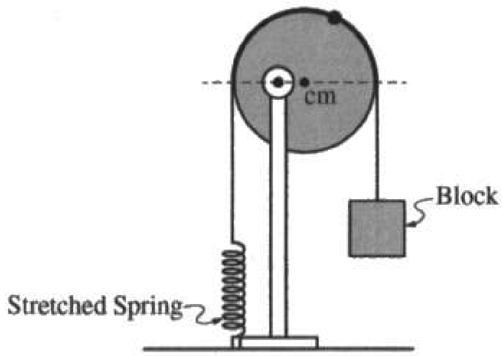

Figure 3

Note: Figure not drawn to scale.

(e) The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

F_{S_B}: magnitude of torque increases because the string was cut in part (b).

F_g: magnitude of torque increases

F_N: magnitude of torque stays the same

F_s: magnitude of torque increases

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Question 3

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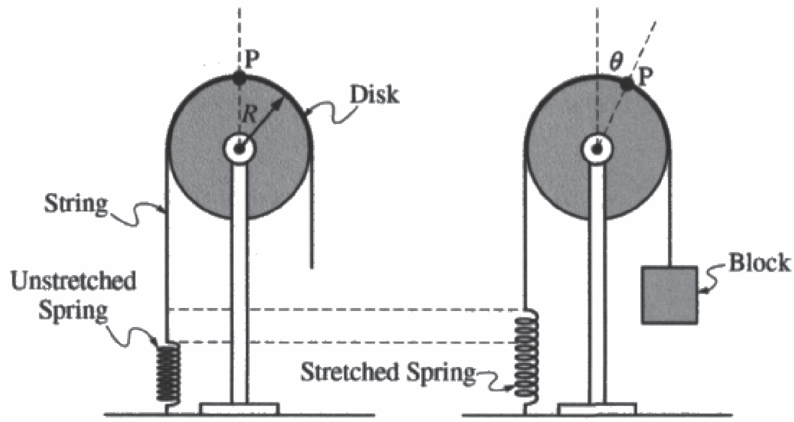


Figure 1 Figure 2

Note: Figures not drawn to scale.

3. A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \frac{1}{2}M_dR^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Question 3

Continue your response to **QUESTION 3** on this page.

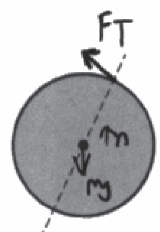
(a) Derive an expression for the mass m_B of the block.

$F_{net} = ma$

$m_b g - -kx = 0$ $x =$ distance for stretched spring

$m_b = \frac{-kx}{g}$

(b) At time $t = 0$, the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.



(c) Derive an expression for the angular acceleration α of the disk immediately after the string is cut.

$\tau_{net} = I\alpha$

$\tau_{net} = T = -kx$

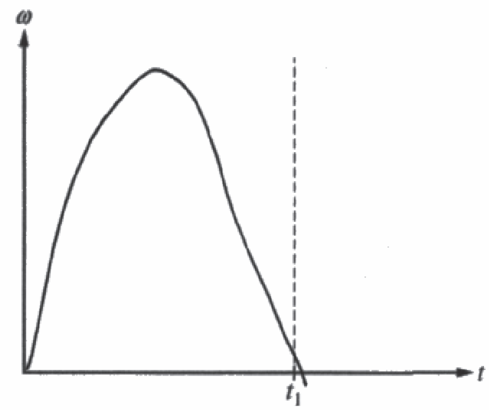
$-kx = \frac{1}{2} m_d r^2 \alpha$

$\alpha = \frac{-2kx}{m_d r^2}$

Question 3

Continue your response to **QUESTION 3** on this page.

(d) At $t = t_1$, the disk has rotated and point P is again directly above the axle. Sketch a graph of the magnitude of the angular velocity ω of the disk as a function of time t from $t = 0$ to $t = t_1$.



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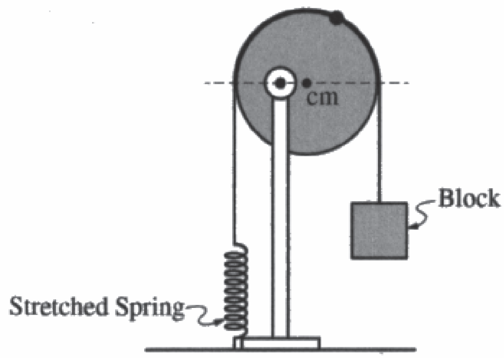


Figure 3

Note: Figure not drawn to scale.

(e) The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

increases

paallel axis therom

$I = I_{cm} + Mr^2 > I_i$

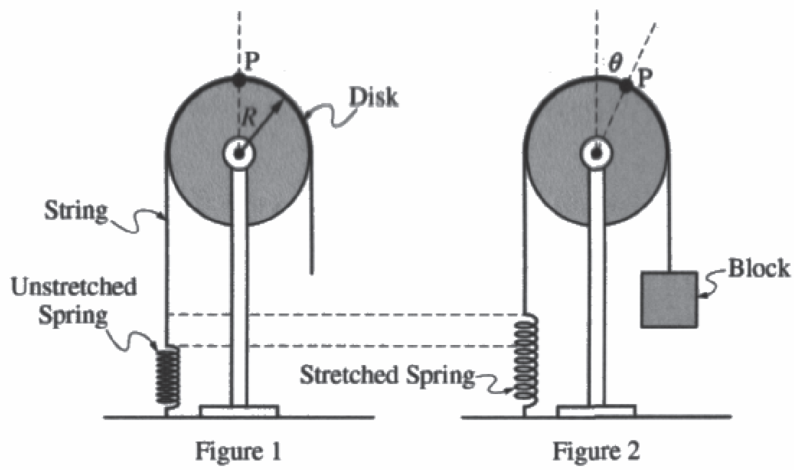
$\tau_{net} = I\alpha$ so torq is bigger

spring is greater

+ tension is greater

Question 3

Begin your response to **QUESTION 3** on this page.



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3. A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \frac{1}{2}M_d R^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

  GO ON TO THE NEXT PAGE.

Question 3

Continue your response to QUESTION 3 on this page.

(a) Derive an expression for the mass m_B of the block.

$\Sigma F = ma$

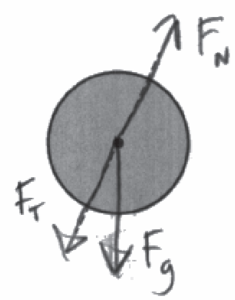
$\Sigma F = T - F_G$

$ma = T - F_G$

$ma = T - mg$

$m_B = \frac{T - mg}{a}$

(b) At time $t = 0$, the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.



(c) Derive an expression for the angular acceleration α of the disk immediately after the string is cut.

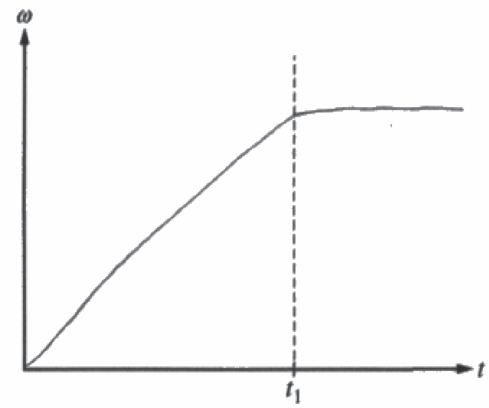
$\omega_f = \omega_i + \alpha t$

GO ON TO THE NEXT PAGE.

Question 3

Continue your response to **QUESTION 3** on this page.

(d) At $t = t_1$, the disk has rotated and point P is again directly above the axle. Sketch a graph of the magnitude of the angular velocity ω of the disk as a function of time t from $t = 0$ to $t = t_1$.



Question 3

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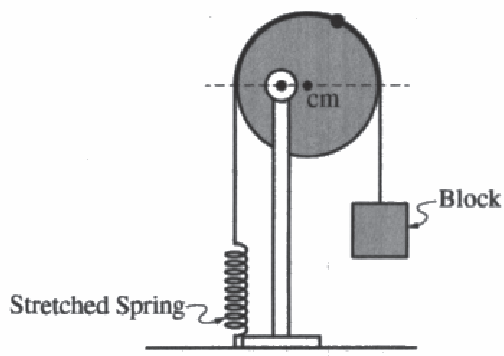

Figure 3

Note: Figure not drawn to scale.

(e) The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

F_G stays the same

F_N stays the same

F_T decreases

  GO ON TO THE NEXT PAGE.

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Read, analyze, and correctly interpret the statement of a prompt, including diagrams showing the apparatus at different moments in the described scenario.
- Apply the concept of both translational and rotational equilibrium.
- Draw forces on a rigid body diagram in the correct location and correct orientation.
- Apply symbolic expressions and algebra to determine the correct relationship between variables within an equation.
- Sketch a graph that shows a functional relationship between angular velocity and time after determining the relationship between angular acceleration and angular position.
- Apply the concept of rotational equilibrium to determine the change in the magnitude of each torque on the rigid body if the axis of rotation changes from its original location.

Sample: 3A
Score: 15

Part (a) earned 3 points. The first point was earned because the response indicates the sum of the forces equals zero. The second point was earned because the response correctly substitutes expressions for the force due to gravity on the block and the force of tension. The third point was earned because the response shows substitution for x in the spring force equation. Part (b) earned 3 points. The first point was earned because the response includes the force caused by the spring at an appropriate location on the diagram. The second point was earned because the response includes the force due to gravity located at the center of mass, pointed straight down, on the diagram. The third point was earned because the response includes the force due to the axle on the diagram. Part (c) earned 4 points. The first point was earned because the response indicates that the net torque is due only to the force exerted on the disk by the force caused by the spring in the rotational form of Newton's second law. The second point was earned because the response expresses the torque on the disk due to the force caused by the spring in terms of the spring force and lever arm. The third point was earned because the response correctly substitutes for the force of the spring. The fourth point was earned because the response correctly substitutes for the rotational inertia of the disk and x. Part (d) earned 2 points. The first point was earned because the graph in the response starts at zero and continually increases until time t_1. The second point was earned because the graph in the response is concave down from time $t = 0$ to $t = t_1$. The response continues past t_1; however, that portion of the graph was not considered for scoring. Part (e) earned 3 points. The first point was earned because the response indicates the torque exerted by the force due to gravity on the disk increased. The second point was earned because the response indicates the torque exerted by the tension caused by the force due to gravity on the block increased. The third point was earned because the response indicates the torque exerted by the tension caused by the spring increased. This response includes a correct statement about the torque due to the axle and can earn all 3 points.

Question 3 (continued)

Sample: 3B
Score: 7

Part (a) earned 2 points. The first point was earned because the response indicates the sum of the forces equals zero. The second point was earned because the response correctly substitutes expressions for the force due to gravity on the block and the force of tension. The third point was not earned because the response does not show substitution for x in the spring force equation. Part (b) earned 2 points. The first point was earned because the response includes the force caused by the spring at an appropriate location on the diagram. The second point was earned because the response includes the force due to gravity located at the center of mass, pointed straight down, on the diagram. The third point was not earned because the response does not include the force due to the axle on the diagram at a correct location nor in the correct direction to keep the disk in translational equilibrium. Part (c) earned 2 points. The first point was earned because the response indicates that the net torque is due only to the force exerted on the disk by the force caused by the spring in the rotational form of Newton's second law. The second point was not earned because the response does not express the torque on the disk due to the force caused by the spring in terms of the spring force and lever arm. The third point was earned because the response correctly substitutes for the force of the spring. The fourth point was not earned because the response does not correctly substitute for the rotational inertia of the disk and x or an expression of x consistent with part (a). Part (d) earned 1 point. The first point was not earned because the graph in the response does not start at zero and continually increases until time t_1. The second point was earned because the graph in the response is concave down from time $t = 0$ to $t = t_1$. Part (e) earned 0 points. The first point was not earned because the response does not indicate the torque exerted by the force due to gravity on the disk increased. The second point was not earned because the response does not indicate the torque exerted by the tension caused by the force due to gravity on the block increased. The third point was not earned because the response does not indicate the torque exerted by the tension caused by the spring increased.

Question 3 (continued)

Sample: 3C
Score: 3

Part (a) earned 0 points. The first point was not earned because the response does not indicate the sum of the forces equals zero. The response does not specify a value for the acceleration. The second point was not earned because the response does not correctly substitute expressions for the force due to gravity on the block and the force of tension. The response does specify the mass of the block in the substitution for the force due to gravity and does not attempt to substitute into the tension force. The third point was not earned because the response does not show substitution for x in the spring force equation. Part (b) earned 2 points. The first point was not earned because the response does not include the force caused by the spring at an appropriate location on the diagram. The second point was earned because the response includes the force due to gravity located at the center of mass, pointed straight down, on the diagram. The third point was earned because the response includes the force due to the axle, labeled F_N, on the diagram. The force due to the axle is directed to counteract downward forces of gravity and tension, as well as the horizontal force introduced via the tension force. Part (c) earned 0 points. The first point was not earned because the response does not indicate that the net torque is due only to the force exerted on the disk by the force caused by the spring in the rotational form of Newton's second law. The second point was not earned because the response does not express the torque on the disk due to the force caused by the spring in terms of the spring force and lever arm. The third point was not earned because the response does not correctly substitute for the force of the spring. The fourth point was not earned because the response does not correctly substitute for the rotational inertia of the disk nor an expression of x consistent with part (a). Part (d) earned 1 point. The first point was earned because the graph in the response starts at zero and continually increases until time t_1. The second point was not earned because the graph in the response is not concave down from time $t = 0$ to $t = t_1$. Part (e) earned 0 points. The first point was not earned because the response does not indicate the torque exerted by the force due to gravity on the disk increased. The second point was not earned because the response does not indicate the torque exerted by the tension caused by the force due to gravity on the block increased. The third point was not earned because the response does not indicate the torque exerted by the tension caused by the spring increased.