2022

AP CollegeBoard

AP® Physics C: Mechanics

Sample Student Responses and Scoring Commentary Set 2

Inside:

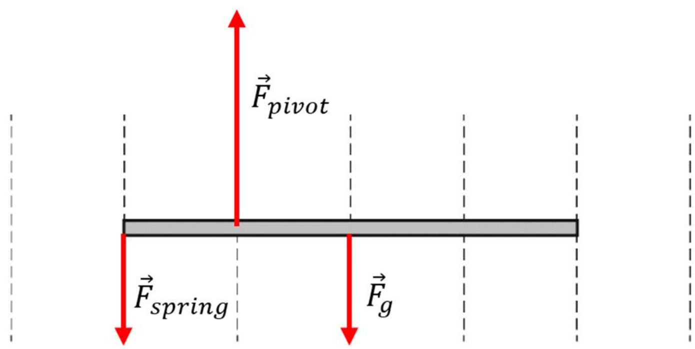
Free-Response Question 3

- ☑ Scoring Guidelines
- ☑ Student Samples
- ☑ Scoring Commentary

Question 3: Free-Response Question — 15 points

(a)	For correct location and label of the force exerted on the board by the spring	**1 point**
	For correct location and label of the force exerted on the board by gravity	**1 point**
	For correct location and label of the force exerted on the board by the pivot	**1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force, which can be used instead of F_{pivot}. F_{spring}, F_S, "spring force," or similar labels may be used for the force exerted by the spring.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) 3 points

(b)	For indicating that the sum of the torques equals zero $\Sigma\tau = 0$	**1 point**
	For substituting mg for the force of gravity and $k\Delta x$ for the spring force into a torque equation $\frac{L}{4}k\Delta x - \frac{L}{4}mg = 0$	**1 point**
	For a correct expression for Δx $\Delta x = \frac{mg}{k}$	**1 point**

Example Response

$\Sigma\tau = 0$

$F_{\text{spring}} d_{\text{spring}} - F_{\text{weight}} d_{\text{weight}} = 0$

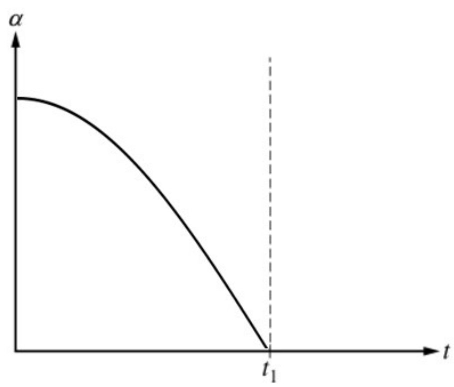
$\frac{L}{4} k\Delta x - \frac{L}{4} mg = 0$

$\Delta x = \frac{mg}{k}$

Scoring Note: This point can be earned regardless of whether a negative sign is present.

	Total for part (b)	**3 points**
(c)(i)	For a sketch that begins at a maximum value and monotonically decreases to a minimum of zero at $t = t_1$	**1 point**
	For a sketch that is concave down between $t = 0$ and $t = t_1$	**1 point**

Example Response



Scoring Note: Any part of the graph beyond t_1 is not considered in scoring.

(c)(ii)	For indicating that the torques are in opposite directions in the rotational form of Newton's second law $\tau_{\text{spring}} - \tau_g = I\alpha$	**1 point**
	For including $\sin(90 - \theta_0)$, $\sin(90 + \theta_0)$, or $\cos\theta_0$ in an expression for the net torque exerted on the rod $F_{spring} d_{spring}(\cos\theta) - F_g d_g(\cos\theta) = I_B\alpha_0$	**1 point**
	For substituting mg for the force due to gravity and substituting $k\Delta x_2$ for the spring force in an expression for the net torque exerted on the rod $d_{spring} k\Delta x_2(\cos\theta_0) - d_g mg(\cos\theta_0) = I_B\alpha_0$	**1 point**
	For substituting correct lever arms in an expression for the net torque exerted on the rod $\frac{L}{4}(\cos\theta_0)k\Delta x_2 - \frac{L}{4}(\cos\theta_0)mg = I_B\alpha_0$	**1 point**

Example Response

$$\tau_{\text{spring}} - \tau_g = I\alpha$$
$$F_{spring} d_{spring} (\cos\theta_0) - F_g d_g (\cos\theta_0) = I_B \alpha_0$$
$$l_{spring} (\cos\theta_0) k\Delta x_2 - l_g (\cos\theta_0) mg = I_B \alpha_0$$
$$\frac{L}{4}(\cos\theta_0) k\Delta x_2 - \frac{L}{4}(\cos\theta_0) mg = I_B \alpha_0$$
$$\alpha_0 = \frac{L}{4I_B}(k\Delta x_2 \cos\theta_0 - mg\cos\theta_0)$$

	Total for part (c)	**6 points**
(d)	For selecting "$\alpha' < \alpha_0$" with an attempt at a relevant justification	**1 point**
	For indicating that the net torque does not change	**1 point**
	For indicating the rotational inertia increases	**1 point**

Example Response

The net torque on the system is the same (the spring and the person exert the same force and the additional torques from the gravitational forces on the masses cancel) but the rotational inertia of the system is now larger. This means that the angular acceleration of the system is now smaller than it was before.

Total for part (d)	**3 points**
Total for question 3	**15 points**

Question 3

Begin your response to **QUESTION 3** on this page.

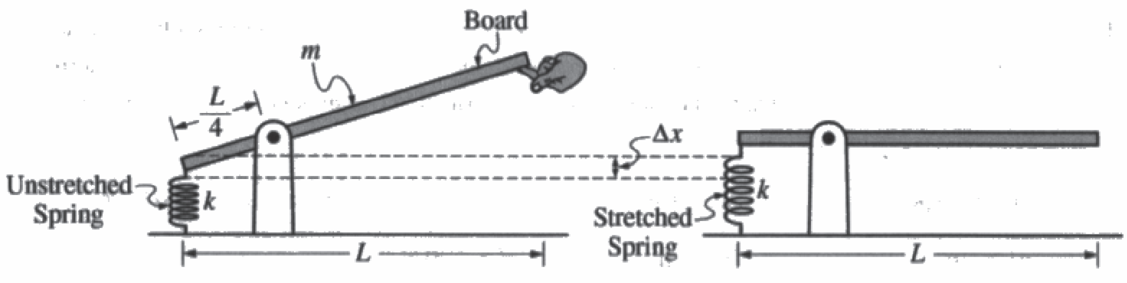


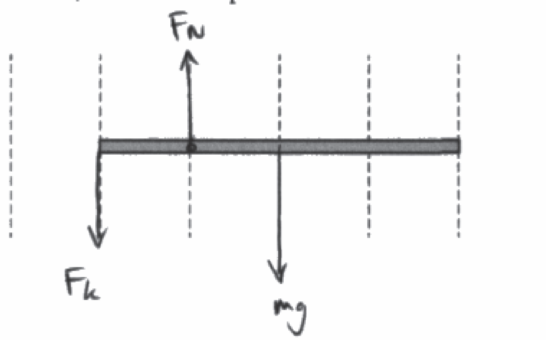
Figure 1

Figure 2

Note: Figures not drawn to scale.

3. A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The right end of the board is initially held by a student so that the spring is unstretched, as shown in Figure 1. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring stretched, as shown in Figure 2. The rotational inertia of the board about the pivot is I.

(a) On the rectangle below, which represents the board, draw and label the forces (not components) that act on the board while the board-spring system is in equilibrium. Each force should be represented by an arrow that starts on, and points away from, the board, and should represent the location at which that force acts.



Question 3

Continue your response to **QUESTION 3** on this page.

(b) Derive an expression for the distance the spring stretches, Δx, when the board is in equilibrium. Express your answer in terms of k, L, m, and physical constants, as appropriate.

$\sum F_{net} = 0, \sum \tau_{net} = 0.$

Let the pivot point be pivot. Clockwise is negative.

$\sum \tau_{net} = r_1 F_k - r_2 mg$

$0 = (\frac{L}{4})(kx) - (\frac{L}{4})(mg).$

$mg = kx$

$\boxed{\Delta x = \frac{mg}{k}}$

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Question 3

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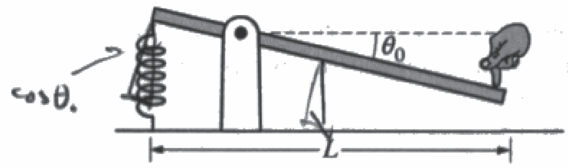
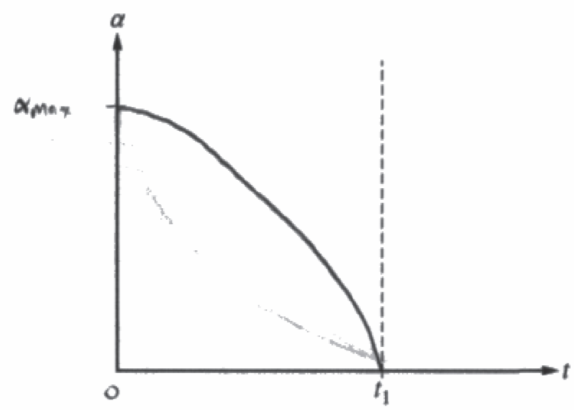


Note: Figure not drawn to scale.

(c) A student pushes the board down on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest.

i. At time $t = 0$, the board is released. At $t = t_1$, the board first crosses the horizontal. Sketch a graph of the magnitude of the angular acceleration α of the board as a function of time t from $t = 0$ to $t = t_1$.



α switches direction at t = t₁ .

Question 3

Continue your response to **QUESTION 3** on this page.

ii. Derive an expression for the angular acceleration α_0 of the board immediately after the board is released. Express your answer in terms of k, L, m, I, Δx_2, θ_0, and physical constants, as appropriate.

$$\tau_{net} = I\alpha$$

$$\tau_{net} = \left(\frac{L}{4}\right)(k\Delta x_2 \cos\theta_0) - \left(\frac{L}{4}\right)(mg\cos\theta_0)$$

$$\alpha = \frac{\tau_{net}}{I} = \boxed{\frac{\left(\frac{L}{4}\right)(k\Delta x_2 \cos\theta_0 - mg\cos\theta_0)}{I}}$$

Question 3

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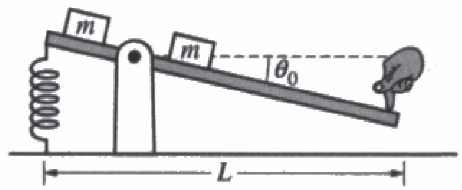

Note: Figure not drawn to scale.

(d) Two blocks of equal mass m are attached to the board equal distances from the pivot point, as shown. The board is again pushed down on the right side so that the spring stretches the same distance Δx_2 as in part (c). The board is then released. How does the new angular acceleration α' when the blocks are attached compare to the angular acceleration α_0 from part (c) ?

__ $\alpha' > \alpha_0$ √ $\alpha' < \alpha_0$ __ $\alpha' = \alpha_0$

Justify your answer.

$I = \int r^2 \, dm$.

with more mass, the moment of inertia of the board-mass system increases.

Since the net torque is the same (the torques of the two masses cancel out each other), the angular acceleration must decrease.

$\tau_{net} = I\alpha$.

If I increases, α must decrease for τ_{net} to ~~be~~ stay constant.

Thus, $\alpha' < \alpha_0$

Question 3

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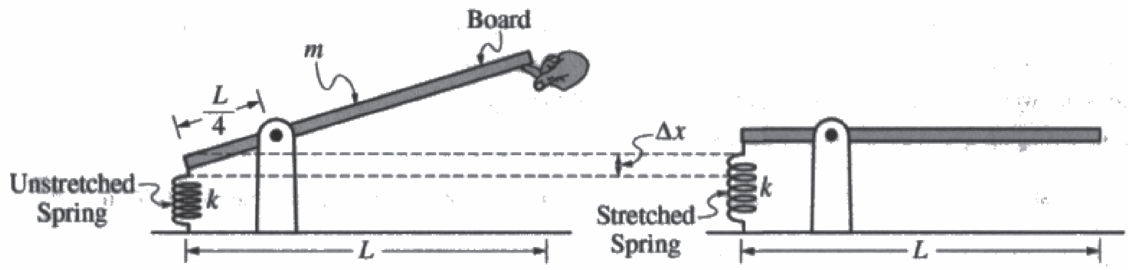
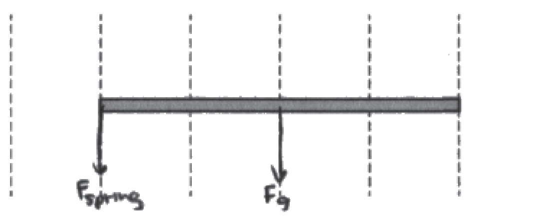


Figure 1 Figure 2

Note: Figures not drawn to scale.

3. A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The right end of the board is initially held by a student so that the spring is unstretched, as shown in Figure 1. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring stretched, as shown in Figure 2. The rotational inertia of the board about the pivot is I.

(a) On the rectangle below, which represents the board, draw and label the forces (not components) that act on the board while the board-spring system is in equilibrium. Each force should be represented by an arrow that starts on, and points away from, the board, and should represent the location at which that force acts.



Question 3

Continue your response to **QUESTION 3** on this page.

(b) Derive an expression for the distance the spring stretches, Δx, when the board is in equilibrium. Express your answer in terms of k, L, m, and physical constants, as appropriate.

$\Sigma\tau = I\alpha = F_g\left(\frac{L}{4}\right)\sin 90 - F_{spring}\left(\frac{L}{4}\right)\sin 90 \qquad \alpha = 0$

$\frac{1}{4}mgL = \frac{1}{4}k\Delta x L$

$\Delta x = \boxed{\frac{mg}{k}}$

Question 3

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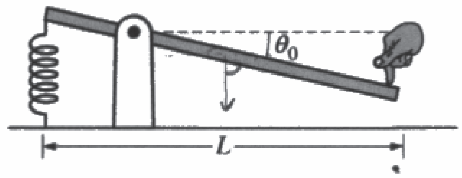
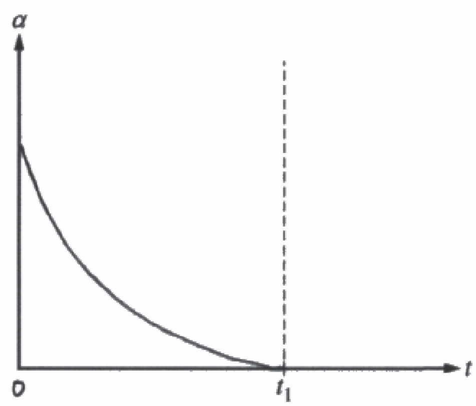


Note: Figure not drawn to scale.

(c) A student pushes the board down on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest.

i. At time $t = 0$, the board is released. At $t = t_1$, the board first crosses the horizontal. Sketch a graph of the magnitude of the angular acceleration α of the board as a function of time t from $t = 0$ to $t = t_1$.



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Question 3

Continue your response to **QUESTION 3** on this page.

ii. Derive an expression for the angular acceleration α_0 of the board immediately after the board is released. Express your answer in terms of k, L, m, I, Δx_2, θ_0, and physical constants, as appropriate.

$$\Sigma \tau = I\alpha_0 = F_g\left(\frac{L}{4}\right)\cos\theta_0 - F_{spring}\left(\frac{L}{4}\right)\cos\theta_0$$

$$\alpha_0 = \frac{\frac{1}{4}mgL\cos\theta_0 - \frac{1}{4}k\Delta x_2 L\cos\theta_0}{I}$$

$$\alpha_0 = (mg - k\Delta x_2)\left(\frac{L\cos\theta_0}{4I}\right)$$

Question 3

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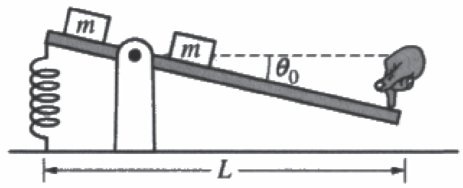

Note: Figure not drawn to scale.

(d) Two blocks of equal mass m are attached to the board equal distances from the pivot point, as shown. The board is again pushed down on the right side so that the spring stretches the same distance Δx_2 as in part (c). The board is then released. How does the new angular acceleration α' when the blocks are attached compare to the angular acceleration α_0 from part (c) ?

___ $\alpha' > \alpha_0$ ___ $\alpha' < \alpha_0$ ✓ $\alpha' = \alpha_0$

Justify your answer.

as the blocks are of equal mass and distance from the pivot, they would exert torques of equal magnitude in opposite directions and cancel eachother out, leaving the angular acceleration unchanged.

$F_{g_1} = F_{g_2}$ $x_1 = x_2$

Question 3

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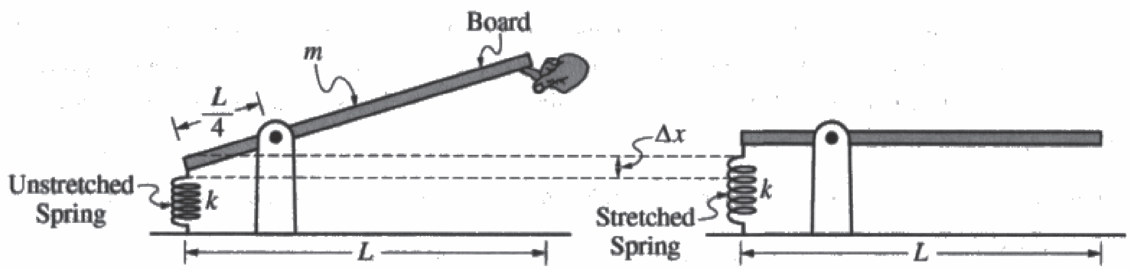


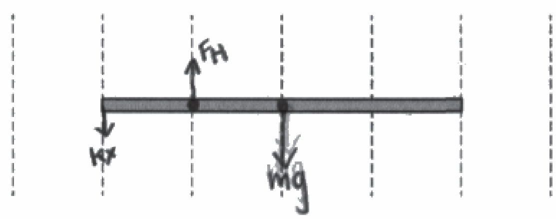
Figure 1

Figure 2

Note: Figures not drawn to scale.

3. A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The right end of the board is initially held by a student so that the spring is unstretched, as shown in Figure 1. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring stretched, as shown in Figure 2. The rotational inertia of the board about the pivot is I.

(a) On the rectangle below, which represents the board, draw and label the forces (not components) that act on the board while the board-spring system is in equilibrium. Each force should be represented by an arrow that starts on, and points away from, the board, and should represent the location at which that force acts.



F_H = Hinge force

Question 3

Continue your response to **QUESTION 3** on this page.

(b) Derive an expression for the distance the spring stretches, Δx, when the board is in equilibrium. Express your answer in terms of k, L, m, and physical constants, as appropriate.

$$U_1 + K_1^{\,0} = U_f + K_f$$ — conservation of energy

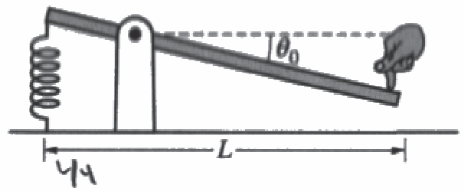
$$\frac{1}{2}k(\Delta x)^2 + mg(2\Delta x) = 0$$

$$\frac{1}{2}k(\Delta x)^2 = -mg(2\Delta x)$$

$$\boxed{\Delta x = \frac{-4mg}{k}}$$

Question 3

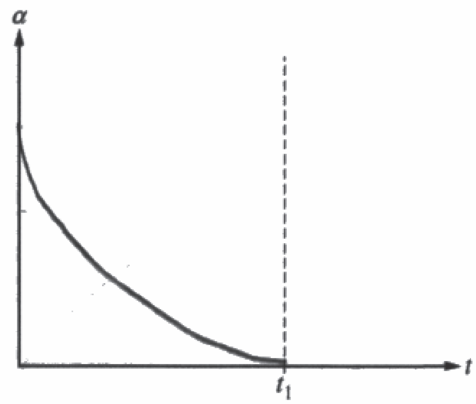
Continue your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

(c) A student pushes the board down on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest.

i. At time $t = 0$, the board is released. At $t = t_1$, the board first crosses the horizontal. Sketch a graph of the magnitude of the angular acceleration α of the board as a function of time t from $t = 0$ to $t = t_1$.



$a = \alpha r$

$\alpha = \frac{a}{r}$

The acceleration keeps decreasing.

Question 3

Continue your response to **QUESTION 3** on this page.

ii. Derive an expression for the angular acceleration α_0 of the board immediately after the board is released. Express your answer in terms of k, L, m, I, Δx_2, θ_0, and physical constants, as appropriate.

$$\Sigma\tau = I\alpha = \Sigma F \cdot r$$

$$(kx)\left(\frac{L}{4}\right) - F\sin\theta_0\left(\frac{3L}{4}\right) = I\alpha$$

$$k(\Delta x_2)\left(\frac{L}{4}\right) - F\sin\theta_0\left(\frac{3L}{4}\right) = I\alpha_0$$

$$\frac{k(\Delta x_2)\left(\frac{L}{4}\right) - F\sin\theta_0\left(\frac{3L}{4}\right)}{I} = \alpha_0$$

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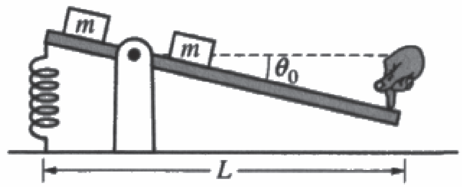
Question 3

Continue your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

(d) Two blocks of equal mass m are attached to the board equal distances from the pivot point, as shown. The board is again pushed down on the right side so that the spring stretches the same distance Δx_2 as in part (c). The board is then released. How does the new angular acceleration α' when the blocks are attached compare to the angular acceleration α_0 from part (c) ?

~~X~~ $\alpha' > \alpha_0$ ✓ $\alpha' < \alpha_0$ ___ $\alpha' = \alpha_0$

Justify your answer.

~~Since there is more mass the object will naturally have more acceleration~~

The thing will be harder to pivot because it has a greater inertia. More inertia the harder to spin.

$I = \Sigma mr^2$

GO ON TO THE NEXT PAGE.

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Draw a force diagram to represent forces exerted on a lever in equilibrium.
- Identify that for a system in rotational equilibrium, the net sum of torques is zero and that the torque exerted by a force applied at the pivot point is zero.
- Substitute appropriate expressions to represent gravitational and spring forces.
- Use multiple steps that follow a logical algebraic pathway to derive a symbolic expression for the displacement from equilibrium of a spring that applies a torque to a lever to balance the torques due to other forces.
- Identify that the angular acceleration of a rotating system is directly proportional to the sum of torques acting on it and inversely proportional to the rotational inertia of the system.
- Apply correct trigonometric substitutions and lever arms into torque expressions.
- Use multiple steps that follow a logical algebraic pathway to derive a symbolic expression for the angular acceleration of a lever oscillating due to opposing torques applied by the lever's weight and a spring force.
- Sketch a graph that shows a functional relationship between angular acceleration and time.
- Identify that the acceleration due to effects of a spring force is maximum at maximum spring displacement, minimum at spring equilibrium, and changes at a non-linear rate.
- Predict how angular acceleration changes when equal masses are added onto an accelerating lever at points equidistant but on opposite sides of the pivot point. Then, justify this prediction using rotational dynamics concepts of torque and rotational inertia.

Sample: 3A
Score: 15

Part (a) earned 3 points. The first point was earned because the spring force is drawn and labeled correctly. The label F_k is an acceptable label for spring force. The second point was earned because the force due to gravity is drawn and labeled correctly. The third point was earned because force exerted on the board by the pivot is drawn and labeled correctly. Normal force is an equivalent label for the force due to the pivot. Part (b) earned 3 points. The first point was earned because the response indicates that the sum of the torques equals zero. The second point was earned because the response includes a substitution of mg for the force of gravity and of $k\Delta x$ for the spring force into a torque equation. The third point was earned because the response includes a correct expression for Δx . Part (c)(i) earned 2 points. The first point was earned because the response includes a sketch that begins at a maximum value and continuously decreases to a minimum of zero. The second point was earned because the response includes a sketch that is concave down between $t = 0$ and $t = t_1$. Part (c)(ii) earned 4 points. The first point was earned because the response indicates that the torques are in opposite directions in the rotational form of Newton's second law. The second point was earned because the response includes a substitution of cos θ (or equivalent expression) in an expression for the net torque exerted on the rod. The third point was earned because the response includes a substitution of mg for the force due to gravity and $k\Delta x_2$ of for the spring force in an expression for the net torque exerted on the rod. The fourth point was earned because the response includes a substitution of the correct lever arm ($L/4$ on gravity and spring force terms) in an expression for the net torque exerted on the rod. Part (d) earned 3 points. The first point was earned because the response includes a correct selection of $\alpha' < \alpha_0$ decrease in angular acceleration with an attempt at a relevant justification. The justification focuses on rotation. The second point was earned because response indicates that the net torque does not change. The third point was earned because the response indicates that rotational inertia increases.

Question 3 (continued)

Sample: 3B
Score: 11

Part (a) earned 2 points. The first point was earned because the spring force is drawn and labeled correctly. The label F_k is an acceptable label for spring force. The second point was earned because the force due to gravity is drawn and labeled correctly. The third was not earned because the force exerted on the board by the pivot is not drawn and labeled correctly. Part (b) earned 3 points. The first point was earned because the response indicates that the sum of the torques equals zero. The second point was earned because the response includes a substitution of mg for the force of gravity and of $k\Delta x$ for the spring force into a torque equation. The third point was earned because the response includes a correct expression for Δx. Part (c)(i) earned 1 point. The first point was earned because the response includes a sketch that begins at a maximum value and continuously decreases to a minimum of zero. The second point was not earned because the response does not include a sketch that is concave down between $t = 0$ and $t = t_1$. The sketch is concave up. Part (c)(ii) earned 4 points. The first point was earned because the response indicates that the torques are in opposite directions in the rotational form of Newton's second law. The second point was earned because the response includes a substitution of cos θ (or equivalent expression) in an expression for the net torque exerted on the rod. The third point was earned because the response includes a substitution of mg for the force due to gravity and $k\Delta x_2$ for the spring force in an expression for the net torque exerted on the rod. The fourth point was earned because the response includes a substitution of the correct lever arm ($L/4$ on gravity and spring force terms) in an expression for the net torque exerted on the rod. Part (d) earned 1 point. The first point was not earned because the response does not include a correct selection of $\alpha' < \alpha_0$ with an attempt at a relevant justification. The incorrect option is selected. The second point was earned because the response indicates that the change in torque is the same in either direction. This point was scored regardless of an incorrect selection. The third point was not earned because the response does not indicate that inertia increases.

Question 3 (continued)

Sample: 3C
Score: 7

Part (a) earned 3 points. The first point was earned because the spring force is drawn and labeled correctly. It is acceptable to use a valid expression that represents the force, such as "kx," as the label for the force due to the spring. The second point was earned because the force due to gravity is drawn and labeled correctly. The third point was earned because the force exerted on the board by the pivot is drawn and labeled correctly. "F_H" for hinge force is an acceptable label for the force due to the pivot. Note that the response clearly defines the label used. Part (b) earned 0 points. The first point was not earned because the response does not indicate that the sum of the torques equals zero. A conservation of energy statement is not appropriate in this context. The second point was not earned because the response does not include a substitution of mg for the force of gravity and of $k\Delta x$ for the spring force into a torque equation. An energy expression is used instead of a force expression. The third point was not earned because the response does not include a correct expression for Δx. Part (c)(i) earned 1 point. The first point was earned because the response includes a sketch that begins at a maximum value and continuously decreases to a minimum of zero. The sketch is always decreasing. The second point was not earned because the response does not include a sketch that is concave down between $t = 0$ and $t = t_1$. The sketch is concave up. Part (c)(ii) earned 1 point. The first point was earned because the response indicates that the torques are in opposite directions in the rotational form of Newton's second law. The second point was not earned because the response does not include a substitution of cos θ (or equivalent expression) in an expression for the net torque exerted on the rod. The third point was not earned because the response does not include a substitution of mg for the force due to gravity and $k\Delta x_2$ for the spring force in an expression for the net torque exerted on the rod. The fourth point was earned because the response does not included a substitution of the correct lever arm ($L/4$ on gravity and spring force terms) in an expression for the net torque exerted on the rod. Part (d) earned 2 points. The first point was earned because the response includes a correct selection of $\alpha' < \alpha_0$ with an attempt at a relevant justification. The response clearly adjusts from an incorrect to a correct selection. The justification relates to rotation and resistance to motion. The second point was not earned because the response does not indicate that the net torque does not change OR that the change in clockwise torque is the same as the counterclockwise torque. The third point was earned because the response indicates that inertia increases.