Chief Reader Report on Student Responses:

2023 AP® Physics C: Mechanics Set 1

Free-Response Questions

• Number of Students Scored	55,602		
• Number of Readers	624 (for all Physics exams)		
• Score Distribution	Exam Score	N	%At
	5	14,703	26.44
	4	14,646	26.34
	3	11,501	20.68
	2	7,803	14.03
	1	6,949	12.50
• Global Mean	3.40		

The following comments on the 2023 free-response questions for AP® Physics C Mechanics were written by the Chief Reader Brian Utter, teaching professor and associate dean of general education at the University of California, Merced. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student preparation in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question 1

Task: Conservation Laws
Topic: Linear and Nonlinear Springs
Max Score: 15
Mean Score: 7.06

What were the responses to this question expected to demonstrate?

The responses were expected to demonstrate understanding of:

- Linear vs. nonlinear elastic forces using functions and F vs. x graphs.
- Work done on an object by both linear and nonlinear springs.
- Conservation of mechanical energy in various situations.
- Conservation of linear momentum.
- The use of calculus to determine the elastic potential energy for a nonlinear spring at maximum compression.

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Responses generally drew and labeled appropriate horizontal and vertical forces on a free-body diagram successfully, including the appropriate magnitude of the applied forces. All forces (not their components) exerted on the object need to be included on an FBD (unless otherwise stated), and the magnitude of these forces should be drawn to scale when directed to do so. Many students neglected to include the vertical forces (force of gravity and normal force) and/or did not draw the spring force for Spring Q twice as large as Spring P.
- Most responses identified that the work done on an object can be estimated by finding the area under a force vs. position graph and that the work done on an object by a spring will be the same if the object reaches the same maximum height on an incline, regardless of the spring used. Some responses incorrectly used $W = F \bullet x$, which does not apply. Furthermore, the work will be equal for the two different springs described in this question when the areas under the curves drawn in the F vs. x graph are the same.
- Responses typically sketched a correct graph of the maximum height reached by an object on an incline vs. the compression distance for both linear and nonlinear springs. Some students incorrectly indicated the graph was linear.
- Most responses applied conservation of mechanical energy to an object moving down an incline. Some mistakenly substituted the sum of the masses of the two blocks into an expression for conservation of energy when Block A is moving down the ramp before the collision.
- Responses generally applied conservation of linear momentum to a collision involving two objects.
- Correct responses applied conservation of mechanical energy to an object compressing a nonlinear spring and successfully identified that the elastic potential energy at the spring's maximum compression is the integral of the force function. Typical errors included not integrating the spring force equation to obtain the elastic potential energy of the compressed spring; instead, many responses incorrectly used the potential energy of a linear spring.
- Some responses incorrectly connected the total energy of the system (or the force exerted on the blocks by the spring) to the maximum compression distance of the springs, and the spring constant. Correct responses related, for two different nonlinear springs, either (1) the total energy of the system, the compression distance of the springs, and the spring constant, or (2) the force exerted on the object by the spring, the maximum compression distance of the springs, and the spring constant.
- Some responses incorrectly justified an answer by merely repeating the prompt statement.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Not drawing the force of gravity, the normal force, or not drawing the spring force for Spring Q twice as large as drawing the spring force for Spring P.	• Drawing the force of gravity, the normal force, and the spring force for Spring Q twice as large as drawing the spring force for Spring P.
• Indicating that $W = F \bullet x$, so the features needed from the graph to estimate the work done are the F and x values.	• The area under the curves in the graphs can be used to estimate the work done.
• The value of $x_0 = 0.04\text{ m}$, because at 0.04 m both springs exert the same force.	• The value of $x_0 > 0.04\text{ m}$. This is because the areas under the curves are not equal until a value of $x_0 > 0.04\text{ m}$.
• Not drawing x_{max} proportional to $x^{1/2}$ for Spring P. • Not drawing x_{max} proportional to $x^{3/2}$ for Spring Q. • Not having the two curves intersect or not intersecting within $0.05\text{ m} < x < 0.10\text{ m}$.	• Drawing x_{max} proportional to $x^{1/2}$ for Spring P. • Drawing x_{max} proportional to $x^{3/2}$ for Spring Q. • Having the two curves intersect at or near $x = 0.07\text{ m}$.
• For Block A moving down the ramp, substituting in the sum of the masses of the two blocks into an expression for conservation of energy: $m_A gH = \frac{1}{2}(m_A + m_B) + v^2$.	• For Block A moving down the ramp, substituting in the mass for Block A only into an expression for conservation of energy: $m_A gH = \frac{1}{2} m_A v^2$.
• For a nonlinear spring, using $U_S = \frac{1}{2}kx^2$ for the elastic potential energy of the compressed spring.	• For a nonlinear spring, using the integral of $F(x)dx$ from 0 to x_{max} to determine the potential energy U_S.
• Indicating that a smaller spring constant results in more compression or that the maximum compression of Spring R is greater than the maximum compression of Spring Q, so $C > D$.	• The potential energy of the two-blocks-spring system must be the same for both springs at maximum compression. So, because Spring Q is compressed less than Spring R, C must be greater than D.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

- Unless otherwise noted, free-body diagram responses should include all forces (but not components) exerted on the object. Make sure that the correct magnitudes are shown if asked, and a grid is provided.
- A justification should include more information than given in the prompt. Restating the prompt as the rationale for a selection is not enough justification to indicate understanding of the material.
- Note when it is appropriate to use $W = F \bullet x$, $U_S = \frac{1}{2}kx^2$, and $U_S = \int F dx$. These are not always interchangeable.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- Teachers should direct students to AP Daily videos from Unit 3: Work, Energy, and Power and Unit 2: Newton's Laws of Motion.
- Teachers should direct students to topic questions as well as personal progress check items to monitor progress being made in the mastery of content.
- Teachers can use the question bank to find items that assess similar content and skills and create practice assignments for students.

Question 2

Task: Experimental Design
Topic: Rotational Dynamics and Energy
Max Score: 15
Mean Score: 5.59

What were the responses to this question expected to demonstrate?

The responses were expected to demonstrate the ability to:

- Derive the period of oscillation for an object where the rotational inertia is due to multiple objects.
- Sketch a graph that shows the functional relationship between the rotational kinetic energy and the number of disks from a torsional pendulum given the equation for the potential energy stored.
- Draw a best-fit line for given data.
- Use a graph to calculate the slope of a linear fit and relate this linear fit to a derived equation.
- Determine and justify a source of error that explains the experimental error in a calculated value of mass.
- Describe the effects of modifying conditions by changing the mass distribution of the disk and justifying the change in slope of the best-fit line.
- Describe the effects of modifying conditions by changing the mass distribution of the disk and justifying how the angular velocity changes consistent with conservation of rotational energy.

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Most responses were able to indicate that the total rotational inertia of the stack of disks is the product of the rotational inertia of one of the disks and the number of stacked disks, and then state the expression for the period of N stacked disks.
- Many responses started the sketch of rotational kinetic energy at a nonzero value, but most responses did not show a constant rotational kinetic energy.
- Most responses were able to draw a best-fit line for the data, although the slope of this line varied significantly.
- Some responses calculated the slope of the line and related it to the derived equation, although many responses used a single data point from the best-fit line and used this to determine the mass of a single disk.
- Most responses properly identified a source of experimental error, although some of these did not justify the effect of the source of error on the results.
- Many responses properly justified how the slope of the best-fit line changes based on the mass distribution of the disk.
- Some responses were able to justify how the angular velocity changes based on the mass distribution of the disk; however, very few recognized that rotational energy was conserved.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Writing a derivation in one line rather than showing multiple steps. • Not stating the derived expression in terms of the variables given.	• $I_{eq} = NI$ • $T = 2\pi\sqrt{\left(N(0.5)MR^2 / \kappa\right)}$
• Not considering conservation of energy and the given potential energy equation. • Responses varied widely with most considering rotational kinetic energy.	• A line drawn indicating a constant energy starting at a nonzero value.
• Lack of understanding that the slope of a line is related to the derived equation for period. • Use of a single point, which sometimes did not fall on the best-fit line.	• A calculation shown for the slope of the line and explicitly relating the slope to the derived equation for period.
• Explaining a relationship that would result in a decrease in a variable, in this case, mass, without relating this quantity to the experiment.	• Explanations that include the measurement of a variable and how this relates to the given value being off. For instance, the period is measured to be too large, which results in the measured value of mass to be smaller.
• Not understanding that the density is greater at larger r from the given function; therefore, the nonuniform disk has more of the mass at larger radius. This means the nonuniform disk has a larger rotational inertia and not a larger mass.	• The nonuniform disk has more mass closer to the edge, resulting in a greater rotational inertia. Therefore, the slope of the best-fit line will be greater because the slope is proportional to the square root of rotational inertia.
• Not justifying the larger angular velocity for the nonuniform disk based on conservation of rotational energy.	• Energy is conserved. Therefore, because rotational kinetic energy is $K = \frac{1}{2}I\omega^2$, the angular velocity of the nonuniform disk is less given that its rotational inertia is larger.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

- Derivations should be shown step-by-step and not in one single line.
- Spend more time on linearization of equations and extracting slope or intercept based on the physics equations to solve for an unknown quantity.
- Focus on conceptual explanations and going beyond just using calculus to justify.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- Teachers should direct students to AP Daily videos from Unit 5: Rotation.
- Teachers should direct students to topic questions as well as personal progress check items to monitor progress being made in the mastery of content.
- Teachers can use the question bank to find items that assess similar content and skills and create practice assignments for students.

Question 3

Task: Theoretical Relationships and Mathematical Routines
Topic: Rotational dynamics and Energy
Max Score: 15
Mean Score: 3.20

What were the responses to this question expected to demonstrate?

The responses were expected to demonstrate the ability to:

- Derive a symbolic expression for the angular speed of the rotating rod after it swings down to the vertical position. This requires selecting either the law of conservation of mechanical energy or Newton's second law in rotational form as the relevant principle to describe the motion of the rod.
- Derive a symbolic expression for the linear speed of the sphere after the collision with the rotating rod. This requires selecting the law of conservation of angular momentum as the relevant principle for describing a rotational collision.
- Create a free-body diagram that appropriately depicts the relevant forces on the sphere. This requires identifying different types of forces, such as the normal force, gravitational force, and kinetic friction force.
- Derive a symbolic expression for the linear speed of the sphere as a function of time. This requires selecting Newton's second law as the relevant principle to describe the linear motion of the sphere between points A and B.
- Derive a symbolic expression for the angular speed of the sphere as a function of time. This requires selecting Newton's second law in rotational form as the relevant principle to describe the rotational motion of the sphere between points A and B.
- Derive a symbolic expression for the time it takes the sphere to move between points A and B. This requires selecting the rolling without sliding condition as the appropriate relationship between the linear and angular speeds of the sphere when it reaches Point B.
- Combine the expressions for the linear speed and for the time to derive a symbolic expression for the linear speed of the sphere at Point B.

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- A large majority of responses chose the relevant principle in part (a) by attempting to apply conservation of mechanical energy. Subsequently, quite a few incorrectly substituted ℓ for the change in height of the center of mass of the rod instead of $\ell/2$, with fewer than 20 percent of all responses successfully completing the derivation of the correct angular speed. About 10 percent of responses attempted to apply Newton's second law in rotational form, which is a difficult approach because the torque on the rod changes as it swings down, and separation of variables plus integration is necessary to arrive at the correct answer. A very small number of responses successfully took this approach. Most responses followed a logical algebraic pathway.
- Many responses incorrectly attempted to apply conservation of linear momentum in part (b), incorrectly set the angular momentum of the rod about its pivot equal to the rotational angular momentum of the sphere (even though it is not rotating immediately after the collision), or failed to assert the condition for rolling without sliding is applicable at point B. This suggests that students had trouble selecting the appropriate model to describe the physical situation.
- The responses generally showed mastery at creating an appropriate free-body diagram to represent the physical situation. Many responses also correctly showed the points of application of the forces, though it was often hard to discern for the vertical forces.
- The majority of responses appropriately applied Newton's second law to the horizontal motion of the sphere. A few responses attempted to apply conservation of mechanical energy to every part of the problem, and quite a few did not write Newton's second law in rotational form where it was necessary. Even fewer correctly applied the condition for rolling without slipping. The responses seemed to reflect the level of understanding of Newton's second law, the second law in rotational form, and the rolling without sliding condition.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Students attempted to find an expression for the angular speed of the rod using Newton's second law in rotational form but did not recognize the torque on the swinging rod is not constant as it swings down.	• The torque on the rod as it swings down is $\frac{\ell}{2}Mgcos\theta$, where θ is the angle with the horizontal and changes from zero to 90°. • Integrating the torque over θ and $I\omega d\omega$ over ω leads to a correct expression of the final angular speed of the rod. An integral must be used because the torque is not constant.
• Reponses often lacked a valid expression for the conservation of angular momentum. Instead, they incorrectly connected the angular speed of the rod and the linear speed of the sphere $(v = \ell\omega)$, or the set the linear momentum of the rod equal to the linear momentum of the sphere after the collision $(Mv_f = mv_0)$, or set the angular momentum of the rod about the pivot equal to the angular momentum of the sphere spinning about its center of mass, $\left(\frac{1}{3}M\ell^2\right)\omega_f = \left(\frac{2}{5}mR^2\right)\omega_{ball}$, even though it is not spinning.	• Only angular momentum is conserved during the collision, and it must be calculated before and after the collision with respect to the pivot: $$\left(\frac{1}{3}M\ell^2\right)\omega_f = \left(m\ell^2\right)\frac{v_0}{\ell} = mv_0\ell$$
• Responses often did not describe the rotational acceleration of the ball in terms of the torque caused by friction.	• Apply Newton's second law in rotational form, recognizing the only torque on the sphere between points A and B is that caused by friction applied to the bottom of the sphere, a distance R from its center of mass: $$\tau = R(\mu mg) = I\alpha = \left(\frac{2}{5}MR^2\right)\alpha$$
• Students did not recognize the need to apply the rolling without slipping condition at point B to find the time it takes the sphere to travel between Point A and Point B. A kinematic equation cannot be used to find the time because neither the distance nor the final velocity is known.	• Begin with the rolling without slipping condition at point B and solve for time: $v_B = R\omega_B$ $v_0 - \mu g t_B = R\omega_B = R\left(\frac{5}{2}\frac{\mu g}{R}t_B\right)$ $v_0 = \frac{7}{2}\mu g t_B \Rightarrow t_B = \frac{2}{7}\frac{v_0}{\mu g}$

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

One common characteristic of a low-scoring response is an attempt to write seemingly relevant expressions that do not relate to the physical situation. For instance, incorrectly setting the angular momentum of the rod about the pivot point equal to the angular momentum of the sphere about its center of mass or attempting to apply the equation for angular frequency of a physical pendulum (or even a simple pendulum), not realizing the angular frequency ω is not the same as the angular speed ω. This sort of mistake might be lessened if we repeatedly emphasize in class that the algebraic expressions we write carry meaning and that writing an algebraic expression is equivalent to writing a description in words. Some teachers introduce new algebraic expressions in class by first writing them out in words, so they are more conceptually relevant, and students are less likely to write nonsensical algebraic expressions.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- Teachers should direct students to AP Daily videos from Unit 4: Systems of Particles and Linear Momentum, and Unit 5: Rotation.
- Teachers should direct students to topic questions as well as personal progress check items to monitor progress being made in the mastery of content.
- Teachers can use the question bank to find items that assess similar content and skills and create practice assignments for students.