Chief Reader Report on Student Responses:

2023 AP® Physics C: Mechanics Set 2

Free-Response Questions

• Number of Students Scored	55,602		
• Number of Readers	624 (for all Physics exams)		
• Score Distribution	Exam Score	N	%At
	5	14,703	26.44
	4	14,646	26.34
	3	11,501	20.68
	2	7,803	14.03
	1	6,949	12.50
• Global Mean	3.40		

The following comments on the 2023 free-response questions for AP® Physics C: Mechanics were written by the Chief Reader, Brian Utter, teaching professor and associate dean of general education at the University of California, Merced. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student preparation in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question 1

Task: Theoretical Relationships and Mathematical Routines
Topic: Momentum and Impulse
Max Score: 15
Mean Score: 8.72

What were the responses to this question expected to demonstrate?

The responses were expected to demonstrate the ability to:

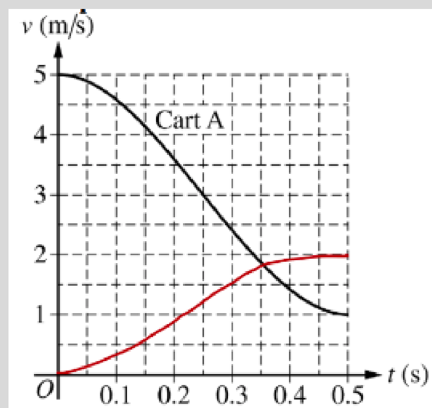
- Identify properties of a graph related to kinematics.
- Use conservation of momentum to solve for the post-collision speed of an object.
- Graph the change in speed versus time of an object during a collision.
- Use calculus to determine the time at which the maximum acceleration occurs during a collision and calculate the maximum force during the collision.
- Sketch the change in force versus time applied to an object during a collision.
- Relate change in momentum and impulse and address the functional dependence between force applied to an object and time.

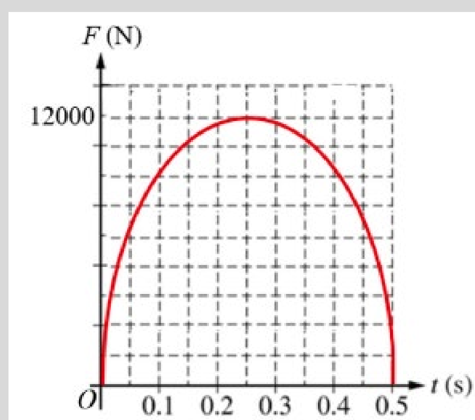
How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Most responses clearly identified the area under a velocity-time graph is the displacement.
- Many responses clearly showed understanding of an increasing speed for Cart B by the sketch of the graph.
- Most responses correctly identified the acceleration function as the derivative of the velocity function and used the function, in some way, to identify when the maximum acceleration occurred.
- Most responses clearly sketched an increasing and then decreasing force over the given time interval and reaching the maximum value calculated in part (b)(i).
- Many responses correctly selected $F_1 < F_2$ and included a response stating a decrease in the time interval requires an increase in the maximum force, although some failed to indicate that the change of momentum was the same for both situations.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Responses not clearly indicating understanding of graphical versus mathematical terms: o using ambiguous terminology, such as "area of the graph," "integral of the curve," and "area/integral of the line." o attempting to use kinematics equations with points from the graph, even though the object is clearly not uniformly accelerating. o using an incorrect relationship between velocity and acceleration (e.g., slope instead of area).	• Responses that indicate "area between the curve and the *x*-axis" or "area under the curve" equals displacement were the clearest identification of a feature of the graph. The integral of the v vs. t curve was accepted as mathematically equivalent but is not strictly a feature of the graph.

○ performing calculations (such as Riemann sum) to determine the displacement rather than identifying what the Riemann sum represents (area).	
• Assuming the collision was perfectly elastic and using conservation of energy to determine the speed of Cart B after the collision. • Assuming the collision was perfectly inelastic and using incorrect mass in a conservation of momentum equation. • Attempting a kinematics approach. • Identifying the final velocity of both carts A and B as being the same after the collision. • Incorrectly assuming Cart A came to rest after the collision, even though the graph clearly shows otherwise.	• Clearly calculating the speed of Cart B after the collision using momentum conservation. • Reasoning based on momentum conservation: The two carts have the same change in momentum magnitude. Cart A's speed decreases by 4 m / s. Because Cart B's mass is twice as much, its speed will increase by $4\text{ m / s} \div 2$. Therefore, Cart B's final speed will be 2 m / s.
• Assuming speed for Cart B undergoes the same changes as Cart A and traces the given line. • Incorrect curvature—simply drawing a straight line between the initial and final speeds, reversing concavity, no inflection point. • Final speed inconsistent with answer determined in part (a)(ii). • Unclear concavity, such as including slight curves at beginning and end with a straight line between.	• 
• Incorrect math relationship between velocity and acceleration, e.g., taking integral of velocity function rather than derivative. • Although prompt states "calculate," responses are not including processes in the solution, e.g., writing $a = 192t^2 - 96t$ and not clearly identifying that acceleration a is the derivative of velocity. • Setting acceleration function (or force function) equal to zero rather than derivative of acceleration (or force function) to determine time of maximum acceleration or maximum acceleration. • Not clearly indicating application of calculus to physics concepts. Responses state maximum force or	• Responses equate acceleration as the derivative of velocity. • The derivative of the acceleration function is set to zero to find the maximum acceleration. Force is given by mass time acceleration. Calculation steps are shown. • Alternatively, responses could show a calculation that determines momentum $\vec{p}$, force as $\vec{F} = d\vec{p} / dt$ and then determine the maximum force. • Responses could indicate that $a(t)$ was graphed and the largest value was found to occur at $t = 0.25\text{ s}$.

acceleration occurs at $t = 0.25\text{ s}$ or maximum acceleration is $a = 12\text{ m / s}^2$ with no corresponding work. • Most responses did not include clear substitution of values into $a = \frac{\sum F}{m}$ (e.g., $F = ma = 12000\text{ N}$ with no intermediate step). • Using an energy or kinematics approach. • Using endpoints instead of critical points (e.g., $a(0) = 0$ and $a(0.5) = 0$ for maximum acceleration). • Stating $a(0.25)$ or $F(0.25)$ with no substitutions into derived or other equation. • Many students used calculators to determine values; however, they did not clearly indicate the process used on the calculator in order to demonstrate the physics content. • Confusing minimums of a graph or function, although prompt asked for magnitude of maximum value. • Using derived function or function labeled as acceleration and calculating values at various time intervals and assuming that the largest value calculated was the maximum value, rather than using underlying physics principles. • Listing multiple time and/or acceleration values but not clearly indicating which was considered the maximum value. • Switching between using momentum and acceleration approaches.	
Errors on the graph included: • Incorrect concavity. • Graphs that do not start and end at $F = 0$. • Sketching a graph that is negative, even with origin provided and part (b)(i) prompt states "magnitude." • Sketched graphs lack symmetry. • Sketched graphs do not include a maximum value for a single point in time (x-value) and instead include a maximum value across the extended time interval.	• 

• Failure to clearly indicate maximum force value calculated in part (b)(i) on the *y*-axis.	• If student is unsure whether curvature on the drawing is clear, they state additional clarifications, such as “concave down throughout” or “curved not linear.”
• Checking box that does not match justification in part (c). • Stating average force is equal to (net) force divided by time. • Not clearly indicating that the impulse or change in momentum of Cart B is the same for both collisions. • Ambiguous phrasing such as “the change is constant.” • Justification using only equations and symbols with no text to explain what is meant by the symbols. • Not clearly indicating that two separate events are being compared rather than before and after a single event (e.g., “the speeds before and after are the same” versus “the speeds before and after each collision are the same”). • Using “conserved,” “the same,” “constant,” and “equal” interchangeably. • Including contradicting statements (e.g., “forces are the same” followed by “F_1 is greater”). • Evaluating internal forces via Newton’s third law rather than Newton’s second for a single cart.	• Indicating that $F_1 < F_2$; because the change in speeds for Cart B is the same for both with and without the foam, the change in momentum will be the same. • Responses include a justification that the impulse on Cart B will be the same for both collisions and because force and time are inversely proportional to each other, a shorter time duration means a larger force must be applied.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

- Remind students of the Task Verbs. For instance, “calculate” requires that all steps in their process be shown.
    - In your classroom, do not allow students to only include a final answer; require they show all their work.
- If students include work in a location other than the space dedicated to that prompt, be sure they clearly indicate where to find the work.
- If students have multiple approaches, they should circle the work that is intended to be scored or cross out the work that is not to be read.
- Students who include a scale on a graph, whether required or not, must be careful with labels regarding orders of magnitude.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- Teachers should direct students to AP Daily videos from Unit 1: Kinematics—Motion in One Dimension and Unit 2: Newton’s Laws of Motion.
- Teachers should direct students to topic questions as well as personal progress check items to monitor progress being made in the mastery of content.
- Teachers can use the question bank to find items that assess similar content and skills and create practice assignments for students.

Question 2

Task: Experimental Design
Topic: Harmonic Oscillation
Max Score: 15
Mean Score: 8.05

What were the responses to this question expected to demonstrate?

The responses were expected to demonstrate the ability to:

- Determine the effective spring constant of springs in parallel.
- Derive a symbolic expression that models the experiment by relating period of the oscillator to the number of springs using the effective spring constant.
- Sketch the curve associated with a relationship.
- Draw a best-fit line when given a set of plotted data points.
- Calculate the slope of the best-fit line.
- Relate the slope of a graph to the given equation.
- Use the slope of experimental data to find a physical value.
- Explain how experimental error could lead to an incorrect conclusion.
- Describe the independence between the period of a simple harmonic oscillator and gravitational force.
- Use the total mechanical energy to determine the effect of changing the effective spring constant on the motion of the oscillator.

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Most recognized that changing the number of springs changed the effective spring constant and, in turn, changed the period of the oscillator.
- Many correctly applied the linear relationship between the number of identical springs and the effective spring constant.
- Many applied the effective spring constant to period of a harmonic oscillator and made the substitution to derive an expression for period.
- Many correctly sketched the curve that represented their model.
- Most drew an appropriate line of best fit to estimate the trend of the data provided, and most responses showed an understanding that a trendline should not simply connect the dots.
- Many were able to find the slope of their line of best fit. However, there were a significant number of responses that chose to use a point on the line of best fit instead of the slope. In this specific case, if their trendline extended through the point $(0,0)$, it was equivalent. Responses that did not earn full points typically chose data points that were not on the line of best fit.
- Many were correctly able to relate the slope of the graph to their model, but some responses struggled to correctly substitute the values represented on the graph into their algebraic expression.
- Some were correctly able to relate experimental error to an accurate result; however, many responses showed that students expected a dependent relationship between maximum displacement and period.
- Many understood that the gravitational force did not impact the period of a mass-spring pendulum, and so changing the orientation of the oscillator did not change the period.
- Many were able to use conservation of energy to relate the effective spring constant and the maximum velocity of an oscillator.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Some responses did not correctly relate the number of springs to the effective spring constant for identical springs added in parallel. Most of the responses that did not receive credit used $k_{eq} = \frac{k}{N}$, $k_{eq} = N$ or $k_{eq} = \frac{k}{N^2}$. • When asked to use a graph after drawing a line of best fit, many responses failed to identify the slope as the best representation of the data. In many cases, responses used a single point to represent the data. In addition, they sometimes used a data point that did not fall on the line of best fit.	• For N identical springs all with a spring constant of k, the equivalent spring constant is the sum of all the spring constants such that $k_{eq} = Nk$. • Using the slope provides the best estimate given the data set. For this data, given two points on the line with the form (a, b): $$\text{slope} = \frac{T_b^2 - T_a^2}{N_b^{-1} - N_a^{-1}} \approx 11.8\ \text{s}^2$$ • Students whose line of best fit passed through the point $(0,0)$ who used a single point in place of the slope still earned credit because the two were equivalent in this case.
• Responses that used a point as a feature of the graph sometimes were confused with what those points meant, choosing to substitute a y-value for T instead of T^2 or an x-value for N instead of $\frac{1}{N}$.	• By squaring the relationship $T = 2\pi\sqrt{\frac{m}{Nk}}$, the points from the graph can be directly substituted, given $T^2 = 4\pi^2\left(\frac{m}{k}\right)N^{-1}$, to calculate k. • Units should be provided for calculated quantities. The spring constant of a single spring has units of $\frac{N}{m}$.
• Some responses indicated incorrectly that changing the displacement of a mass-spring system would affect the period of the oscillator. • Some responses did not recognize the relationship between period and spring constant or between mass and spring constant in order to determine what error might artificially lower the experimental spring constant.	• Mass is directly related to the spring constant, so underestimating the mass in the experiment will result in an underestimation of the spring constant as well. • A calibration error with the balance may result in representing the mass as heavier than it is. Using the measured period of a lighter mass will result in calculating a smaller spring constant than the manufacturer claims.
• Many responses incorrectly indicated that the period was affected by the gravitational force when the orientation was changed from vertical to frictionless horizontal.	• The period of a mass-spring oscillator depends only on the mass and spring constant of the oscillator. Because neither mass nor the spring constant have changed, the period of oscillation is the same if the mass oscillates horizontally on a frictionless surface.

• Responses were unable to quantify how increasing the number of springs affected the speed of the block. • Many responses provided an incomplete justification for an increasing v_{max} with an increase in N, for example, successfully connecting an increase k_{eq} to an increase in spring force but failing to include acceleration as part of the relationship. • Responses plugged values and stated the outcome but did not adequately justify the path from N to v_{max}.	• Increasing N increases the equivalent spring constant, and because $F_s = k\Delta x$ and the displacement is constant, increases the spring force as well. Using Newton's second law, $\Sigma F = ma$, increasing the force with a constant mass increases the acceleration. An increase in acceleration leads to an increased velocity. • Another acceptable response: Based on conservation of energy, increasing the spring constant increases the elastic potential energy. $U_{s(max)} = K_{max}$ $\frac{1}{2}k_{eq}A^2 = \frac{1}{2}mv_{max}^2$ $v_{max} = \sqrt{\frac{k_{eq}A^2}{m}}$ Thus, at equilibrium, the kinetic energy increases, and because mass is constant, the maximum velocity increases. • Another acceptable response: The spring constant is directly related to the angular speed, such that $\omega = \sqrt{\frac{k_{eq}}{m}} = \frac{v}{r} = \frac{v_{max}}{A}$ If m and r are constant for an oscillator, then increasing k will increase v_{max}.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

- Practice using the equation sheet and prior knowledge to derive relationships between variables. Students must clearly show their logical progression in a derivation without skipping steps. Prompts that use derive and calculate require multiple steps that provide evidence of the initial physics principles and the physics reasoning that led to the final relationship to earn full credit.
- Provide opportunities for students to graph and linearize different relationships. Students should recognize the general shape of linear, inverse, parabolic, inverse-square, and inverse-square root relationships including concavity and trend.
- Use lab data collection as an opportunity for students to collect and graph different relationships. Allowing students to analyze data and look for trends before algebraically deriving the answers will connect the relationships with graphs and physical behavior.
- Practice justification and reasoning skills in the classroom. Using the Claim-Evidence-Response model will allow the teacher to demonstrate what a valid and adequate justification looks like.
- Use ranking tasks and student discussion to provide practice to concisely and clearly justify understanding to classmates using solid and appropriate reasoning.
- Practice drawing lines of best fit and using the slope to gather information. Remind students that lines of best fit should be drawn with a straight edge and do not necessarily go through the origin.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- Teachers should direct students to AP Daily videos from Unit 6: Oscillations.
- Teachers should direct students to topic questions as well as personal progress check items to monitor progress being made in the mastery of content.
- Teachers can use the question bank to find items that assess similar content and skills and create practice assignments for students.

Question 3

Task: Theoretical Relationships and Mathematical Routines
Topic: Rotation
Max Score: 15
Mean Score: 5.79

What were the responses to this question expected to demonstrate?

The responses were expected to demonstrate the ability to:

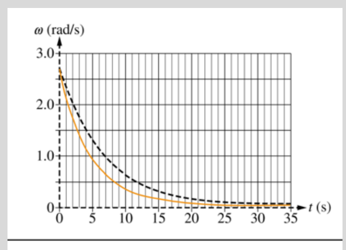
- Identify and use the parallel axis theorem to find one blade's rotational inertia for a point parallel to the center of mass.
- Determine the total rotational inertia of a three-blade system, using logical algebraic pathways.
- Recognize that time for one revolution is period and select relevant given values to calculate an unknown quantity with units.
- Relate amount of dissipated energy to the total initial rotational kinetic energy, select relevant given values, and calculate the unknown quantity with units.
- Derive an expression for torque using the appropriate equation and a derivative of the angular speed decay equation.
- Use integral calculus to derive angular displacement from the known exponential decay expression for angular speed.
- Sketch a graph of a new three-blade system's exponential decay where a variable was changed.

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Many responses recognized a need to utilize the parallel axis theorem and made appropriate substitutions of the rotational inertia of one blade about its center of mass and the distance from the center of mass.
- Most responses attempted to take a sum of the rotational inertia values of three blades but incorrectly just multiplied the single-blade's rotational center of mass equation by three.
- Most responses correctly calculated the time for one rotation; a small number did not extract relevant quantities from the narrative.
- Most responses indicated that initial rotational kinetic energy was the energy dissipated, and some correctly calculated the dissipated energy with correct units. A small number of students did not recognize that they had been given the final and initial angular speed values and tried to use the graph to extract information.
- Many responses attempted to use Newton's second law in rotational form, but some did not recognize the need for a derivative to correctly determine angular acceleration. Some students used rotational kinematic equations to find the angular acceleration.
- Some responses recognized the general calculus relationship for rotational kinematics but made little or no attempt to integrate the exponential decay equation that was given or incorrectly attempted to use constant rotational kinematic formulas.
- Most responses recognized to draw a graph of similar exponential decay curve.
- Most responses recognized the graphs would start at the same angular speed, but only some of the responses were drawn correctly below the original curve. Some of the responses showed an understanding that the change in decay constant should move the new graph below the original curve.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Responses used an incorrect equation for parallel axis theorem, for example, missed that d is squared.	• $I_{\text{blade}} = I_{\text{cm}} + Md^2$ or $I_1 = I_{\text{cm}} + Mh^2$ • $I_{\text{blade}} = \frac{1}{18}ML^2 + M\left(\frac{L}{3}\right)^2$ • $I_{\text{blade}} = \frac{1}{18}ML^2 + \frac{1}{9}ML^2$
• Responses multiplied I_{cm} by 3 instead of the I for the blade. • Responses didn't multiply all parts of the parallel axis theorem by 3.	• $I_{\text{total}} = 3I_{\text{blade}}$ • $I_{\text{total}} = I_1 + I_2 + I_3$ • $I = \frac{3}{18}ML^2 + \frac{3}{9}ML^2$
• Responses did not demonstrate understanding of how to find time for one rotation. • Some were distracted by the length value and attempted to find an equation mixing up length with angular momentum L.	• $T = \frac{2\pi}{\omega_o}$ and $T = \frac{2\pi}{2.6\,\frac{\text{rad}}{\text{s}}}$ • $v = \frac{d}{t}$ turns into $t = \frac{2\pi L}{L\omega_o}$
• Responses indicate that some did not realize they had the values for I_{total} and ω_o needed to calculate the initial rotational kinetic energy or did not identify that they were told "the system comes to rest." • Responses incorrectly attempted to use the graph area as a tool to find energy or attempted to solve for β using the graph, which was unnecessary.	• $E_{\text{diss}} = K_o$ $K_{\text{f}} = 0\text{ J}$ $K_o = \frac{1}{2}I\omega_o^2$ • $E = K_{\text{rotational}}$ $Kr = \frac{I\omega^2}{2}$ • Responses picked and used values from the graph for ω_o in a visually reasonable range.
• Responses attempted to use the torque cross product equation, or just didn't apply the appropriate law. • Inability to differentiate an exponential decay equation. • Responses incorrectly used constant rotational acceleration equations for ω.	• $\Sigma\tau = I\alpha$ $\tau = I \cdot \omega'(t)$ • $\alpha = \frac{d\omega}{dt}$ $\alpha = \omega'(t)$ $\alpha = -\beta\omega_o e^{-\beta t}$ • $\tau = I_{\text{total}}\left(-\beta\omega_o e^{-\beta t}\right)$ $\tau = -\frac{1}{2}ML^2\beta_o\omega_o e^{-\beta_o t}$

• Responses incorrectly used constant acceleration rotational kinematic equations for θ. • Responses that do not demonstrate knowledge about how to integrate the given ω decay equation for $\Delta\theta$. • Missing that when integrating the decay equation to get $\Delta\theta$, evaluating at the limit $t = 0$ seconds yields a value of $\frac{\omega_o}{\beta_o}$.	• $\Delta\theta = \int \omega dt$ • $\Delta\theta = \int_0^t \omega dt \qquad \Delta\theta = \left(\frac{\omega_o}{-\beta} e^{-\beta t}\right)_0^t$ • $\Delta\theta = \frac{\omega_o}{-\beta_o}\left(-1 + e^{-\beta_o t}\right)$ • $\Delta\theta = -\frac{\omega_o}{\beta_o} e^{-\beta_o t} + C \qquad C = \frac{\omega_o}{\beta_o}$ • $\Delta\theta = -\frac{\omega_o}{\beta_o}\left(\frac{1}{e^{\beta_o t}} - 1\right)$
• Responses missed that both situations have the same given ω_o, so the new curve has the same starting point. • Responses drew a curve showing slower decay instead of faster decay.	• Curve started in the same position and appeared qualitatively similar to the original curve but falling off faster. 

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

- Prompts that use "derive" expect evidence of where the final product originates: practice starting from fundamental physics principles or an equation from the equation sheet and provide steps for physics reasoning that lead to the final relationship to earn full credit. As a general strategy, suggest the following process:
    - Indicate a starting law or equation from the equation sheet.
    - Substitute variables and constants from the problem explicitly.
    - Algebraically manipulate to arrive at the final expression.
- Prompts that use "calculate" similarly expect work where the final value originates and the physics reasoning that led to the final relationship to earn full credit. Again, practice starting from fundamental physics principles or an equation from the equation sheet and provide steps for physics reasoning that lead to the final calculated quantity to earn full credit.
- Provide students practice work scoring old AP student responses.
- Provide practice for students for rotational motion situations, such as nonconstant acceleration, where a calculus-based approach is required.
- Practice derivatives of many functions, including exponential growth functions.
- Remind students of the importance of showing limits of integration on definite integrals.
- Provide practice with multi-section questions where values given in earlier sections are needed to solve later sections.
- Provide practice finding rotational inertia of multi-object systems by summing inertias of individual objects.

- Provide opportunities for students to graph relationships. Have students practice things like drawing a new graph shape if you change an existing variable.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- Teachers should direct students to AP Daily videos from Unit 5: Rotation.
- Teachers should direct students to topic questions as well as personal progress check items to monitor progress being made in the mastery of content.
- Teachers can use the question bank to find items that assess similar content and skills and create practice assignments for students.