2023

AP® Physics C: Mechanics

Free-Response Questions
Set 2

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1\ \text{u} = 1.66 \times 10^{-27}\ \text{kg} = 931\ \text{MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25}\ \text{J}\cdot\text{m} = 1.24 \times 10^3\ \text{eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^9\ (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1\ \text{atm} = 1.0 \times 10^5\ \text{N/m}^2 = 1.0 \times 10^5\ \text{Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$\left|\vec{F}_f\right| \le \mu \left|\vec{F}_N\right|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{\max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi \sqrt{\frac{m}{k}}$

$T_p = 2\pi \sqrt{\frac{\ell}{g}}$

$\left|\vec{F}_G\right| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0} \left|\frac{q_1 q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = N e v_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L \frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

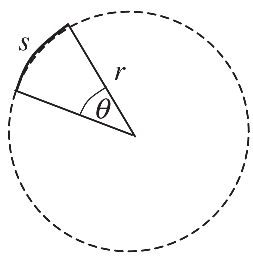
$a^2 + b^2 = c^2$

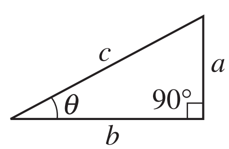
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$\vec{A}\bullet\vec{B} = AB\cos\theta$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

Begin your response to **QUESTION 1** on this page.

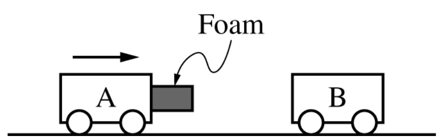
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.

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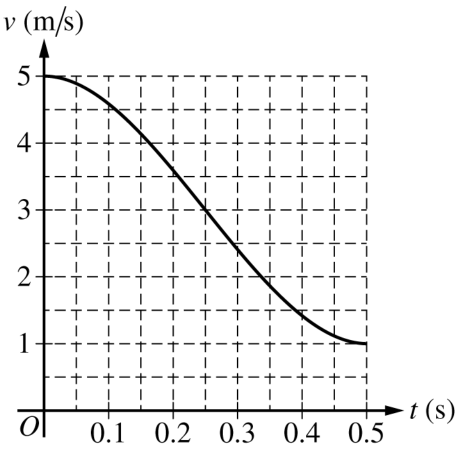
Continue your response to **QUESTION 1** on this page.

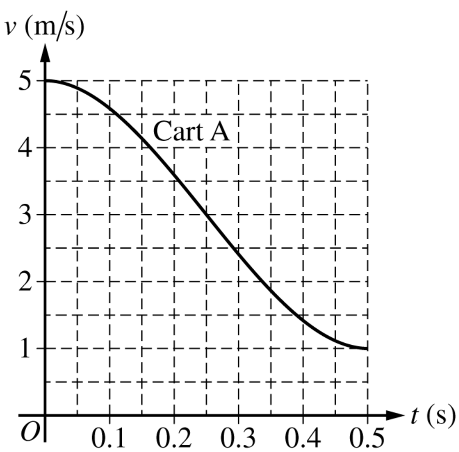


(a) The graph shows the velocity v of Cart A as a function of time t for the time interval when the foam and Cart B are in contact.

i. What feature(s) of the graph could be used to estimate the displacement of Cart A during the collision?

ii. Using the information shown in the graph, determine the speed of Cart B at $t = 0.5$ s.

iii. On the following grid, draw a smooth curve of the velocity of Cart B as a function of time.



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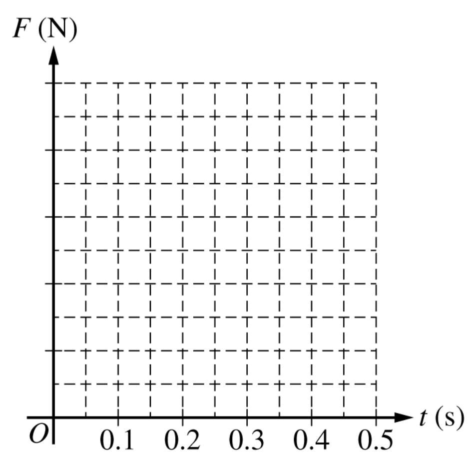
Continue your response to **QUESTION 1** on this page.

(b) For $0 \le t \le 0.50$ s, the velocity v of Cart A can be described by the function $v(t) = 64t^3 - 48t^2 + 5$.

i. Calculate the magnitude of the maximum net force acting on Cart A during this interval.

ii. On the following grid, draw a smooth curve of the magnitude of the force acting on Cart A as a function of time. Clearly indicate the value of the maximum force on the vertical axis.



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Continue your response to **QUESTION 1** on this page.

The foam is removed from the front of Cart A and the experiment is repeated. The carts collide, with both Cart A and Cart B having the same initial and final velocities as in the original collision. The time intervals during which the carts are in contact are different in the collision with the foam and the collision without the foam. In the collision without the foam, Cart A is in contact with Cart B for a shorter duration than in the original collision, when the foam was present.

For the original collision when the foam is present, the magnitude of the average net force exerted on Cart B is F_1. For the collision without the foam, the magnitude of the average net force exerted on Cart B is F_2.

(c) What is the relationship between the magnitude of the average net force F_1 exerted on Cart B for the collision with the foam and the magnitude of the average net force F_2 exerted on Cart B for the collision without the foam?

_____ $F_1 > F_2$ _____ $F_1 < F_2$ _____ $F_1 = F_2$

Justify your answer.

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Begin your response to **QUESTION 2** on this page.

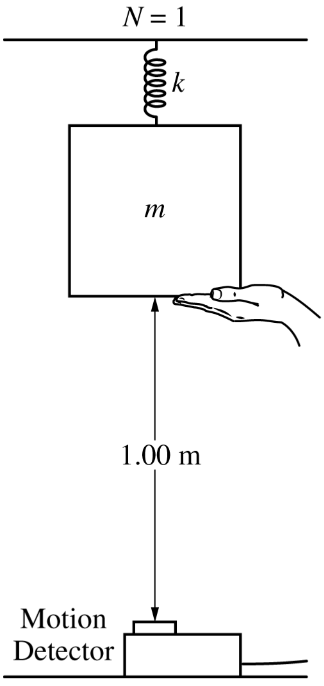

Figure 1

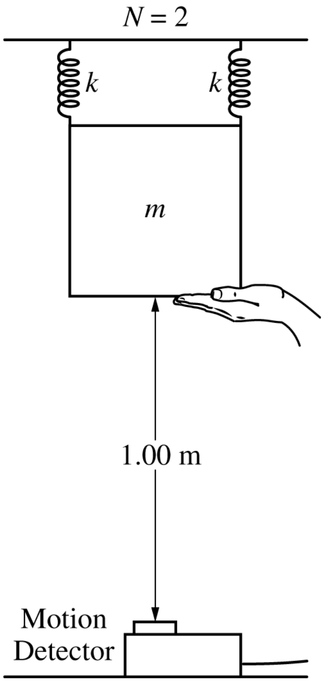

Figure 2

2. A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height 1.00 m above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

(a) Derive an expression for T as a function of N. Express your answer in terms of m, k, N, and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, sketch a graph of T as a function of N for $N \geq 1$.

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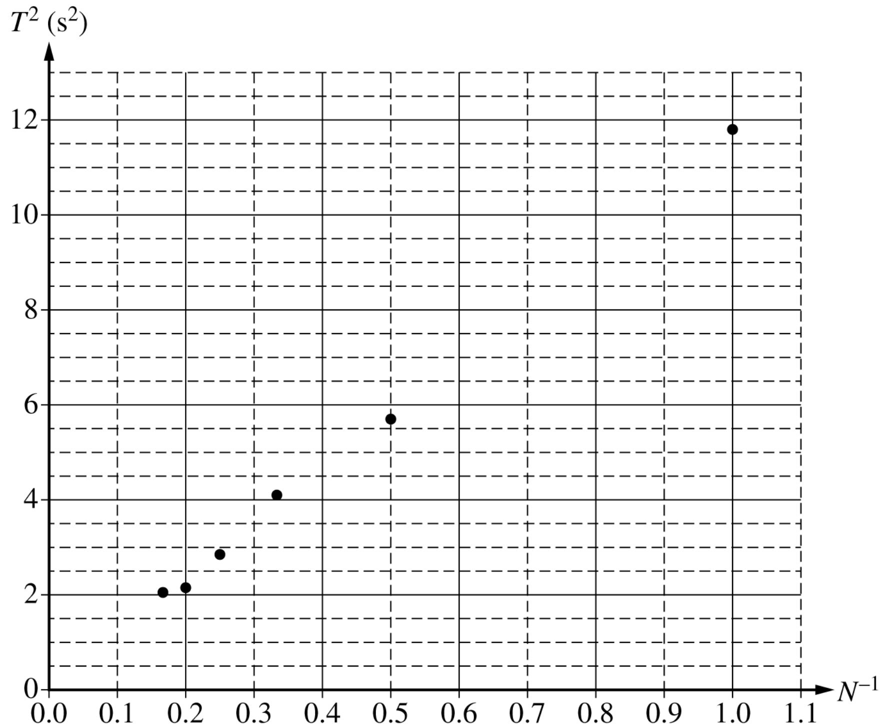
Continue your response to **QUESTION 2** on this page.

(c) The student plots the data for T^2 as a function of N^{-1}, as shown.



i. Draw the best-fit line for the data.

ii. The mass of the block is measured to be $m = 1.5$ kg. Using the graph, calculate an experimental value for the spring constant k for a single spring.

iii. The student finds that the value given by the manufacturer for the spring constant is larger than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value for k. Briefly justify your answer.

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$N = 2$

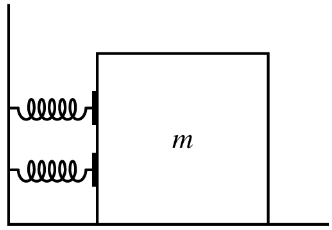

Figure 3

(d) The student conducts a similar investigation to determine the relationship between the period of oscillation T of a system consisting of a block and a number N of identical springs but arranges the block-spring system horizontally on a table, as shown in Figure 3. Frictional forces between the table and the block are negligible. In each trial, the block is displaced the same horizontal distance from equilibrium and released from rest.

i. The student plots T^2 as a function of N^{-1} for this new data. Would the slope of the best-fit line from this new investigation be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than   ___ less than   ___the same as

Briefly justify your answer.

ii. When $N = 1$, the maximum speed of the block is found to be v_{max}. When N increases, will v_{max} increase, decrease, or stay the same?

____ increase   ____ decrease   ____ stay the same

Justify your answer.

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Begin your response to **QUESTION 3** on this page.

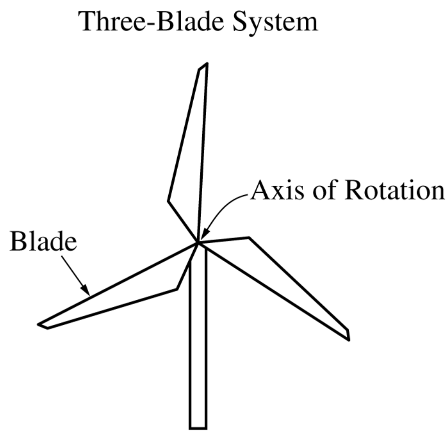

Figure 1

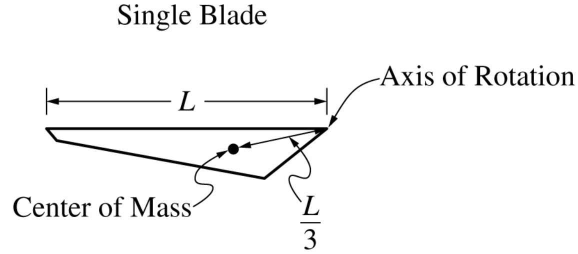

Figure 2

3. A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

(a) Derive an expression for the rotational inertia of the three-blade system. Express your answer in terms of M, L, and physical constants, as appropriate. The rotational inertia of each blade about an axis through its center of mass is given by the equation $I_{\text{cm}} = \frac{1}{18}ML^2$.

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Continue your response to **QUESTION 3** on this page.

(b) While the wind blows, the three-blade system operates at a constant angular speed $\omega_0 = 2.6$ rad/s. The length of one blade is $L = 36$ m. The numerical value of the rotational inertia of the system is $I_{sys} = 6.7 \times 10^6$ kg·m^2. Calculate the time T it takes the outer edge of a single blade to complete one revolution.

(c) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

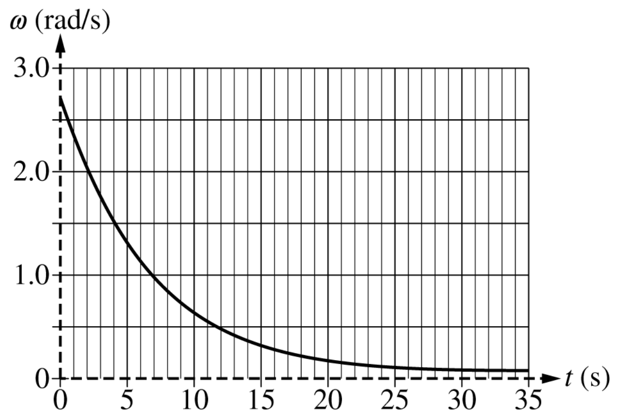

Figure 3

i. Calculate the amount of energy dissipated from $t = 0$, when the wind stops blowing, until the system comes to rest.

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Continue your response to **QUESTION 3** on this page.

ii. Derive an expression for the net torque exerted on the system as a function of t as the system slows down. Express your answers in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.

iii. Derive an expression for the angular displacement of the system $\Delta\theta$ as a function of t. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.

The three-blade system is now replaced with a second three-blade system identical to the first, except that the second three-blade system slows down according to the equation $\omega = \omega_0 e^{-\beta t}$, where $\omega_0 = 2.6$ rad/s and $\beta > \beta_0$. The original angular speed function is shown as a dashed line in Figure 4.

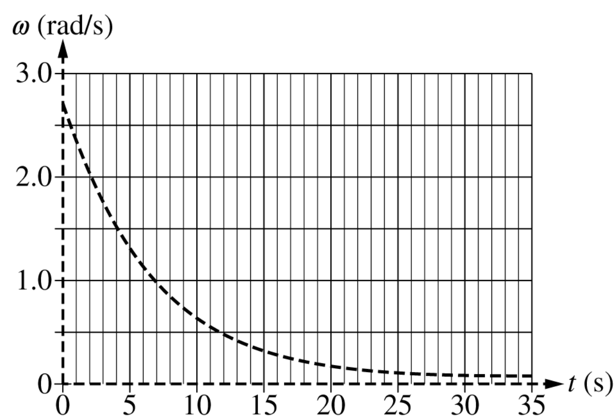

Figure 4

(d) On the graph in Figure 4, sketch the angular speed of the second three-blade system as a function of time t.

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STOP

END OF EXAM