2023

AP® Physics C: Mechanics

Sample Student Responses and Scoring Commentary Set 1

Inside:

Free-Response Question 2

☑ Scoring Guidelines

☑ Student Samples

☑ Scoring Commentary

Question 2: Free-Response Question — 15 points

(a) For indicating the rotational inertia is the sum of the rotational inertia for all the stacked disks — **1 point**

Example Response

$$I_{eq} = \sum_{i=1}^{N} I_i$$

$$I_{eq} = N\left(\frac{1}{2}MR^2\right)$$

For an expression for the period consistent with the previous rotational inertia expression — **1 point**

Example Response

$$T = 2\pi\sqrt{\frac{N\left(\frac{1}{2}MR^2\right)}{\kappa}}$$

Example Solution

$$I_{eq} = \sum_{i=1}^{N} I_i$$

$$I_{eq} = N\left(\frac{1}{2}MR^2\right)$$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

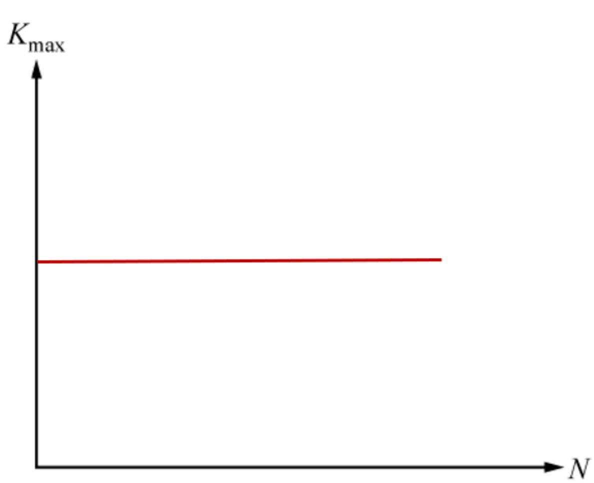
$$T = 2\pi\sqrt{\frac{N\left(MR^2\right)}{2\kappa}}$$

Total for part (a) — **2 points**

(b) For a sketch that begins at a non-zero value — **1 point**

For a sketch that is constant with slope equal to zero — **1 point**

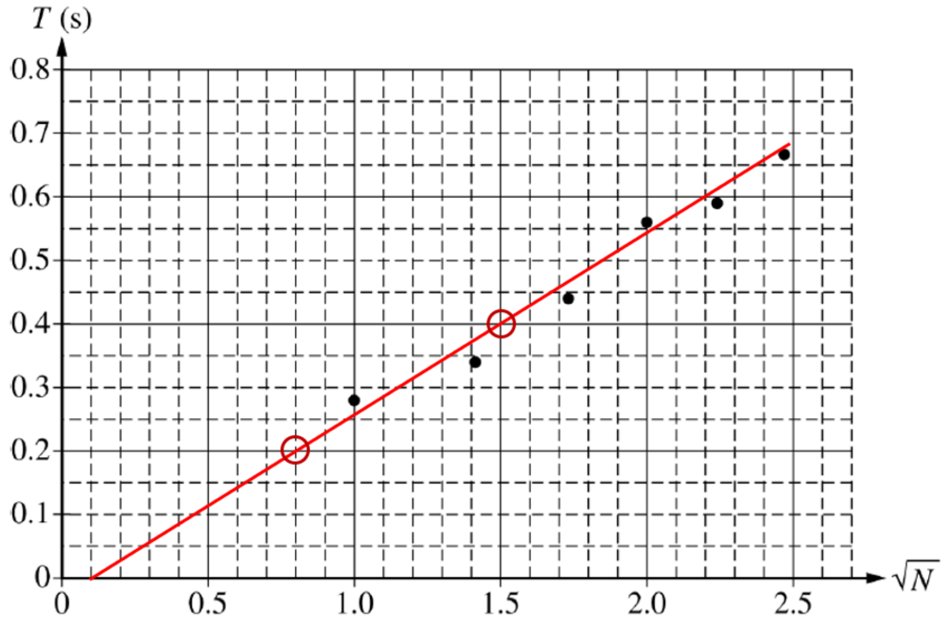
Example Solution



Total for part (b) — **2 points**

(c)(i)	For drawing an appropriate line or curve of best fit that approximates the data	1 point

Example Solution



(c)(ii) For using two points on the line to calculate the slope **1 point**

Example Response

$$\text{slope} = \frac{T_2 - T_1}{\sqrt{N_2} - \sqrt{N_1}}$$

$$\text{slope} = \frac{0.4\,\text{s} - 0.2\,\text{s}}{1.5 - 0.8} = 0.29\,\text{s}$$

Scoring Note: Slope values may range from .22 s to .33 s.

For correctly relating the slope to the period of the torsional pendulum consistent with part (a) **1 point**

Example Response

$$T = 2\pi\sqrt{\frac{NI}{\kappa}}$$

$$\rightarrow \text{slope} = 2\pi\sqrt{\frac{I}{\kappa}}$$

For substituting the slope into the equation to determine the mass of the disk **1 point**

Example Response

$$\rightarrow M = \frac{\kappa(\text{slope})^2}{2\pi^2 R^2}$$

Example Solution

$$T = 2\pi\sqrt{\frac{NI}{\kappa}}$$

$$\rightarrow \text{slope} = 2\pi\sqrt{\frac{I}{\kappa}}$$

$$(\text{slope})^2 = 4\pi^2\frac{I}{\kappa}$$

$$(\text{slope})^2 = 4\pi^2\frac{\left(\frac{1}{2}MR^2\right)}{\kappa}$$

$$\rightarrow M = \frac{\kappa(\text{slope})^2}{2\pi^2 R^2}$$

$$M = \frac{(1.6\,\text{N}\cdot\text{m})\left(\frac{0.4\,\text{s} - 0.2\,\text{s}}{1.5 - 0.8}\right)^2}{2\pi^2(0.2\,\text{m})^2}$$

$$\therefore M = 0.17\,\text{kg}$$

(c)(iii)	For indicating a source of error that could correctly explain the observed difference with an attempt at a relevant justification	**1 point**
	For a correct justification that links the source of experimental error to the larger experimental value for M. Accept one of the following: • Experimental uncertainties for the period, for example the period is measured to be larger • The mass is more concentrated to the edge, or some other distribution that results in a larger rotational inertia **Scoring Note:** Responses that indicate the given κ is too large or the given radius is too small may earn the second point.	**1 point**

Example Solution

The experimental value of the mass could be too large because the period measured by the student is too large.

OR

The experimental value of the mass could be too large because the mass of the disk is concentrated at the edge of the disk, causing the rotational inertia of the disk to be larger than that used to determine the experimental value of mass.

Total for part (c) 6 points

(d)(i)	For indicating that the slope would be greater with an attempt at a relevant justification	**1 point**
	For indicating that using disks with densities that increase with r will increase the rotational inertia	**1 point**
	For indicating the functional relationship between slope and rotational inertia: $\sqrt{I} \propto slope$	**1 point**

Example Solution

The disks with a density that increases towards the edge of the disk will have a greater proportion of their mass farther from the axis of rotation, so their rotational inertia will be larger than that of a uniform disk. Therefore, the slope of the line will be greater because the slope is proportional to $\sqrt{I}$.

(d)(ii)	For selecting $\omega_U > \omega_{non\text{-}U}$ with an attempt at a relevant justification, or a selection consistent with part (d)(i)	**1 point**
	For using energy conservation to justify the relationship	**1 point**

Example Solution

Energy in a torsion pendulum is conserved, $\frac{1}{2}I\omega^2 = \frac{1}{2}\kappa\theta^2$*, where* $\frac{1}{2}\kappa\theta^2$ *is a constant.*

Therefore, since $I_U < I_{non\text{-}U}$, $\omega_U > \omega_{non\text{-}U}$ *to keep energy conserved.*

Total for part (d) 5 points

Total for question 2 15 points

Question 2

Begin your response to **QUESTION 2** on this page.

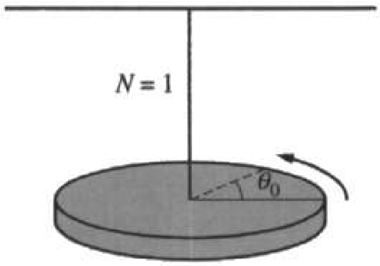


Figure 1

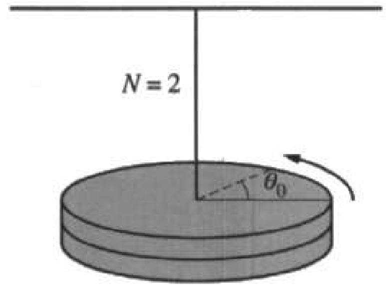


Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(a) Using $T = 2\pi\sqrt{\frac{I}{\kappa}}$, derive an expression for T as a function of N. Express your answer in terms of M, R, κ, N, and physical constants, as appropriate.

$$T = 2\pi\sqrt{\frac{N \cdot \frac{1}{2}MR^2}{k}} = \boxed{2\pi\sqrt{\frac{NMR^2}{2k}}}$$

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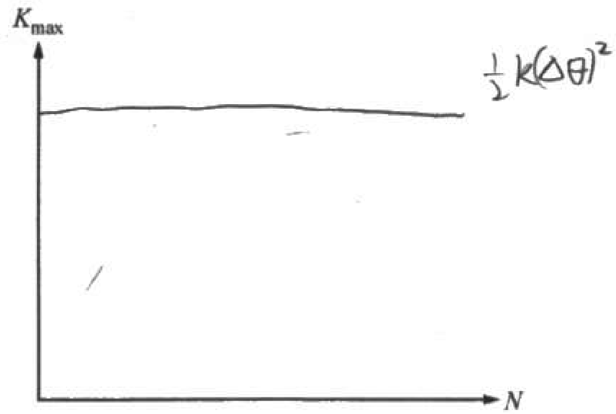
Question 2

Continue your response to QUESTION 2 on this page.

(b) The potential energy stored in the torsional pendulum when the disks are displaced is $U = \frac{1}{2}\kappa(\Delta\theta)^2$. On the following axes, sketch a graph of the maximum kinetic energy K_{max} of the torsional pendulum as a function of N for $N \geq 1$.

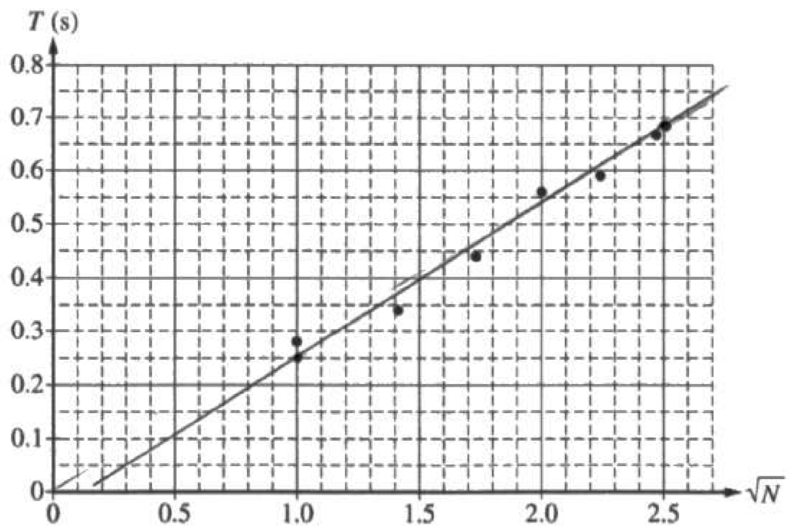
Kmax doesn't depend on N



Question 2

Continue your response to **QUESTION 2** on this page.

(c) The student plots the data for T as a function of $\sqrt{N}$, as shown.



i. Draw the best-fit line for the data.

ii. The student previously determined that the radius of a disk is $R = 0.2$ m and found that $\kappa = 1.6$ N·m. Using the graph, calculate the mass M of a single disk.

$$\text{slope} = 2\pi\sqrt{\frac{MR^2}{2k}} = \frac{0.69-0.25}{2.5-1.0} = 0.29$$

$$\rightarrow M = \left(\frac{\text{slope}}{2\pi}\right)^2 \cdot \frac{2k}{R^2} = \boxed{0.17\text{ kg}}$$

iii. The student finds that the value given by the manufacturer for the mass of the disk is less than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value of M. Justify your answer.

If the torsion constant measured is higher than the true value, then the calculated mass would be higher than the true value.

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Question 2

Continue your response to **QUESTION 2** on this page.

(d) The student repeats the experiment, but now the disks have a density that varies as a function of the radius of the disk according to $\rho = 0.3r$.

i. Would the slope of the best-fit line for this new data be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

_√_ greater than ___less than ___the same as

Justify your answer.

Since the mass distribution is higher towards the edges, I is higher, for each disk resulting in a higher slope.

ii. When $N = 1$, the maximum angular speed of the torsional pendulum with a uniform disk is found to be ω_U. When $N = 1$, the maximum angular speed of the torsional pendulum with a nonuniform disk is $\omega_{non\text{-}U}$. Which of the following correctly compares ω_U and $\omega_{non\text{-}U}$?

_√_ $\omega_U > \omega_{non\text{-}U}$ _√_ $\omega_U < \omega_{non\text{-}U}$ ___ $\omega_U = \omega_{non\text{-}U}$

Briefly justify your answer.

The maximum kinetic energy is still the same, so since the non-uniform disk has a higher moment of inertia, it has a lower angular speed.

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Question 2

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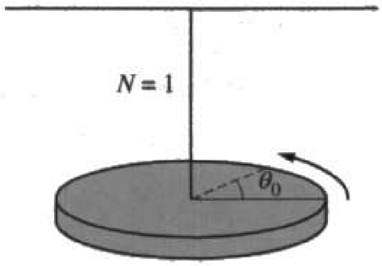


Figure 1

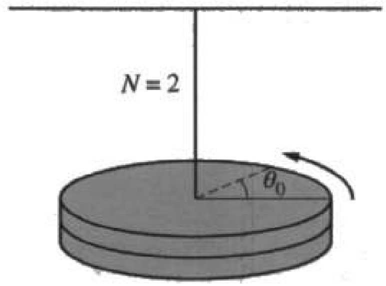


Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(a) Using $T = 2\pi\sqrt{\frac{I}{\kappa}}$, derive an expression for T as a function of N. Express your answer in terms of M, R, κ, N, and physical constants, as appropriate.

$$T = 2\pi\sqrt{\frac{N \cdot I}{K}}$$

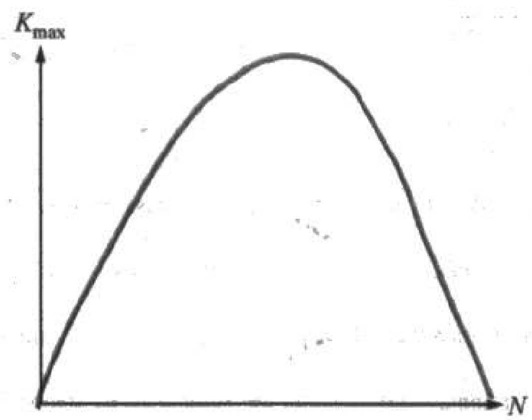
$$T = 2\pi\sqrt{\frac{N \cdot MR^2}{2K}}$$

$$T = 2\pi R\sqrt{\frac{NM}{2k}}$$

Question 2

Continue your response to **QUESTION 2** on this page.

(b) The potential energy stored in the torsional pendulum when the disks are displaced is $U = \frac{1}{2}\kappa(\Delta\theta)^2$. On the following axes, sketch a graph of the maximum kinetic energy K_{max} of the torsional pendulum as a function of N for $N \geq 1$.



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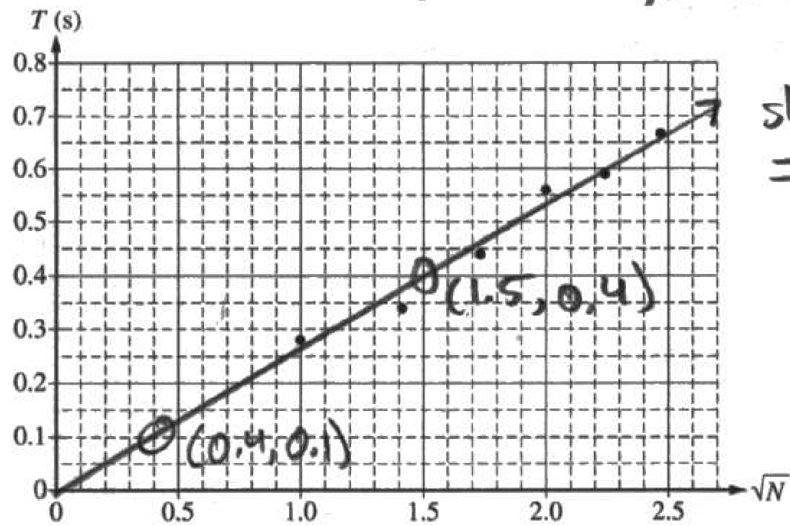
Question 2

Continue your response to **QUESTION 2** on this page.

(c) The student plots the data for T as a function of $\sqrt{N}$, as shown.

$T = 2\pi R \sqrt{\frac{NM}{2K}}$



slope $= 2\pi R \sqrt{\frac{M}{2K}}$

i. Draw the best-fit line for the data.

ii. The student previously determined that the radius of a disk is $R = 0.2$ m and found that $\kappa = 1.6$ N·m. Using the graph, calculate the mass M of a single disk.

slope $= \frac{0.4-0.1}{1.5-0.4} = 0.273 = 2\pi(0.2)\sqrt{\frac{M}{2(1.6)}}$

$0.273 = 1.257\sqrt{\frac{M}{3.2}}$

$\frac{M}{3.2} = 0.047$

$\boxed{M = 0.151 \text{ kg}}$

iii. The student finds that the value given by the manufacturer for the mass of the disk is less than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value of M. Justify your answer.

They mis measured the value of K, and its actual value is larger than 1.6 N·m.

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2

Continue your response to **QUESTION 2** on this page.

(d) The student repeats the experiment, but now the disks have a density that varies as a function of the radius of the disk according to $\rho = 0.3r$.

i. Would the slope of the best-fit line for this new data be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

✓ greater than ___less than ___the same as

Justify your answer.

Because ρ is a function of r, the slope would be greater b/c the T values would be higher.

ii. When $N = 1$, the maximum angular speed of the torsional pendulum with a uniform disk is found to be ω_U. When $N = 1$, the maximum angular speed of the torsional pendulum with a nonuniform disk is $\omega_{non\text{-}U}$. Which of the following correctly compares ω_U and $\omega_{non\text{-}U}$?

✓ $\omega_U > \omega_{non\text{-}U}$ ____ $\omega_U < \omega_{non\text{-}U}$ ____ $\omega_U = \omega_{non\text{-}U}$

Briefly justify your answer.

less mass means a higher speed b/c T would be smaller.

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Question 2

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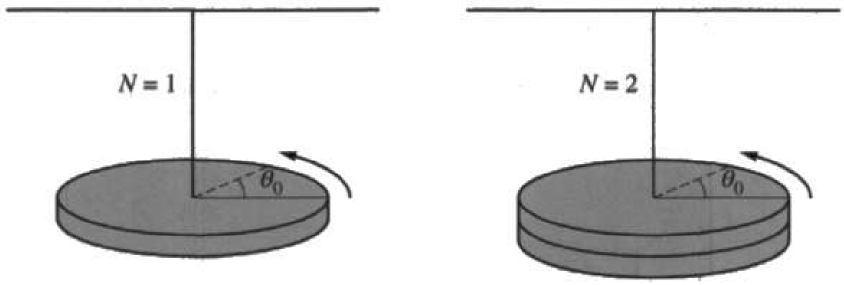

Figure 1 Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

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(a) Using $T = 2\pi\sqrt{\frac{I}{\kappa}}$, derive an expression for T as a function of N. Express your answer in terms of M, R, κ, N, and physical constants, as appropriate.

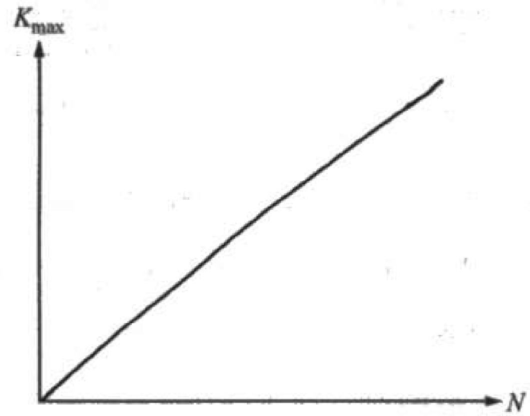
$I = \frac{1}{2}(NM)R^2$ because $M_{total} = M \cdot N$

$T = 2\pi\sqrt{\frac{\frac{1}{2}NMR^2}{K}} = \boxed{2\pi\sqrt{\frac{NMR^2}{2K}}}$

Question 2

Continue your response to QUESTION 2 on this page.

(b) The potential energy stored in the torsional pendulum when the disks are displaced is $U = \frac{1}{2}\kappa(\Delta\theta)^2$. On the following axes, sketch a graph of the maximum kinetic energy K_{max} of the torsional pendulum as a function of N for $N \geq 1$.



$\cancel{\frac{1}{2}k(\theta)^2 = \frac{1}{2}(\frac{1}{3}NM)}$

$\cancel{\frac{1}{2}}k(\theta)^2 = \cancel{\frac{1}{2}}(\frac{1}{3}NMR^2)\omega^2$

$3K\cancel{\theta^2 = NMR^2}\omega^2$

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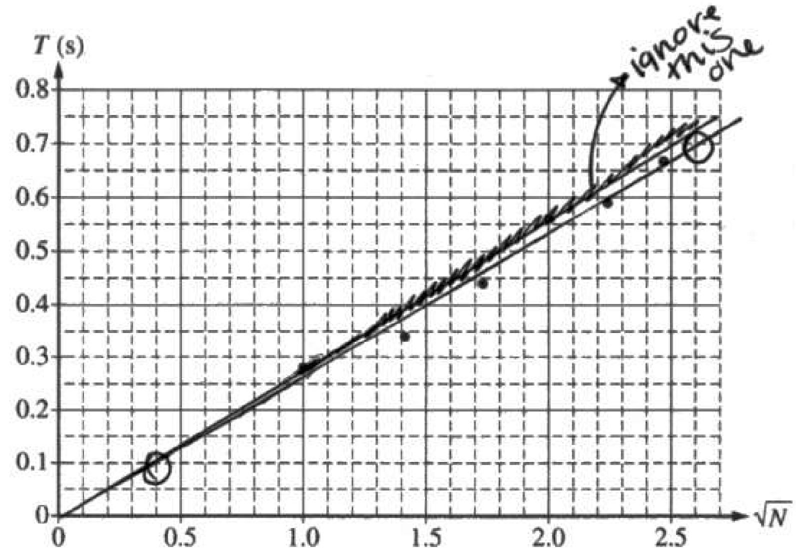
Question 2

Continue your response to **QUESTION 2** on this page.

(c) The student plots the data for T as a function of $\sqrt{N}$, as shown.



i. Draw the best-fit line for the data.

ii. The student previously determined that the radius of a disk is $R = 0.2$ m and found that $\kappa = 1.6$ N·m. Using the graph, calculate the mass M of a single disk.

~~M = √slope using~~ ~~M = (slope)² using part a~~

$T = 2\pi\sqrt{\frac{NMR}{2K}}$

$\left(\frac{T}{2\pi}\right)^2 = \frac{NMR}{2K}$

$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{.7 - .1}{2.6 - .4} = .2727\ \sqrt{m}$

$\frac{2K\left(\frac{T^2}{2\pi}\right)}{NR}$

~~M = .0743 g~~

iii. The student finds that the value given by the manufacturer for the mass of the disk is less than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value of M. Justify your answer.

a greater angle could have been used, resulting in larger measured T, leading to a higher mas

Question 2

Continue your response to **QUESTION 2** on this page.

(d) The student repeats the experiment, but now the disks have a density that varies as a function of the radius of the disk according to $\rho = 0.3r$.

i. Would the slope of the best-fit line for this new data be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than ✓ less than ___ the same as

Justify your answer.

the mass will be decreased, leading to a lower slope

ii. When $N = 1$, the maximum angular speed of the torsional pendulum with a uniform disk is found to be ω_U. When $N = 1$, the maximum angular speed of the torsional pendulum with a nonuniform disk is $\omega_{non\text{-}U}$. Which of the following correctly compares ω_U and $\omega_{non\text{-}U}$?

✓ $\omega_U > \omega_{non\text{-}U}$ ___ $\omega_U < \omega_{non\text{-}U}$ ___ $\omega_U = \omega_{non\text{-}U}$

Briefly justify your answer.

the nonuniform disc will have a greater moment of inertia due to lack of symmetry, which will cause a lower angular speed.

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Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Derive the period of oscillation for an object where the rotational inertia is due to multiple objects.
- Sketch a graph that shows the functional relationship between the rotational kinetic energy and the number of disks from a torsional pendulum given the equation for the potential energy stored.
- Draw a best-fit line for given data.
- Use a graph to calculate the slope of a linear fit and relate this linear fit to a derived equation.
- Determine and justify a source of error that explains the experimental error in a calculated value of mass.
- Describe the effects of modifying conditions by changing the mass distribution of the disk and justifying the change in slope of the best-fit line.
- Describe the effects of modifying conditions by changing the mass distribution of the disk and justifying how the angular velocity changes consistent with conservation of rotational energy.

Sample: 2A
Score: 15

Part (a) earned 2 points. The first point was earned for correctly indicating that the rotational inertia for the stacked disks is the sum of the rotational inertia for all the stacked disks from the expression: "$N\left(\frac{1}{2}MR^2\right)$." The second point was earned for correctly deriving the expression for the period consistent with the expression for rotational inertia: "$2\pi\sqrt{\frac{NMR^2}{2\kappa}}$." Part (b) earned 2 points. The first point was earned for drawing a curve that starts at a nonzero value. The second point was earned for drawing a curve that shows a constant value for maximum kinetic energy with a slope of zero. Part (c) earned 6 points. The first point was earned for showing a straight line that reasonably approximates the points on the graph. Note: The line of best fit does not have to extend to the axis to earn this point. The second point was earned for relating the slope to the model developed in part (a) for the period of the torsional pendulum. The third point was earned for relating the slope to the model developed in part (a) for the period of the torsional pendulum. The fourth point was earned for substituting the value calculated for slope to determine the mass of the disk. The fifth point was earned for identifying an experimental error and attempting a relevant justification that could explain the observed difference in mass. The sixth point was earned for identifying an error that justifies the relationship between experimental error and a difference in experimental value of mass: "If the torsion constant measured is higher than the true value, then the calculated mass would be higher than the true value." Part (d) earned 5 points. The first point was earned for selecting "greater than" and attempting a relevant justification. The second point was earned for indicating that the rotational inertia will increase when disks with densities that increase with r are used: "Since the mass distribution is higher towards the edges, I is higher for each disk." The third point was earned for describing a correct functional relationship between the slope and the rotational inertia: "I is higher for each disk, resulting in a higher slope." The fourth point was earned for selecting $\omega_{\mathrm{U}} > \omega_{\text{non-U}}$ and attempting a relevant justification consistent with part (d)(i). The fifth point was earned for including energy conservation in the justification.

Question 2 (continued)

Sample: 2B
Score: 9

Part (a) earned 2 points. The first point was earned for correctly indicating that the rotational inertia for the stacked disks is the sum of the rotational inertia for all the stacked disks from the expression: "$N\left(\frac{1}{2}MR^2\right)$." The second point was earned for correctly deriving the expression for the period consistent with the expression for rotational inertia: "$2\pi\sqrt{\frac{NMR^2}{2\kappa}}$." Part (b) earned no points. The first point was not earned because the graph drawn in the response starts at zero on the y-axis. The second point was not earned because the graph drawn in the response has a curve, not a constant value of maximum kinetic energy with a slope of zero. Part (c) earned 5 points. The first point was earned for drawing a straight line that reasonably approximates the points on the graph. The second point was earned for using two points from the line of best fit drawn in part (c)(i) to calculate the slope. Note: This point was not earned for the final answer but for the process of calculating the slope. The third point was earned for relating the slope to the model developed in part (a) for the period of the torsional pendulum. The fourth point was earned for substituting the value calculated for slope to determine the mass of the disk. The fifth point was earned for identifying an experimental error and attempting a relevant justification that could explain the observed difference in mass. The sixth point was not earned because the response does not relate the source of error identified and the difference in the experimental value of mass. The response states that the actual value of κ is larger than the previously found value. The actual value of κ should be smaller to have a mass that is smaller than the experimental value. Part (d) earned 2 points. The first point was earned for selecting "greater than" and attempting a relevant justification. The second point was not earned because the response does not indicate that using nonuniform disks will increase the rotational inertia. The third point was not earned because the response does not indicate a correct relationship between slope and rotational inertia. The fourth point was earned for selecting $\omega_{\mathrm{U}} > \omega_{\text{non-U}}$ and attempting a relevant justification consistent with part (d)(i). The fifth point was not earned because response does not use energy conservation in the justification.

Question 2 (continued)

Sample: 2C
Score: 5

Part (a) earned 2 points. The first point was earned for correctly indicating that the rotational inertia for the stacked disks is the sum of the rotational inertia for all the stacked disks from the expression: "$I = \frac{1}{2}(NM)R^2$." The second point was earned for correctly deriving the expression for the period consistent with the expression for rotational inertia: "$2\pi\sqrt{\frac{NMR^2}{2\kappa}}$." Part (b) earned no points. The first point was not earned because the graph drawn in the response starts at zero on the y-axis. The second point was not earned because the graph drawn in the response increases linearly and does not have a constant value for kinetic energy with a slope of zero. Part (c) earned 2 points. The first point was earned for showing a straight line that reasonably approximates the points on the graph. Note: The line of best fit does not have to extend to the axis to earn this point. The second point was earned for using two points from the line of best fit drawn in part (c)(i) to calculate the slope. The third point was not earned because the response does not relate the slope to the derived expression for the period of the torsional pendulum. The fourth point was not earned because the response does not substitute the value calculated for slope to determine the mass of the disk. The fifth point was not earned because the response does not identify a source of experimental error that could explain the observed difference in mass: "a greater angle could have been used." The sixth point was not earned because the response does not relate the identified error to a difference in the experimental value of mass. Part (d) earned 1 point for selecting $\omega_{\text{U}} > \omega_{\text{non-U}}$ and attempting a relevant justification consistent with part (d)(i). The second point was not earned because the response selects "less than." The third point was not earned because the response does not indicate that using disks with densities that increase with larger radius will increase the rotational inertia. The fourth point was not earned because the response does not include energy conservation in the justification.