2023

AP

AP® Physics C: Mechanics

Sample Student Responses and Scoring Commentary Set 2

Inside:

Free-Response Question 2

☑ Scoring Guidelines

☑ Student Samples

☑ Scoring Commentary

Question 2: Free-Response Question — 15 points

(a) For indicating the equivalent spring constant k_{eq} is the sum of the spring constants of all the springs arranged in parallel — **1 point**

Example Response

$$k_{eq} = Nk$$

For a correct expression for period consistent with k_{eq} above — **1 point**

Example Response

$$T = 2\pi\sqrt{\frac{m}{Nk}}$$

Example Solution

$$k_{eq} = \sum_{i=1}^{N} k_i$$

$$k_{eq} = Nk$$

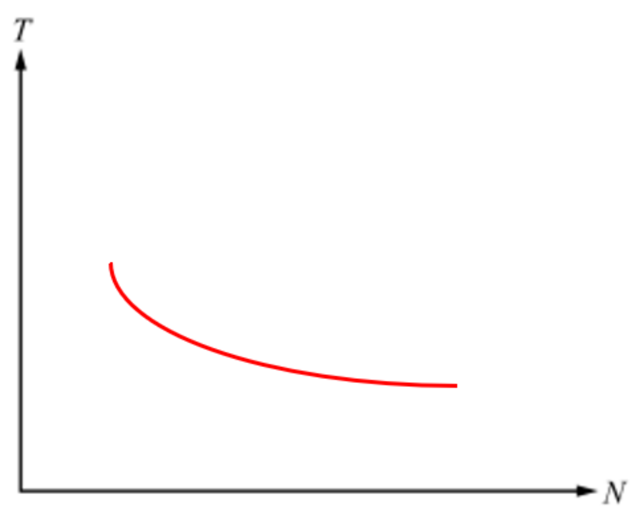
$$T = 2\pi\sqrt{\frac{m}{Nk}}$$

Total for part (a) — 2 points

(b) For a graph that increases or decreases with N consistent with the expression derived in part (a) — **1 point**

For a concavity that is consistent with the expression derived in part (a) — **1 point**

Example Solution



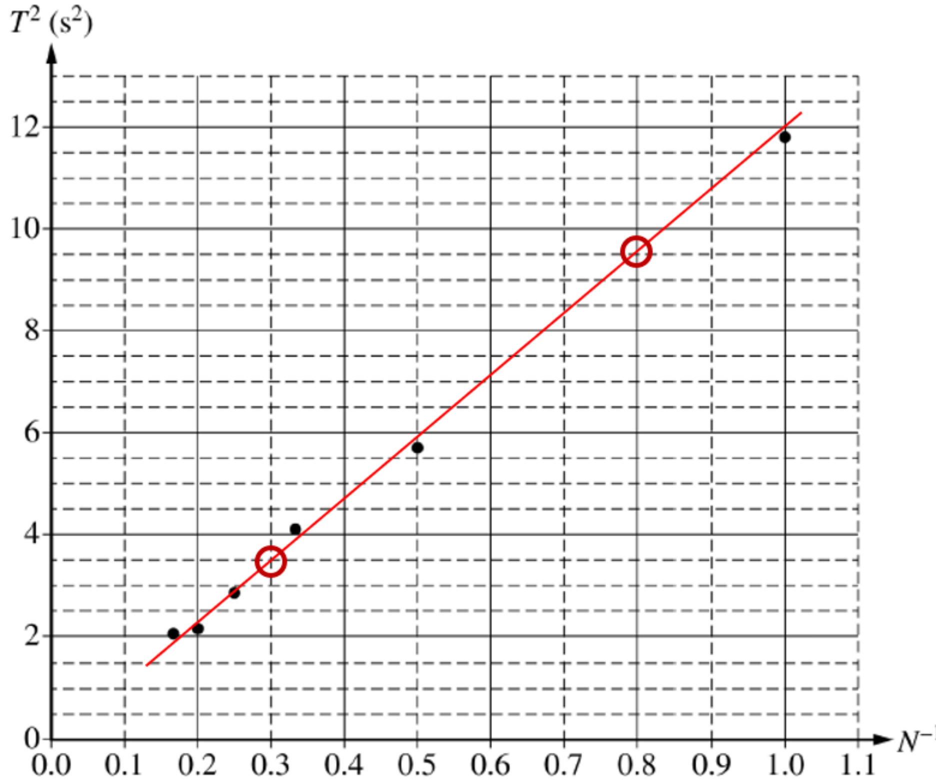
Scoring Note: The student is not required to label $N = 1$ on the horizontal axis, so the curve can start at the vertical axis, implying that $N = 1$ is at the origin.

Scoring Note: Discrete points following the appropriate curve will earn this point.

Total for part (b) — 2 points

(c)(i) For drawing an appropriate line of best fit that approximates the data **1 point**

Example Solution



(c)(ii) For using two points from the best-fit line to calculate slope **1 point**

Scoring Note: Using data points that fall on the best-fit line is acceptable.

Example Response

$$\text{slope} = \frac{\Delta\left(T^2\right)}{\Delta\left(N^{-1}\right)}$$

$$\text{slope} = \frac{(9.5\text{ s}^2 - 3.5\text{ s}^2)}{(0.8 - 0.3)}$$

$$\therefore \text{slope} = 12\text{ s}^2$$

For relating the slope of the line of the T^2 vs. N^{-1} graph consistent with the expression derived in part (a) **1 point**

Example Response

$$T^2 = 4\pi^2 \frac{m}{Nk}$$

$$\text{slope} = T^2 N$$

$$\text{slope} = 4\pi^2 \frac{m}{k}$$

For a calculated answer that has units of N/m **1 point**

Example Response

$$k = 4.93\text{ N/m}$$

Example Solution

$$T^2 = 4\pi^2 \frac{m}{Nk}$$

$$\text{slope} = 4\pi^2 \frac{m}{k}$$

$$k = \frac{4\pi^2 m}{\text{slope}}$$

$$k = \frac{4\pi^2 (1.5\text{ kg})}{\frac{(9.5\text{ s}^2 - 3.5\text{ s}^2)}{(0.8 - 0.3)}}$$

$$\rightarrow k = \frac{4\pi^2 (1.5\text{ kg})}{12\text{ s}^2}$$

$$k = 4.93\text{ N/m}$$

(c)(iii) For indicating a source of error that could result in the observed difference with an attempt at a relevant justification **1 point**

Examples include:

- Experimental uncertainties in the mass of the system
- Motion detector miscalibration
- Timing error due to reaction time

For a correct justification that links the source of experimental error to the smaller experimental value of k **1 point**

Examples include:

- Experimental uncertainties in the mass of the system (e.g., mass of block is too small, not accounting for the mass of the spring)
- Calibration of the motion sensor produces a graph from which too large a period is measured
- A larger period due to measurement error will result in a smaller value of k

Total for part (c) 6 points

(d)(i) For indicating that the slope would not change with an attempt at a relevant justification **1 point**

For a justification that the period depends on the mass and spring constant which remain unchanged **1 point**

OR

For a justification that indicates that the period is independent of gravitational force

Example Solution

The period of oscillation of the spring-block system depends on the mass and the effective spring constant, $T = 2\pi\sqrt{\frac{m}{Nk}}$. *The slope is equal to the square of the period over the inverse number of identical springs,* $\text{Slope} = \frac{T^2}{N^{-1}}$. *This means that the slope is proportional to the mass divided by the spring constant. Since neither the mass nor the spring constant change, the slope will remain unchanged.*

OR

The period of oscillation of the spring-block system depends on the mass and the effective spring constant, $T = 2\pi\sqrt{\frac{m}{Nk}}$. *The slope is equal to the square of the period over the inverse number of identical springs,* $\text{Slope} = \frac{T^2}{N^{-1}}$. *Since the period is independent of the gravitational force, and the only change was arranging the block-spring system horizontally instead of vertically, the period is not affected by the change in the orientation of the system. Therefore, the slope will remain unchanged.*

(d)(ii)	For a relationship between v_{max} and N that is consistent with the expression from part (a) with an attempt at relevant justification	**1 point**
	For using energy conservation to justify the relationship **OR** For using the relationship between v_{max} and ω to justify the relationship	**1 point**
	For indicating that the effective spring constant changes the elastic potential energy of the spring block system U_s therefore changing v_{max} in a manner consistent with the expression from part (a) **OR** For an inverse relationship between ω and period, indicating a change in v_{max} consistent with the expression from part (a)	**1 point**

Example Solution

An increase in the number of identical springs attached in parallel to each other causes the effective spring constant to increase, $k_{eff} = \sum_{i=1}^{N} k_i$ *. Since the effective spring constant is proportional to potential energy of the block-spring system,* $U_s = \frac{1}{2} k_{eff} x^2$ *, an increase in the effective spring constant causes the potential energy of the block-spring system to increase for a given displacement. Therefore, based on conservation of energy,* $\frac{1}{2} k_{eff} x^2 = \frac{1}{2} m v_{max}^2$ *, an increase in the potential energy will cause an increase in the kinetic energy, resulting in a greater maximum velocity. Therefore, the maximum velocity will increase with an increase in the number of springs added.*

OR

An increase in the number of identical springs attached in parallel to each other causes the effective spring constant to increase, $k_{eff} = \sum_{i=1}^{N} k_i$ *. Since the effective spring constant is inversely proportional to the period of oscillation of the block-spring system,* $T = 2\pi\sqrt{\frac{m}{k_{eff}}}$ *, an increase in the effective spring constant causes the period to decrease. Since the period is inversely proportional to the angular frequency of the block-spring system,* $\omega = \frac{2\pi}{T}$ *, the angular frequency increases. Since the maximum velocity is proportional to the angular frequency,* $v_{max} = x_0\omega$ *, this results in an increase in the maximum velocity. Therefore, the maximum velocity will increase with an increase in the number of springs added.*

Total for part (d) 5 points

Total for question 2 15 points

Question 2

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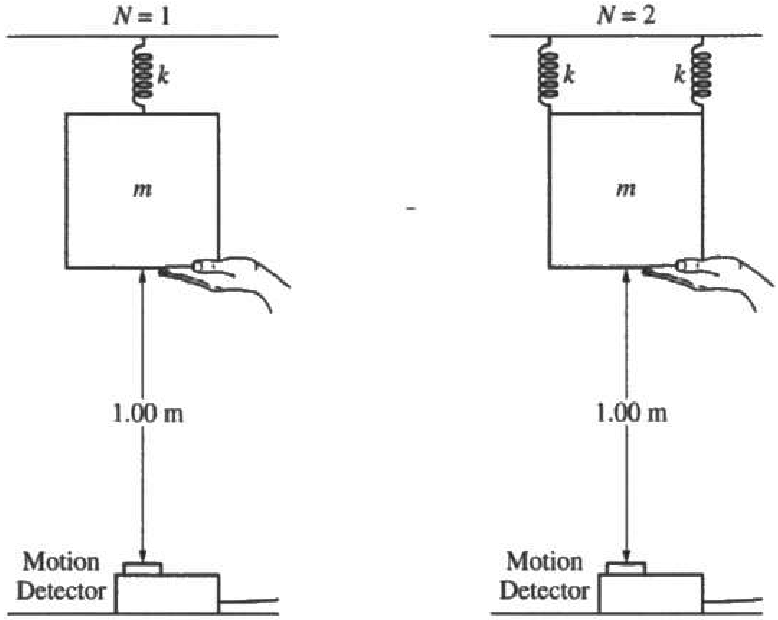


Figure 1 Figure 2

2. A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height 1.00 m above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

(a) Derive an expression for T as a function of N. Express your answer in terms of m, k, N, and physical constants as appropriate.

$T = 2\pi\sqrt{m/Nk}$

$N = 2$

$2k$

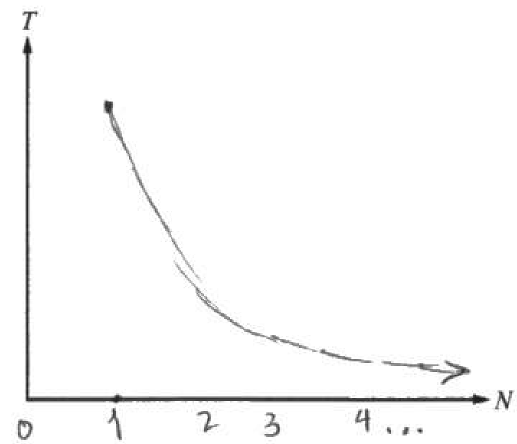
number of springs

$N \cdot k$

Question 2

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, sketch a graph of T as a function of N for $N \geq 1$.



Question 2

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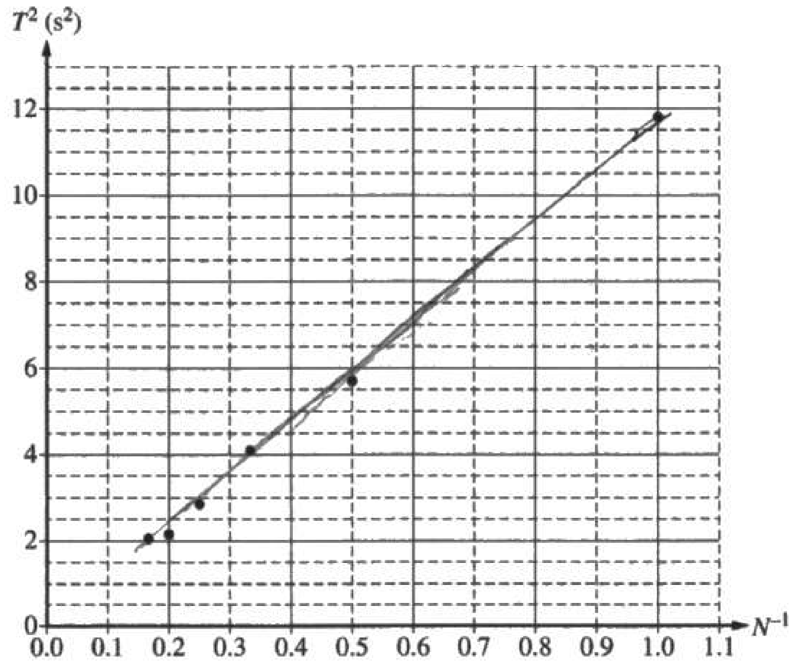
(c) The student plots the data for T^2 as a function of N^{-1}, as shown.



i. Draw the best-fit line for the data.

ii. The mass of the block is measured to be $m = 1.5$ kg. Using the graph, calculate an experimental value for the spring constant k for a single spring.

$\frac{T}{1/N} = 4\pi^2 \frac{1}{k}$ $\frac{11.8 - 2.1}{1.0 - 0.17} = 29.855$

$k = 1.322$ N/m

iii. The student finds that the value given by the manufacturer for the spring constant is larger than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value for k. Briefly justify your answer.

The mass used is too large, causing spring deformation which would reduce k

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Question 2

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$N = 2$

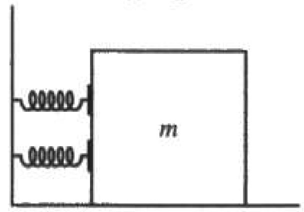

Figure 3

(d) The student conducts a similar investigation to determine the relationship between the period of oscillation T of a system consisting of a block and a number N of identical springs but arranges the block-spring system horizontally on a table, as shown in Figure 3. Frictional forces between the table and the block are negligible. In each trial, the block is displaced the same horizontal distance from equilibrium and released from rest.

i. The student plots T^2 as a function of N^{-1} for this new data. Would the slope of the best-fit line from this new investigation be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than ___ less than X the same as

Briefly justify your answer.

By making the system perpendicular to gravity nothing about the T changes as the formula doesn't involve g, just m, N kk

ii. When $N = 1$, the maximum speed of the block is found to be v_{max}. When N increases, will v_{max} increase, decrease, or stay the same?

X increase ___ decrease ___ stay the same

Justify your answer.

There is greater plastic U, & when it is all converted to KE at the midpoint of oscillation, the v_{max} will be greater

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Question 2

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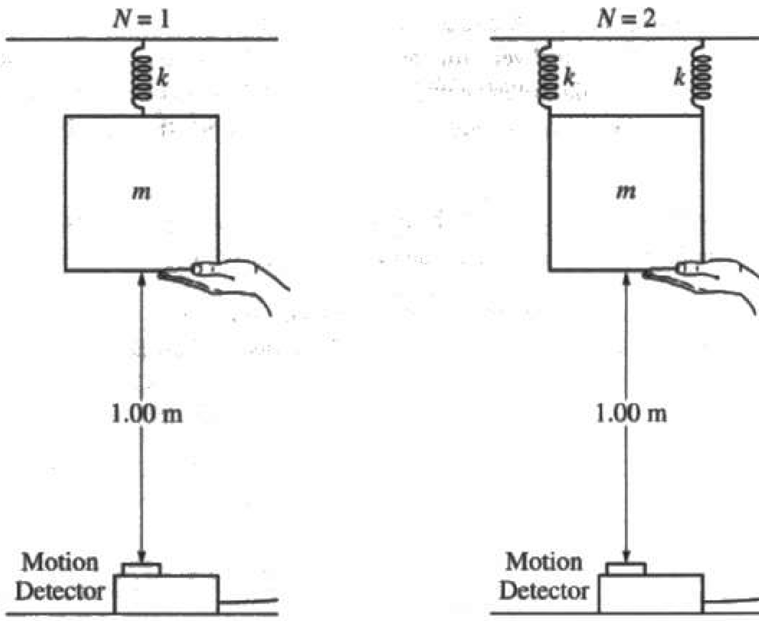


Figure 1   Figure 2

2. A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height 1.00 m above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

   (a) Derive an expression for T as a function of N. Express your answer in terms of m, k, N, and physical constants as appropriate.

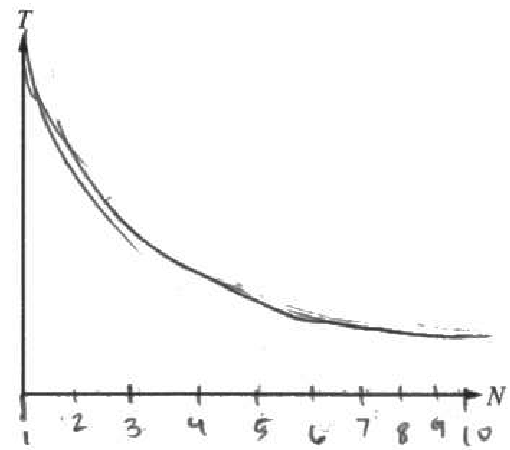
$T_s = ~~2\pi~~ \sqrt{\frac{m}{K}}$

$T_s = 2\pi \sqrt{\frac{m}{NK}}$

Question 2

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, sketch a graph of T as a function of N for $N \geq 1$.



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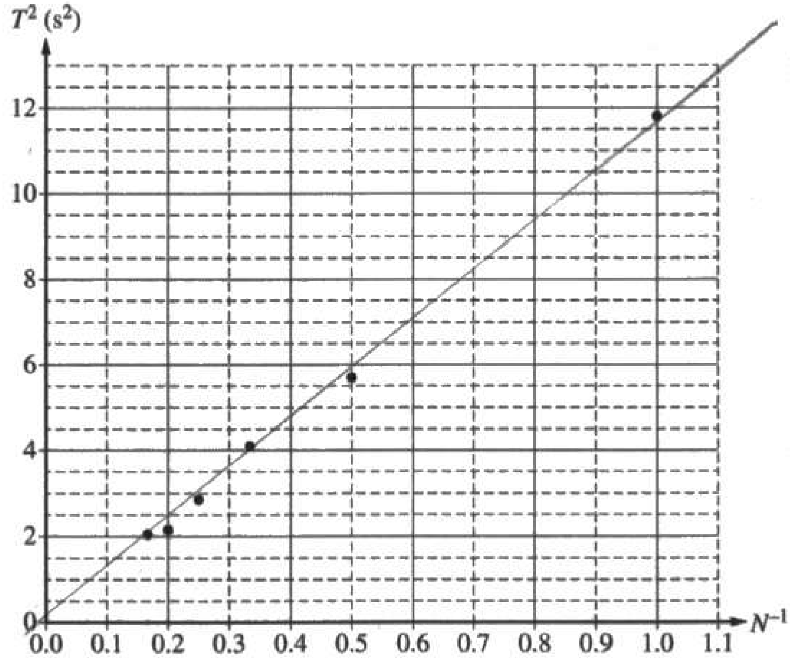

i. Draw the best-fit line for the data.

ii. The mass of the block is measured to be $m = 1.5$ kg. Using the graph, calculate an experimental value for the spring constant k for a single spring.

$T_s = 2\pi\sqrt{\frac{m}{K}}$ $K = m / \left(\frac{T_s}{2\pi}\right)^2$ $T_s^2 \approx 11.8$

$T_s \approx \sqrt{11.8}$

$\left(\frac{T_s}{2\pi}\right)^2 / m = \frac{1}{K}$ $K \approx 5.0184$

iii. The student finds that the value given by the manufacturer for the spring constant is larger than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value for k. Briefly justify your answer.

The motion detector may have registered a smaller Δx than expected, making K a larger quantatity to balance it out, as K determines how easy it is to remove a spring from equilibrium.

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Question 2

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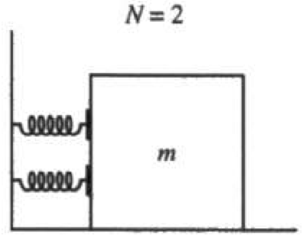

Figure 3

(d) The student conducts a similar investigation to determine the relationship between the period of oscillation T of a system consisting of a block and a number N of identical springs but arranges the block-spring system horizontally on a table, as shown in Figure 3. Frictional forces between the table and the block are negligible. In each trial, the block is displaced the same horizontal distance from equilibrium and released from rest.

i. The student plots T^2 as a function of N^{-1} for this new data. Would the slope of the best-fit line from this new investigation be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than ✓ less than ___ the same as

Briefly justify your answer.

The force of gravity provides a constant acceleration downwards, making the previous experiment's spring take longer to complete a period

ii. When $N = 1$, the maximum speed of the block is found to be v_{max}. When N increases, will v_{max} increase, decrease, or stay the same?

✓ increase ___ decrease ___ stay the same

Justify your answer.

The more springs you add, the greater they will want to turn back to its equilibrium.

$T_s = 2\pi\sqrt{\frac{m}{k}} > T_s = 2\pi\sqrt{\frac{m}{3k}}$ shorter period, bigger restoration force

Question 2

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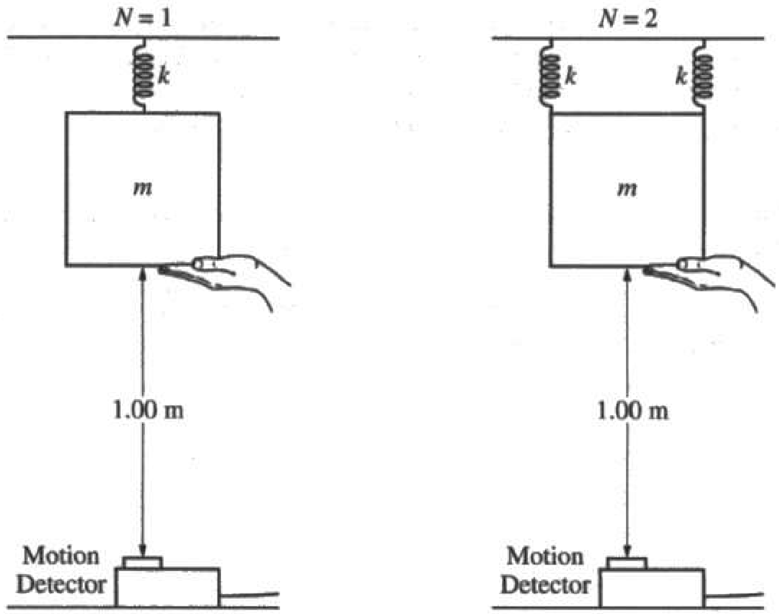

Figure 1 Figure 2

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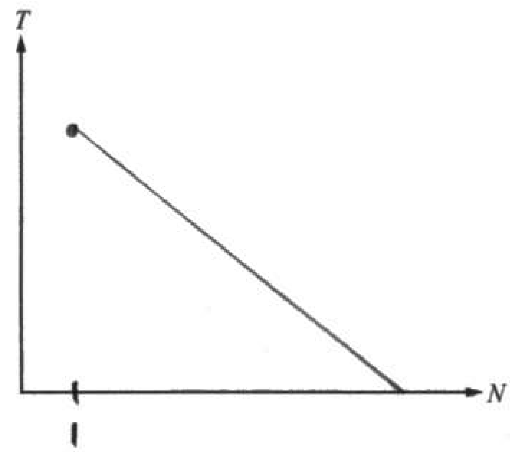
(a) Derive an expression for T as a function of N. Express your answer in terms of m, k, N, and physical constants as appropriate.

$$T = \frac{2\pi\sqrt{\frac{m}{k}}}{N}$$

Question 2

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, sketch a graph of T as a function of N for $N \geq 1$.



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Question 2

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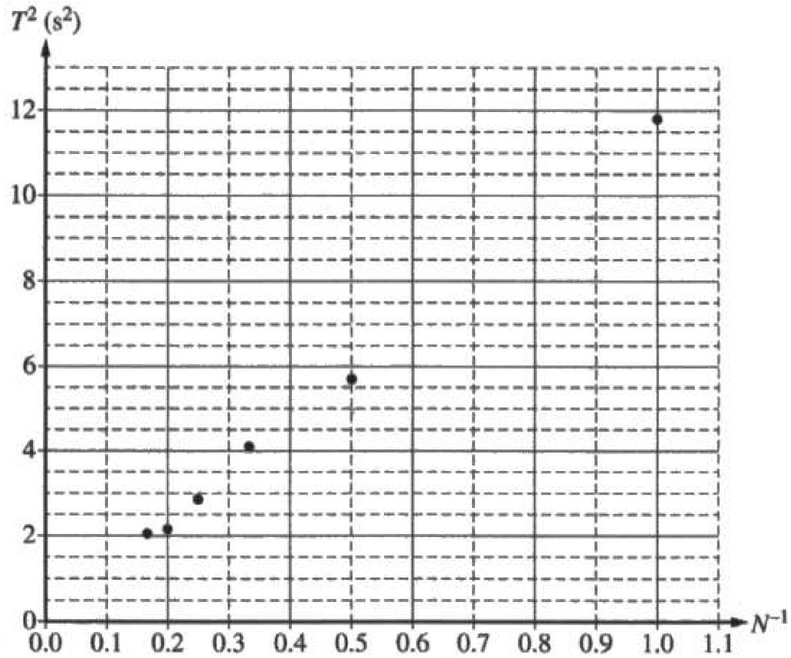

i. Draw the best-fit line for the data.

ii. The mass of the block is measured to be $m = 1.5$ kg. Using the graph, calculate an experimental value for the spring constant k for a single spring.

$k = \frac{m}{\left(\frac{NT}{2\pi}\right)^2}$ $T = \frac{2\pi\sqrt{m}}{N}$

$= 2.5747$

iii. The student finds that the value given by the manufacturer for the spring constant is larger than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value for k. Briefly justify your answer.

approximation of t^2. b/c the exact point is not given

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Question 2

Continue your response to **QUESTION 2** on this page.

$N = 2$

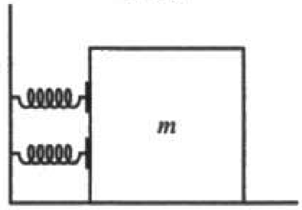

Figure 3

(d) The student conducts a similar investigation to determine the relationship between the period of oscillation T of a system consisting of a block and a number N of identical springs but arranges the block-spring system horizontally on a table, as shown in Figure 3. Frictional forces between the table and the block are negligible. In each trial, the block is displaced the same horizontal distance from equilibrium and released from rest.

i. The student plots T^2 as a function of N^{-1} for this new data. Would the slope of the best-fit line from this new investigation be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

_✓_ greater than   _✓_ less than   ___the same as

Briefly justify your answer.

Less b/c gravity is no longer acting infavor of the ocilation.

ii. When $N = 1$, the maximum speed of the block is found to be v_{max}. When N increases, will v_{max} increase, decrease, or stay the same?

____ increase   _✓_ decrease   ____ stay the same

Justify your answer.

there will be less force on the black causing it to decreas

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Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Determine the effective spring constant of springs in parallel.
- Derive a symbolic expression that models the experiment by relating period of the oscillator to the number of springs using the effective spring constant.
- Sketch the curve associated with a relationship.
- Draw a best-fit line when given a set of plotted data points.
- Calculate the slope of the best-fit line.
- Relate the slope of a graph to the given equation.
- Use the slope of experimental data to find a physical value.
- Explain how experimental error could lead to an incorrect conclusion.
- Describe the independence between the period of a simple harmonic oscillator and gravitational force.
- Use the total mechanical energy to determine the effect of changing the effective spring constant on the motion of the oscillator.

Sample: 2A
Score: 14

Part (a) earned 2 points. The first point was earned because the response correctly states that the relationship for the equivalent spring constant for N springs is Nk. The second point was earned because the response substitutes the equivalent spring constant into the expression for the period of a mass-spring oscillator: "$T = 2\pi\sqrt{\frac{m}{Nk}}$". Part (b) earned 2 points. The first point was earned because the graph in the response is decreasing, consistent with the expression derived in part (a). The second point was earned because the graph in the response is concave up, consistent with the expression derived in part (a). Part (c)(i) earned 1 point because the response has a straight line that reasonably approximates the points on the graph. Note: The line of best fit does not have to extend to the axis to earn this point. Part (c)(ii) earned 2 points. The first point was earned because the response uses two points from the line of best fit drawn in part (c)(i) to calculate the slope. Note: This point is not earned for the final answer but for the process of calculating the slope. The second point was not earned because the response does not relate the slope to the model from part (a). The third point was earned because the response calculates a value for k with units of N/m. Part (c)(iii) earned 2 points. The first point was earned because the response identifies an experimental error that could impact the period: "The mass used is too large." The second point was earned because the error identified shows the correct relationship between experimental error and a reduced experimental value of k: "causing spring deformation which would reduce k." Part (d)(i) earned 2 points. The first point was earned because the response correctly selects "the same as" and includes an attempt at a relevant justification. The second point was earned because the response correctly shows independence between period and gravitational acceleration: "By making the system perpendicular to gravity nothing about the T changes as the formula doesn't involve g, just m, N, k." Part (d)(ii) earned 3 points. The first point was earned because the response correctly selects "increase" and includes an attempt at a relevant justification. The second point was earned because the response uses conservation of energy to justify the relationship. The third point was earned because the response indicates that increasing N increases the elastic potential energy of the system, increasing kinetic energy at equilibrium and, therefore, increasing v_{max}.

Question 2 (continued)

Sample: 2B
Score: 8

Part (a) earned 2 points. The first point was earned because the response shows the correct substitution of $k_{eq} = Nk$ into the period equation. The second point was earned because the response substitutes the equivalent spring constant into the expression for the period of a mass-spring oscillator. Part (b) earned 2 points. The first point was earned because the graph in the response is decreasing, consistent with the expression derived in part (a). The second point was earned because the graph in the response is concave up, consistent with the expression derived in part (a). Part (c)(i) earned 1 point because the response has a straight line that reasonably approximates the points on the graph. Part (c)(ii) earned 2 points. The first point was earned because the line of best fit has a y-intercept of $(0,0)$, a single point from the best-fit line can be used with $(0,0)$ to implicitly calculate the slope of the line. The second point was earned because the response consistently substitutes the implicitly calculated value of slope into the model developed in part (a). The third point was not earned because the response calculates a value for k but does not include units of N/m. Part (c)(iii) earned no points. The first point was not earned because the response identifies an experimental error that would not impact the period because the change in displacement has no effect on the period, so it will not affect the measured spring constant. The second point was not earned because the error identified does not indicate a correct relationship between experimental error and a reduced experimental value of k. Part (d)(i) earned no points. The first point was not earned because the response selects "less than." The second point was not earned because the response indicates that the period T is dependent on the gravitational force. Part (d)(ii) earned 1 point. The first point was earned because the response correctly selects "increase" and includes an attempt at a relevant justification. The second point was not earned because the response does not use physics principles to justify the relationship between N and v_{max}. The third point was not earned because the response does not connect the increased spring constant to the maximum velocity, consistent with the model developed in part (a).

Sample: 2C
Score: 1

Part (a) earned no points. The first point was not earned because the response does not include a relationship between N and the equivalent spring constant. The second point was not earned because the response does not include a correct relationship between N and period. Part (b) earned 1 point. The first point was earned because the graph in the response is decreasing, consistent with the expression in part (a). The second point was not earned because the concavity of the graph drawn in the response is inconsistent with the expression derived from part (a). Part (c)(i) earned no points because the response does not include a line of best fit. Part (c)(ii) earned no points. The first point was not earned because the response does not identify or calculate features from the graph. The second point was not earned because the response does not relate the period to a feature of the graph. The third point was not earned because the response calculates a value for k but does not include units of N/m. Part (c)(iii) earned no points. The first point was not earned because the response does not relate the experiment to the value of k. The second point was not earned because the response does not relate experimental error to the value of k. Part (d)(i) earned no points. The first point was not earned because the response selects "less than." The second point was not earned because the response does not indicate independence between period and gravitational acceleration. Part (d)(ii) earned no points. The first point was not earned because the response selects "decrease." The second point was not earned because the response does not use physics principles to justify the relationship between N and v_{max}.The third point was not earned because the response does not connect the increased spring constant to the maximum velocity, consistent with the expression for part (a).