2023

AP® Physics C: Mechanics

Sample Student Responses and Scoring Commentary

Set 1

Inside:

Free-Response Question 3

- ☑ Scoring Guidelines
- ☑ Student Samples
- ☑ Scoring Commentary

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Question 3: Free-Response Question 15 points

(a) For using a correct expression for conservation of energy of the rod-Earth system **1 point**

Example Response

$$\Delta U + \Delta K = 0$$

$$\left(0 - Mgh_{\mathrm{CM}}\right) + \left(\frac{1}{2}I\omega_f^2 - 0\right) = 0$$

$$Mgh_{\mathrm{cm}} = \frac{1}{2}I\omega_f^2$$

For correctly substituting h and I into the correct energy expression **1 point**

Example Response

$$Mg\frac{\ell}{2} = \frac{1}{2}\left(\frac{1}{3}M\ell^2\right)\omega_f^2$$

Example Solution

$$\Delta U + \Delta K = 0$$

$$\left(0 - Mgh_{\mathrm{CM}}\right) + \left(\frac{1}{2}I\omega_f^2 - 0\right) = 0$$

$$Mgh_{\mathrm{cm}} = \frac{1}{2}I\omega_f^2$$

$$Mg\frac{\ell}{2} = \frac{1}{2}\left(\frac{1}{3}M\ell^2\right)\omega_f^2$$

$$\therefore \omega_f = \sqrt{\frac{3g}{\ell}}$$

Alternate Solution **1 point**

For using a correct expression of Newton's second law in rotational form

Alternate Example Response

$$\tau_{net} = I\alpha$$

For correctly substituting expressions for net torque and rotational inertia of the rod **1 point**

Alternate Example Response

$$\frac{\ell}{2}Mg\cos\theta = \frac{1}{3}M\ell^2\alpha$$

Alternate Example Solution

$$\tau_{net} = I\alpha$$

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \frac{d\omega}{d\theta}\omega$$

$$\frac{\ell}{2}Mg\cos\theta = \frac{1}{3}M\ell^2\alpha$$

$$\frac{\ell}{2}Mg\cos\theta = \frac{1}{3}M\ell^2\frac{d\omega}{d\theta}\omega$$

$$\frac{\ell}{2}Mg\cos\theta d\theta = \frac{1}{3}M\ell^2\omega d\omega$$

$$\int_0^{\frac{\pi}{2}}\frac{\ell}{2}Mg\cos\theta d\theta = \int_0^{\omega_f}\frac{1}{3}M\ell^2\omega d\omega$$

$$\frac{\ell}{2}Mg\left[\sin\frac{\pi}{2} - \sin 0\right] = \frac{1}{3}M\ell^2\left[\frac{1}{2}\omega_f^2 - 0\right]$$

$$3\frac{g}{\ell}[1-0] = \left[\omega_f^2\right]$$

$$\omega_f = \sqrt{\frac{3g}{\ell}}$$

Scoring Note: *The full integration is not needed to earn points but is presented for clarity.*

Total for part (a) 2 points

(b)	For using a correct expression for conservation of angular momentum	**1 point**

Example Response

$$L_i = L_f$$
$$I\omega = mvr$$

For correctly substituting the expression for ω_f from part (a) **1 point**

Example Response

$$\frac{2}{3}m\ell^2\sqrt{\frac{3g}{\ell}} = mv_0\ell$$

Example Solution

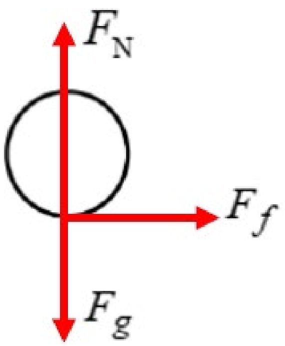
$$L_i = L_f$$
$$I\omega = mvr$$
$$\frac{2}{3}m\ell^2\sqrt{\frac{3g}{\ell}} = mv_0\ell$$
$$\therefore v_0 = \sqrt{\frac{4}{3}g\ell}$$

Scoring Note: The last equation is not needed for scoring the item but is presented for clarity.

Total for part (b) 2 points

(c)	For correctly drawing and labeling the force due to gravity and the normal force on the sphere	**1 point**
	For drawing the frictional force horizontally to the right at the bottom of the sphere	**1 point**

Example Solution



Scoring note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, “grav force”, “F Earth on sphere”, “F on sphere by Earth”, $F_{\text{Earth on sphere}}$, $F_{\text{E,Sphere}}$, $F_{\text{Sphere,E}}$. The labels G or g are not appropriate labels for the force due to gravity. F_n, F_N, N, “normal force”, “ground force”, or similar labels may be used for the normal force.

Scoring Note: A response that includes extraneous vectors can earn a maximum of 1 point.

Scoring Note: Horizontally displacing the F_N and F_g vectors slightly is permitted in order to show the distinct points at which those forces are exerted on the sphere.

Total for part (c) 2 points

(d)(i) For using Newton's second law in the horizontal direction **1 point**

Example Response

$$\Sigma F = ma$$

$$-\mu mg = ma$$

For a correct derivation of acceleration **1 point**

Example Response

$$a = -\mu g$$

For a correct expression for the velocity **1 point**

Example Response

$$\therefore v = v_0 - \mu gt$$

Example Solution

$$\Sigma F = ma$$

$$-\mu mg = ma$$

$$a = -\mu g$$

$$v = v_0 + at$$

$$\therefore v = v_0 - \mu gt$$

Scoring Note: Only the final expression for velocity must have correct signs.

(d)(ii) For using Newton's second law in rotational form with substitutions for rotational inertia and torque **1 point**

Example Response

$$\tau = I\alpha$$

$$F_f R = \frac{2}{5} m R^2 \alpha$$

For correctly substituting for friction and solving for α **1 point**

Example Response

$$F_f = \mu m g$$

$$\alpha = \frac{5\mu g}{2R}$$

For correctly substituting α into a rotational kinematic equation and solving for ω **1 point**

Example Response

$$\omega = \omega_0 + \alpha t$$

$$\therefore \omega = \frac{5\mu g}{2R} t$$

Example Solution

$$\tau = I\alpha$$

$$F_f R = \frac{2}{5} m R^2 \alpha$$

$$F_f = \mu m g$$

$$\alpha = \frac{5\mu g}{2R}$$

$$\omega = \omega_0 + \alpha t$$

$$\therefore \omega = \frac{5\mu g}{2R} t$$

Total for part (d) 6 points

(e)(i) For indicating the linear speed is equal to $R\omega$ when slipping stops at Point B **1 point**

Example Response

$$v = R\omega$$

For correctly substituting v and ω from parts (d)(i) and (d)(ii) **1 point**

Scoring Note: Substituting the acceleration from part (d)(i) into a valid kinematic equation that includes time can earn this point.

Example Response

$$v_0 - \mu g t = R\frac{5\mu g}{2R}t$$

Example Solution

$$v = R\omega$$

$$v_0 - \mu g t = R\frac{5\mu g}{2R}t$$

$$\therefore t = \frac{2v_0}{7\mu g}$$

(e)(ii) For correctly substituting the expression for time from (e)(i) into the expression for velocity in (d)(i) **1 point**

Example Solution

$$v = v_0 - \mu g t$$

$$v = v_0 - \mu g\frac{2v_0}{7\mu g}$$

$$\therefore v = \frac{5}{7}v_0$$

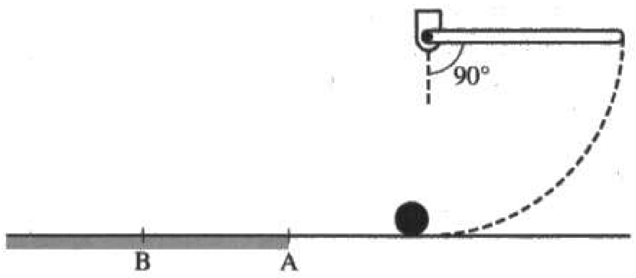
Scoring Note: The last equation is not needed for scoring the item but is presented for clarity.

Total for part (e) 3 points

Total for question 3 15 points

Question 3

Begin your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

$$mg\Delta h = \left(\frac{1}{3}ML^2\right)\cdot\frac{w^2}{2}, \quad \Delta h = \frac{L}{2}.$$

$$\frac{mgL}{2} = \frac{1}{3}ML^2\frac{w^2}{2}, \quad \frac{3gL}{L^2} = w^2, \quad \boxed{w = \sqrt{3g/L}}$$

Question 3

Continue your response to **QUESTION 3** on this page.

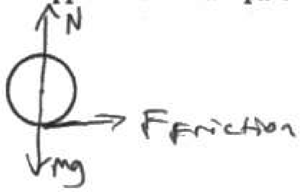
(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate.

$$I\omega = mvL, \quad \frac{2m}{3}L^2\left(\sqrt{3g/L}\right) = mvL$$

$$v = \frac{2mL^2}{3mL}\sqrt{3g/L} = \boxed{\frac{2}{3}L\sqrt{\frac{3g}{L}}}$$

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.



(d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

i. The linear velocity v of the center of mass of the sphere as a function of time t

$$a = \frac{\mu mg}{m} = \mu g$$

$$v(t) = v_0 - at = \boxed{v_0 - \mu g t}$$

ii. The angular velocity ω of the sphere as a function of time t

$$w(t): \quad I\alpha = \tau = \mu mgR, \quad \alpha = \frac{\mu mgR}{\left(\frac{2}{5}mR^2\right)} = \frac{5\mu g}{2R}$$

$$w(t) = \alpha t = \boxed{\frac{5\mu g}{2R}t}$$

Question 3

Continue your response to **QUESTION 3** on this page.

(e)

i. Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

$\omega R = v$, $\frac{5\mu g}{2} t = v_0 - \mu g t$

$t\left(\frac{5\mu g}{2} + \mu g\right) = v_0 = t\left(\frac{7}{2}\mu g\right) = v_0$

$$\boxed{t = \frac{2v_0}{7\mu g}}$$

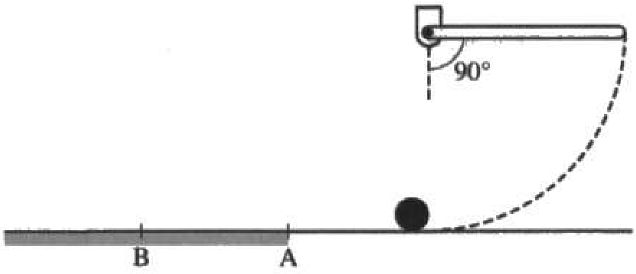
ii. Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

$v\left(\frac{2v_0}{7\mu g}\right) = v_0 - \mu g\left(\frac{2v_0}{7\mu g}\right)$

$= v_0 - \frac{2v_0}{7} = \boxed{\frac{5v_0}{7}}$

Question 3

Begin your response to QUESTION 3 on this page.



Note: Figure not drawn to scale.

Figure 1

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(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

$U_i + \overset{0}{\cancel{K_i}} = \overset{0}{\cancel{U_f}} + K_f$

$mgh = \frac{1}{2}I\omega^2$

$\cancel{2m}g\cancel{\ell} = \frac{1}{2}\left(\frac{1}{3}\cancel{2m}\ell^{\cancel{2}}\right)\omega^2$

$\frac{6g}{\ell} = \omega^2 \qquad \omega = \sqrt{\frac{6g}{\ell}}$

Question 3

Continue your response to **QUESTION 3** on this page.

(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate.

$$m_R v_R + \overset{0}{\cancel{m_s v_s}} = \overset{0}{\cancel{m_R v_{Rf}}} + m_s v_{sf}$$

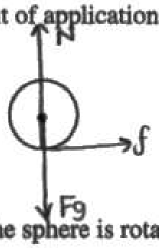
$$= \sqrt{\frac{6g}{\ell}}$$

$$v_{sf} = \frac{m_R v_R}{m_s} = \frac{2\cancel{m}\sqrt{\frac{6g}{\ell}}}{\cancel{m}}$$

$$v_0 = 2\sqrt{\frac{6g}{\ell}}$$

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.



(d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

i. The linear velocity v of the center of mass of the sphere as a function of time t

Σ

$$\Sigma F_y = 0 = N - F_g$$

$$N = F_g$$

$$N = mg$$

$$\Sigma F_x = ma = -f$$

$$\cancel{m}a = -\mu \cancel{m}g$$

$$a = -\mu g$$

$$V = V_0 - \mu g t$$

ii. The angular velocity ω of the sphere as a function of time t

$$\omega = \omega_0 + \alpha t$$

Question 3

Continue your response to **QUESTION 3** on this page.

(e)

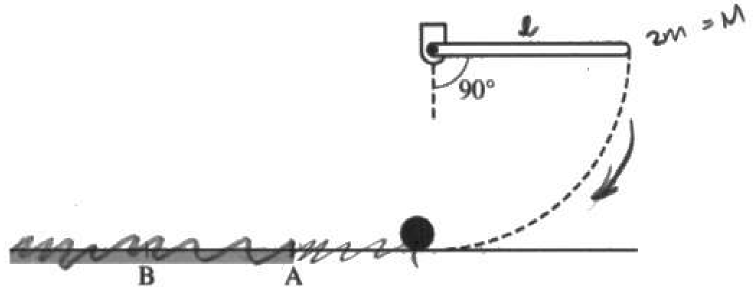
i. Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

$\Delta X = V_0 t + \frac{1}{2} a t^2$

ii. Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

Question 3

Begin your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

$I = \frac{1}{2} M \ell^2$

$\frac{\theta}{sec}$

Question 3

Continue your response to **QUESTION 3** on this page.

(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate.

$\Delta K_{rod} = \Delta K_{v_0}$

conservation of momentum

$m_1 v_1 = m_2 v_2$

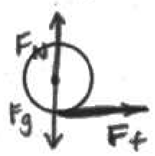
$2m(r\omega) = m v_2$

$V_0 = 2r\omega$

m/sec

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.



(d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

i. The linear velocity v of the center of mass of the sphere as a function of time t

linear velocity decreases as kinetic energy transfers to angular

$mv^2 -$

meters/sec

ii. The angular velocity ω of the sphere as a function of time t

$r\omega = v$

$\omega =$

$F_{friction} \times r = \text{Torque}$

rad/sec

Question 3

Continue your response to **QUESTION 3** on this page.

(e)

i. Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

$\Delta x = V_0 t + \frac{1}{2}ame($

____________________ sec

frictional force causes rotation

$T = r \times F$

ii. Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

$V = \frac{1}{2} mv^2 - \frac{1}{2} I \omega^2$

$V = \frac{1}{2} mv^2 - \frac{1}{2} (\frac{2}{5} mR^2) \omega^2$

$K = \frac{1}{2} mv^2$

$= \frac{1}{2} I \omega^2$

conservation of energy

m/sec

conservation of momentum

$p_1 = p_2$

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Derive a symbolic expression for the angular speed of the rotating rod after it swings down to the vertical position. This requires selecting either the law of conservation of mechanical energy or Newton's second law in rotational form as the relevant principle to describe the motion of the rod.
- Derive a symbolic expression for the linear speed of the sphere after the collision with the rotating rod. This requires selecting the law of conservation of angular momentum as the relevant principle for describing a rotational collision.
- Create a free-body diagram that appropriately depicts the relevant forces on the sphere. This requires identifying different types of forces, such as the normal force, gravitational force, and kinetic friction force.
- Derive a symbolic expression for the linear speed of the sphere as a function of time. This requires selecting Newton's second law as the relevant principle to describe the linear motion of the sphere between points A and B.
- Derive a symbolic expression for the angular speed of the sphere as a function of time. This requires selecting Newton's second law in rotational form as the relevant principle to describe the rotational motion of the sphere between points A and B.
- Derive a symbolic expression for the time it takes the sphere to move between points A and B. This requires selecting the rolling without sliding condition as the appropriate relationship between the linear and angular speeds of the sphere when it reaches Point B.
- Combine the expressions for the linear speed and for the time to derive a symbolic expression for the linear speed of the sphere at Point B.

Question 3 (continued)

Sample: 3A
Score: 15

Part (a) earned 2 points. The first point was earned for including a correct expression for conservation of energy for the rod-Earth system: "$mg\Delta h = \left(\frac{1}{3}ML^2\right) \cdot \frac{\omega^2}{2}$". The second point was earned for correctly substituting the change in height and the rotational inertia of the rod: "$mg\frac{L}{2} = \frac{1}{3}ML^2\frac{\omega^2}{2}$". Part (b) earned 2 points. The first point was earned for equating the angular momentum of the rod just before the collision to the angular momentum of the sphere about the pivot after the collision. The sphere is treated as a particle. The second point was earned for correctly substituting the expression for ω_f from part (a) into the angular momentum equation. Note: This point was earned even though the linear speed is represented as v instead of v_0. Part (c) earned 2 points. The first point was earned for including an upward normal force and a downward gravitational force acting on the sphere. The second point was earned for including a horizontal frictional force applied to the bottom of the sphere. Part (d) earned 6 points. The first point was earned for using Newton's second law in the horizontal direction. The second point was earned for correctly deriving the acceleration. Note: This point was earned regardless of the sign on the acceleration expression. The third point was earned for deriving a correct expression for the velocity: "$v_0 - \mu gt$". The fourth point was earned for using Newton's second law in rotational form. The fifth point was earned for correctly substituting the friction force into the expression for torque to solve for angular acceleration. The sixth point was earned for substituting the angular acceleration into a correct kinematic equation for ω. Part (e) earned 3 points. The first point was earned for indicating that the final velocity is determined by the condition for rolling without sliding. The second point was earned for substituting the expression for v from part (d)(i) and the expression for ω from part (d)(ii) into the expression for rolling without sliding. The third point was earned for substituting the time from part (e)(i) into the expression for a decreasing velocity: "$v_0 - mg\left(\frac{2v_0}{7mg}\right)$".

Sample: 3B
Score: 6

Part (a) earned 1 point for including a correct expression for conservation of energy for the rod-Earth system. The second point was not earned because the response substitutes L as the change in height of the rod. Part (b) earned no points. The first point was not earned because the response begins with conservation of linear momentum. The second point was not earned because the response substitutes the expression for the angular speed ω of the rod from part (a) into the linear speed v_R of the rod. Part (c) earned 2 points. The first point was earned for including an upward normal force and a downward gravitational force acting on the sphere. The second point was earned for including a horizontal frictional force applied to the bottom of the sphere. Part (d) earned 3 points. The first point was earned for using Newton's second law in the horizontal direction. The second point was earned for correctly deriving the acceleration. The third point was earned for deriving a correct expression for the velocity: "$v = v_0 - \mu gt$". Note: The sign in the response is a negative sign with an erasure mark. The fourth point was not earned because the response does not use Newton's second law in rotational form. The fifth point was not earned because the response does not substitute the friction force into the expression for torque to solve for angular acceleration. The sixth point was not earned because the response does not substitute the angular acceleration into a correct kinematic equation for ω. Part (e) earned no points. The first point was not earned because the response does not indicate the final velocity is determined by the condition for rolling without sliding. The second point was not earned because the response does not substitute the expression for v from part (d)(i) and the expression for ω from part (d)(ii) into the expression for rolling without sliding. The third point was not earned because the response does not substitute the time from part (e)(i) into the expression for a decreasing velocity.

Question 3 (continued)

Sample: 3C
Score: 2

Part (a) earned no points. The first point was not earned because the response does not include an expression for conservation of energy. The second point was not earned because the response does not substitute the change in height and the rotational inertia of the rod into a conservation of energy expression. Part (b) earned no points. The first point was not earned because the response does not equate the angular momentum of the rod just before the collision to the angular momentum of the sphere about the pivot after the collision. The second point was not earned because the response does not substitute the expression for ω_f from part (a) into the angular momentum equation.

Part (c) earned 2 points. The first point was earned for including an upward normal force and a downward gravitational force acting on the sphere. The second point was earned for including a horizontal frictional force applied to the bottom of the sphere. Part (d) earned no points. The first point was not earned because the response does not use Newton's second law in the horizontal direction. The second point was not earned because the response does not derive an expression for the acceleration of the sphere. The third point was not earned because the response does not derive a correct expression for the velocity. The fourth point was not earned because the response does not use Newton's second law in rotational form. The fifth point was not earned because the response does not substitute the friction force into the expression for torque. The sixth point was not earned because the response does not substitute the angular acceleration into a correct kinematic equation for ω. Part (e) earned no points. The first point was not earned because the response does not indicate the final velocity is determined by the condition for rolling without sliding. The second point was not earned because the response does not substitute appropriate quantities into $v = R\omega$ or a kinematic equation that contains time. The third point was not earned because the response does not substitute the time from part (e)(i) into an expression for velocity.