2023

AP

AP® Physics C: Mechanics

Sample Student Responses and Scoring Commentary Set 2

Inside:

Free-Response Question 3

- ☑ Scoring Guidelines
- ☑ Student Samples
- ☑ Scoring Commentary

Question 3: Free-Response Question — 15 points

(a) For stating the parallel axis theorem — **1 point**

Example Response

$$I_{\text{blade}} = I_{\text{CM}} + Md^2$$

For using correct substitutions of the rotational inertia of one blade about its center of mass and substituting the distance from the center of mass — **1 point**

Example Response

$$I_{\text{blade}} = \frac{1}{18}ML^2 + M\left(\frac{L}{3}\right)^2$$

For multiplying the rotational inertia of one blade by 3 — **1 point**

Example Response

$$I_{\text{rotor}} = 3I_{\text{blade}} = \frac{1}{2}ML^2$$

Example Solution

$$I_{\text{blade}} = I_{\text{CM}} + Md^2$$

$$I_{\text{blade}} = \frac{1}{18}ML^2 + M\left(\frac{L}{3}\right)^2$$

$$I_{\text{blade}} = \frac{1}{6}ML^2$$

$$I_{\text{rotor}} = 3I_{\text{blade}} = \frac{1}{2}ML^2$$

Total for part (a) — 3 points

(b) For calculating the correct answer with correct units (2.4 s) — **1 point**

Example Solution

$$v = r\omega$$

$$v = L\omega_0$$

$$\frac{d}{t} = L\omega_0$$

$$t = \frac{d}{L\omega_0} = \frac{2\pi L}{L\omega_0} = \frac{2\pi}{\omega_0}$$

$$t = \frac{2\pi}{(2.6\ \text{rad/s})}$$

$$\therefore t = 2.4\text{ s}$$

Total for part (b) — 1 point

(c)(i) For indicating that the total initial rotational kinetic energy is dissipated **1 point**

Example Response

$$\Delta K_{\text{rot}} = E_{\text{dis}}$$
$$0 - \frac{1}{2} I \omega_0^{\,2} = E_{\text{dis}}$$

For substituting correct values for the rotational inertia and initial angular speed of the system **1 point**

Example Response

$$E_{\text{dis}} = -\frac{1}{2}\left(6.7\times10^{6}\ \text{kg}\cdot\text{m}^{2}\right)\left(2.6\ \text{rad/s}\right)^{2}$$

For an answer consistent with substitutions above and with correct units **1 point**

Example Response

$$E_{\text{dis}} = -2.3\times10^{7}\ \text{J}$$

Example Solution

$$\Delta K_{\text{rot}} = E_{\text{dis}}$$
$$0 - \frac{1}{2} I \omega_0^{\,2} = E_{\text{dis}}$$
$$E_{\text{dis}} = -\frac{1}{2}\left(6.7\times10^{6}\ \text{kg}\cdot\text{m}^{2}\right)\left(2.6\ \text{rad/s}\right)^{2}$$
$$E_{\text{dis}} = -2.3\times10^{7}\ \text{J}$$

Scoring Note: A response may earn full credit for positive or negative values of dissipated energy.

(c)(ii)	For using Newton's second law in rotational form	**1 point**
	For attempting to differentiate the equation for ω	**1 point**

Example Response

$$\tau = I_{\text{sys}} \frac{d}{dt}\left(\omega_0 e^{-\beta_0 t}\right)$$

For a correct expression for the torque on the system	**1 point**

Example Response

$$\tau = -\beta_0 I_{\text{sys}} \omega_0 e^{-\beta_0 t}$$

Example Solution

$$\tau = I\alpha$$

$$\tau = I_{\text{sys}} \frac{d\omega}{dt}$$

$$\tau = I_{\text{sys}} \frac{d}{dt}\left(\omega_0 e^{-\beta_0 t}\right)$$

$$\therefore \tau = -\beta_0 I_{\text{sys}} \omega_0 e^{-\beta_0 t}$$

(c)(iii)	For attempting to integrate the expression for angular speed	**1 point**

Example Response

$$\Delta\theta = \int \omega(t)\,dt$$

For using the correct limits of integration	**1 point**

Example Response

$$\Delta\theta = \int_0^t \omega_0 e^{-\beta_0 t}\,dt$$

For a correct expression for angular displacement	**1 point**

Example Response

$$\Delta\theta = \frac{\omega_0}{\beta_0}\left(1 - e^{-\beta_0 t}\right)$$

Example Solution

$$\Delta\theta = \int \omega(t)\,dt$$

$$\Delta\theta = \int_0^t \omega_0 e^{-\beta_0 t}\,dt$$

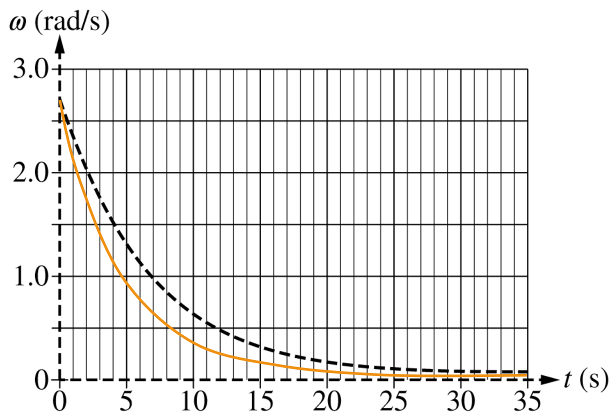
$$\Delta\theta = \left(-\frac{\omega_0}{\beta_0} e^{-\beta_0 t}\right)\Bigg|_0^t = -\frac{\omega_0}{\beta_0} e^{-\beta_0 t} + \frac{\omega_0}{\beta_0}(1)$$

$$\therefore \Delta\theta = \frac{\omega_0}{\beta_0}\left(1 - e^{-\beta_0 t}\right)$$

Total for part (c) 9 points

(d)	For drawing a continuous curve showing an exponential decay	**1 point**
	For starting at $\omega = 2.6$ rad/s and drawing a curve below the original curve	**1 point**

Example Solution



Total for part (d) 2 points

Total for question 3 15 points

Question 3

Begin your response to QUESTION 3 on this page.

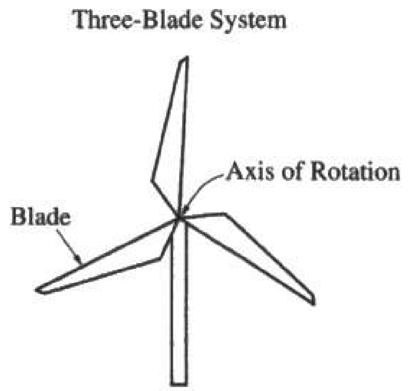


Figure 1

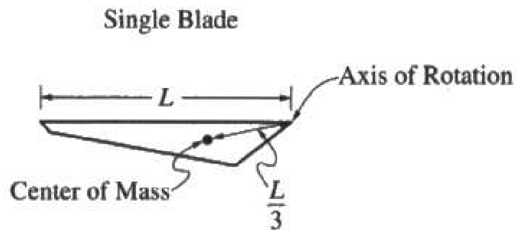


Figure 2

3. A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

(a) Derive an expression for the rotational inertia of the three-blade system. Express your answer in terms of M, L, and physical constants, as appropriate. The rotational inertia of each blade about an axis through its center of mass is given by the equation $I_{cm} = \frac{1}{18}ML^2$.

$I_{sys} = 3 \times I_{blade}$

$I_{blade} = I_{cm} + mx^2 \leftarrow$ parallel axis theorem

$= \frac{1}{18}ML^2 + M\left(\frac{L}{3}\right)^2$

$= \frac{1}{18}ML^2 + \frac{1}{9}ML^2$

$= \frac{1}{6}ML^2$

Thus, $I_{sys} = 3\left(\frac{1}{6}ML^2\right) = \boxed{\frac{1}{2}ML^2}$

  GO ON TO THE NEXT PAGE.

Question 3

Continue your response to **QUESTION 3** on this page.

(b) While the wind blows, the three-blade system operates at a constant angular speed $\omega_0 = 2.6$ rad/s. The length of one blade is $L = 36$ m. The numerical value of the rotational inertia of the system is $I_{sys} = 6.7 \times 10^6$ kg·m^2. Calculate the time T it takes the outer edge of a single blade to complete one revolution.

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2.6 \text{ rad/s}} = 2.42 \text{ seconds}$$

(c) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

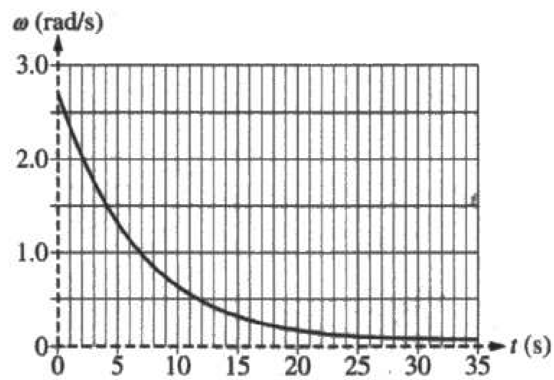


Figure 3

i. Calculate the amount of energy dissipated from $t = 0$, when the wind stops blowing, until the system comes to rest.

$$E_{loss} = K_{rotation} = \frac{1}{2}I\omega^2 = \frac{1}{2}(6.7 \times 10^6)(2.6)^2 = 2.26 \times 10^7 \text{ J}$$

Question 3

Continue your response to **QUESTION 3** on this page.

ii. Derive an expression for the net torque exerted on the system as a function of t as the system slows down. Express your answers in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.

$$\tau = I\alpha = I \cdot \frac{d}{dt}\omega(t)$$

$$= I_{sys} \frac{d}{dt}\left[\omega_0 e^{-\beta_0 t}\right]$$

$$= -\beta_0 I_{sys}\, \omega_0\, e^{-\beta_0 t}$$

iii. Derive an expression for the angular displacement of the system $\Delta\theta$ as a function of t. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.

$$\Delta\theta = \int_0^t \omega(x)\,dx$$

$$= \int_0^t \omega_0 e^{-\beta_0 x}\,dx = \left[\frac{-\omega_0}{\beta_0} e^{-\beta_0 x}\right]_0^t$$

$$= \frac{-\omega_0}{\beta_0}\left(e^{-\beta_0 t} - 1\right) = \boxed{\frac{\omega_0}{\beta_0}\left(1 - e^{-\beta_0 t}\right)}$$

The three-blade system is now replaced with a second three-blade system identical to the first, except that the second three-blade system slows down according to the equation $\omega = \omega_0 e^{-\beta t}$, where $\omega_0 = 2.6$ rad/s and $\beta > \beta_0$. The original angular speed function is shown as a dashed line in Figure 4.

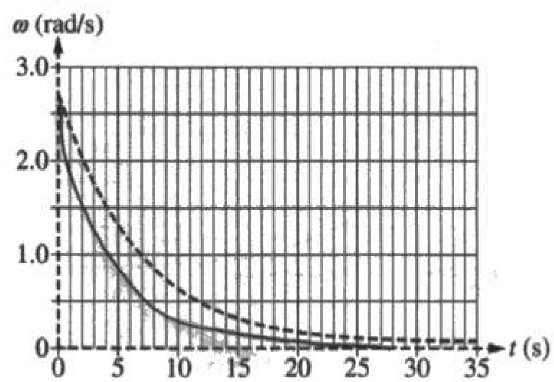


Figure 4

(d) On the graph in Figure 4, sketch the angular speed of the second three-blade system as a function of time t.

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 3

Begin your response to QUESTION 3 on this page.

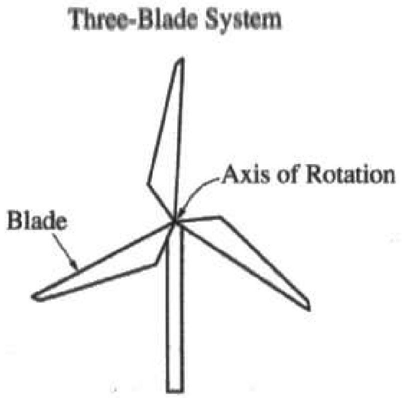


Figure 1

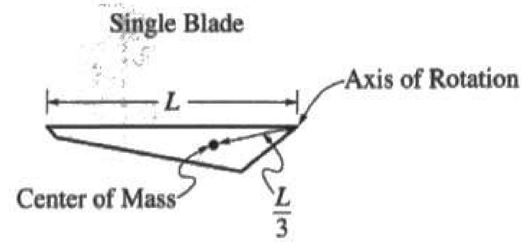


Figure 2

3. A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

(a) Derive an expression for the rotational inertia of the three-blade system. Express your answer in terms of M, L, and physical constants, as appropriate. The rotational inertia of each blade about an axis through its center of mass is given by the equation $I_{cm} = \frac{1}{18}ML^2$.

$I_{new} = I_{cm} + MD^2$

$\frac{1}{18}ML^2 + M\left(\frac{1}{3}L\right)^2$

$\frac{1}{18}ML^2 + \frac{1}{9}ML^2 = \frac{3}{18}ML^2$

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 3

Continue your response to QUESTION 3 on this page.

(b) While the wind blows, the three-blade system operates at a constant angular speed $\omega_0 = 2.6$ rad/s. The length of one blade is $L = 36$ m. The numerical value of the rotational inertia of the system is $I_{sys} = 6.7 \times 10^6$ kg·m^2. Calculate the time T it takes the outer edge of a single blade to complete one revolution.

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2.6} = 2.42$ seconds

(c) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

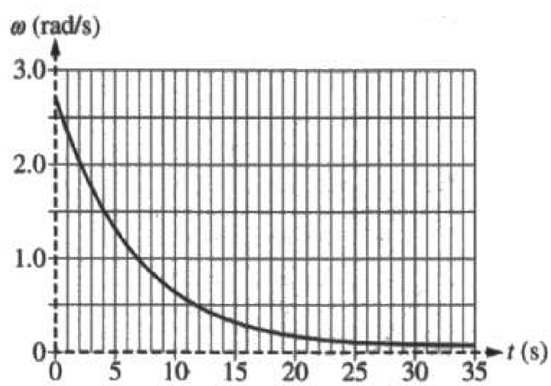


Figure 3

i. Calculate the amount of energy dissipated from $t = 0$, when the wind stops blowing, until the system comes to rest.

$K = \frac{1}{2} I \omega^2$

$\Delta E = K$

because when it stops $E = 0$

$= \frac{1}{2}(6.7 \times 10^6)(2.6)^2 = 22646000$ J

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 3

Continue your response to **QUESTION 3** on this page.

ii. Derive an expression for the net torque exerted on the system as a function of t as the system slows down. Express your answers in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.

$\alpha = \frac{\tau_{net}}{I}$ $\quad \Sigma\tau = \tau = I\alpha$

$\frac{W_0}{t} I_{sys} = \tau_{net}$ $\quad 0 = w_i + \alpha t$ $\quad \alpha = \frac{w_i}{t}$

iii. Derive an expression for the angular displacement of the system $\Delta\theta$ as a function of t. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.

$\theta_f = \theta_i + w_0 t + \frac{1}{2}\alpha t^2$ $\quad \Delta\theta = w_i t + \frac{1}{2} w_i t$

$\Delta\theta = w_i t + \frac{1}{2}\alpha t^2$

$\Delta\theta = W_0 t + \frac{1}{2}\frac{w_i}{t} t^2 =$

The three-blade system is now replaced with a second three-blade system identical to the first, except that the second three-blade system slows down according to the equation $\omega = \omega_0 e^{-\beta t}$, where $\omega_0 = 2.6$ rad/s and $\beta > \beta_0$. The original angular speed function is shown as a dashed line in Figure 4.

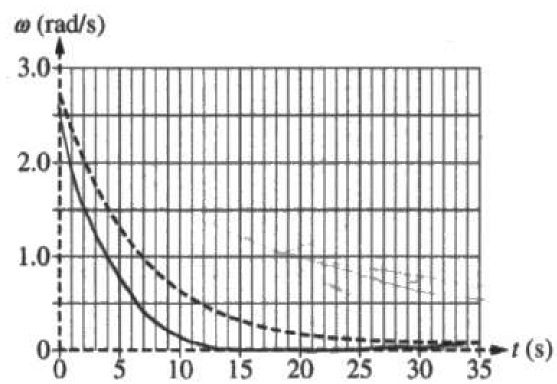


Figure 4

(d) On the graph in Figure 4, sketch the angular speed of the second three-blade system as a function of time t.

Question 3

Begin your response to QUESTION 3 on this page.

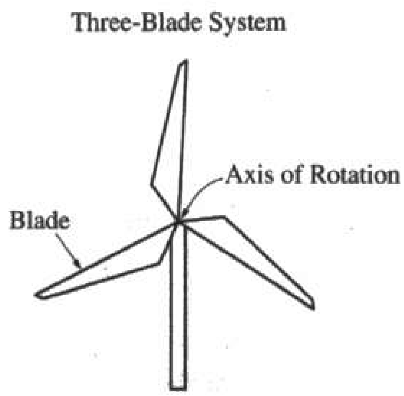


Figure 1

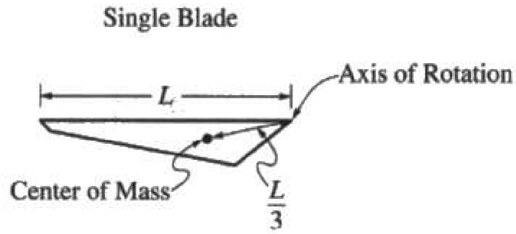


Figure 2

3. A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

(a) Derive an expression for the rotational inertia of the three-blade system. Express your answer in terms of M, L, and physical constants, as appropriate. The rotational inertia of each blade about an axis through its center of mass is given by the equation $I_{cm} = \frac{1}{18}ML^2$.

$I = \int r^2 dm$

$I_{cm} = \frac{1}{18} ML^2$

$3(I_{cm}) = 3\left(\frac{1}{18} ML^2\right) = \frac{3}{18} ML^2 = \boxed{\frac{1}{6} ML^2}$

$Cm_{sys} = X = 0$

axis of ~~rotation~~

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 3

Continue your response to **QUESTION 3** on this page.

(b) While the wind blows, the three-blade system operates at a constant angular speed $\omega_0 = 2.6$ rad/s. The length of one blade is $L = 36$ m. The numerical value of the rotational inertia of the system is $I_{sys} = 6.7 \times 10^6$ kg·m^2. Calculate the time T it takes the outer edge of a single blade to complete one revolution.

$\omega_0 = 2.6$ r/s
$L_{c.o.m} = 36$m
$I_{sys} = 6.7 \times 10^6$ kgm^2
$2\pi r = 2\pi(36) = 72\pi$ = total circumference of path. $= \theta$

$\theta = \omega t$
$= 2.6T$

$\frac{72\pi}{2.6} = \pi$
$= \boxed{86.9979\ s}$

(c) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

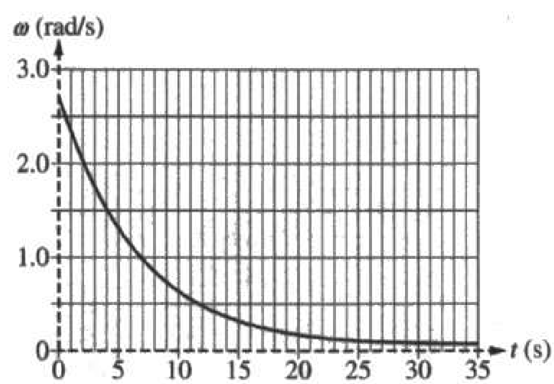


Figure 3

i. Calculate the amount of energy dissipated from $t = 0$, when the wind stops blowing, until the system comes to rest.

$\omega(0) = 2.7$
$\omega(35) = 0.1$

$2.7 - 0.1 = \boxed{2.6\ J}$

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 3

Continue your response to **QUESTION 3** on this page.

ii. Derive an expression for the net torque exerted on the system as a function of t as the system slows down. Express your answers in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.

$\tau = IR = I_{sys}L$

$I_{sys} = 6.7 \times 10^6 \text{ kgm}^2$

$R = 36\text{m} = L$

$\omega(t) = \omega_0 e^{-B_0 t}$

$\boxed{\tau = \omega_0 e^{-B_0 t} I_{sys} L}$

iii. Derive an expression for the angular displacement of the system $\Delta\theta$ as a function of t. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.

$\Delta\theta = \theta_0 + \omega_0 t + \frac{1}{2}at^2$

The three-blade system is now replaced with a second three-blade system identical to the first, except that the second three-blade system slows down according to the equation $\omega = \omega_0 e^{-\beta t}$, where $\omega_0 = 2.6$ rad/s and $\beta > \beta_0$. The original angular speed function is shown as a dashed line in Figure 4.

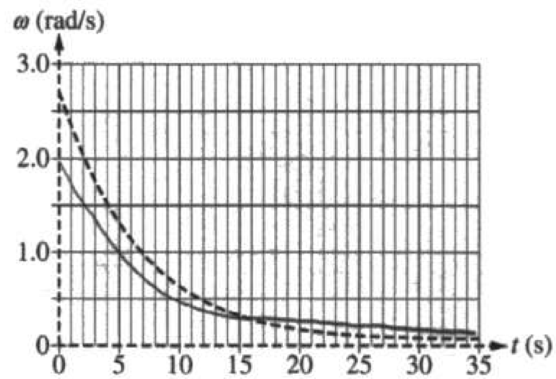


Figure 4

(d) On the graph in Figure 4, sketch the angular speed of the second three-blade system as a function of time t.

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Identify and use the parallel axis theorem to find one blade's rotational inertia for a point parallel to the center of mass.
- Determine the total rotational inertia of a three-blade system, using logical algebraic pathways.
- Recognize that time for one revolution is period and select relevant given values to calculate an unknown quantity with units.
- Relate amount of dissipated energy to the total initial rotational kinetic energy, select relevant given values, and calculate the unknown quantity with units.
- Derive an expression for torque using the appropriate equation and a derivative of the angular speed decay equation.
- Use integral calculus to derive angular displacement from the known exponential decay expression for angular speed.
- Sketch a graph of a new three-blade system's exponential decay where a variable was changed.

Sample: 3A
Score: 15

Part (a) earned 3 points. The first point was earned because the response correctly states the parallel axis theorem. The second point was earned because the response correctly substitutes both the rotational inertia of one blade about its center of mass and the distance from the center of mass. The third point was earned because the response correctly multiplies the rotational inertia of one blade by three to derive the rotational inertia of the three-blade system. Part (b) earned 1 point because the response correctly calculates the time T with correct units for one revolution of the outer edge of a blade. Part (c)(i) earned 3 points. The first point was earned because the response correctly indicates that the total rotational kinetic energy is dissipated. The second point was earned because the response correctly substitutes the correct values for rotational inertia and the initial angular speed into the rotational kinetic energy equation. The third point was earned because the response correctly calculates the numerical value for the energy dissipated with the appropriate units. Part (c)(ii) earned 3 points. The first point was earned because the response uses Newton's second law in rotational form. The second point was earned because the response correctly differentiates the equation for ω. The third point was earned because the response has a correct expression for the torque on the system. Part (c)(iii) earned 3 points. The first point was earned because the response correctly integrates the expression for angular speed. The second point was earned because the response shows correct limits of integration. The third point was earned because the response has a correct expression for angular displacement. Part (d) earned 2 points. The first point was earned because the response correctly graphs a continuous curve showing exponential decay. The second point was earned because the response correctly starts at ω=2.6 rad/s and is drawn below the original curve.

Question 3 (continued)

Sample: 3B
Score: 9

Part (a) earned 2 points. The first point was earned because the response correctly states the parallel axis theorem. The second point was earned because the response correctly substitutes both the rotational inertia of one blade about its center of mass and the distance from the center of mass. The third point was not earned because the response does not multiply the rotational inertia of one blade by three to derive the rotational inertia of the three-blade system. Part (b) earned 1 point because the response correctly calculates the time T with correct units for one revolution of the outer edge of a blade. Part (c)(i) earned 3 points. The first point was earned because the response correctly indicates that the total rotational kinetic energy is dissipated. The second point was earned because the response correctly substitutes the correct values for rotational inertia and the initial angular speed into the rotational kinetic energy equation. The third point was earned because the response correctly calculates the numerical value for the energy dissipated with the appropriate units. Part (c)(ii) earned 1 point. The first point was earned because the response uses Newton's second law in rotational form. The second point was not earned because the response does not differentiate the equation for $\boldsymbol{\omega}$. The third point was not earned because the response does not have the correct expression for the torque on the system. Part (c)(iii) earned 0 points. The first point was not earned because the response does not integrate the expression for angular speed. The second point was not earned because the response does not show correct limits of integration. The third point was not earned because the response does not have a correct expression for angular displacement. Part (d) earned 2 points. The first point was earned because the response correctly graphs a continuous curve showing exponential decay. The second point was earned because the response correctly starts at $\boldsymbol{\omega}$=2.6 rad/s and is drawn below the original curve.

Sample: 3C
Score: 2

Part (a) earned 1 point. The first point was not earned because the response does not state the parallel axis theorem. The second point was not earned because the response does not substitute both the rotational inertia of one blade about its center of mass and the distance from the center of mass. The third point was earned because the response multiplies the rotational inertia of one blade by three. Part (b) earned 0 points because the response does not calculate the time T for one revolution of the outer edge of a blade. Part (c)(i) earned 0 points. The first point was not earned because the response does not indicate that the total rotational kinetic energy is dissipated. The second point was not earned because the response does not substitute the correct values for rotational inertia and the initial angular speed into the rotational kinetic energy equation. The third point was not earned because the response does not calculate the numerical value for the energy dissipated. Part (c)(ii) earned 0 points. The first point was not earned because the response does not use Newton's second law in rotational form. The second point was not earned because the response does not differentiate the equation for $\boldsymbol{\omega}$. The third point was not earned because the response does not have the correct expression for the torque on the system. Part (c)(iii) earned 0 points. The first point was not earned because the response does not integrate the expression for angular speed. The second point was not earned because the response does not show correct limits of integration. The third point was not earned because the response does not have a correct expression for angular displacement. Part (d) earned 1 point. The first point was earned because the response correctly graphs a continuous curve showing exponential decay. The second point was not earned because the response does not start at $\boldsymbol{\omega}$=2.6 rad/s nor is it drawn entirely below the original curve.