2023

AP® Physics C: Mechanics

Scoring Guidelines

Set 1

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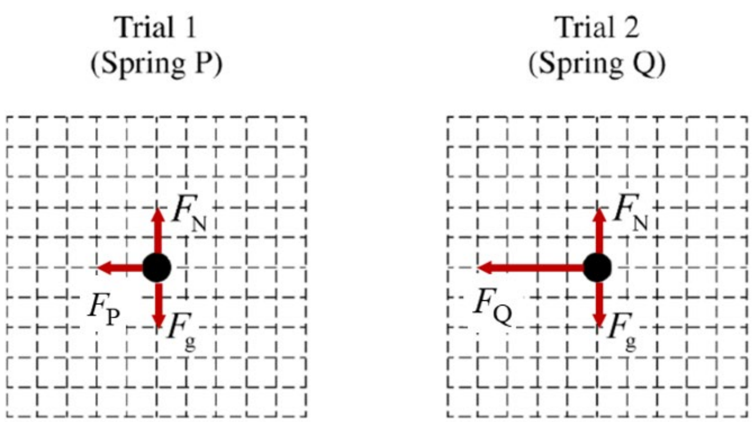
Question 1: Free-Response Question — 15 points

(a)	For correctly drawing and labeling the force of gravity and the normal force on both dots	**1 point**
	For drawing and labeling the spring force to the left on both dots, where the force for Spring Q (Trial 2) is twice as long as the force for Spring P (Trial 1)	**1 point**

Example Solution



Scoring Note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force", "F Earth on block", "F on block by Earth", $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force", "ground force", or similar labels may be used for the normal force.

Scoring Note: A response that includes extraneous vectors can earn a maximum of 1 point.

Total for part (a) — 2 points

(b)(i)	For a statement that the work is equal to the area under the curve	**1 point**
(b)(ii)	For stating the spring compression will be greater than 0.040 m with an attempt at a relevant justification	**1 point**
	For a justification that indicates that the heights are equal when the area between each function and the horizontal axis are equal, which happens after $x = 0.040 \text{ m}$	**1 point**

Scoring Note: While a mathematical solution is not required to earn credit for this point, students may reference a mathematical solution.

Example Solution

The work done by each spring is equal to the area under their respective curves. This work is converted to the change in potential energy and therefore relates to the maximum height reached by the block. At $x = 0.040 \text{ m}$, the area under the curve for Spring Q is greater than that of Spring P. Therefore, it is not until a compression greater than $x = 0.040 \text{ m}$ that the areas under the two curves are equal, have the same work, and convert to the same maximum height.

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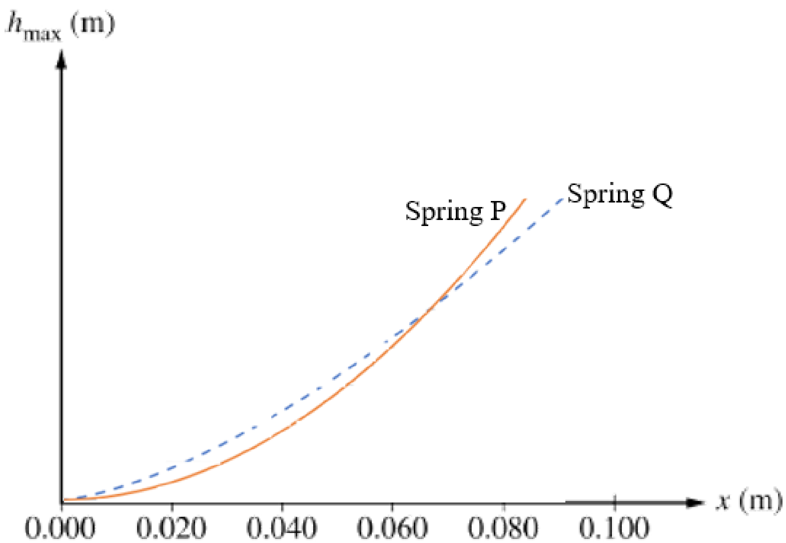
(b)(iii)	For drawing smooth concave upward curves for both springs P and Q	**1 point**
	For showing that the curves intersect after $x = 0.05$ m, but before $x = 0.10$ m	**1 point**
	For showing the curve for Spring P below Spring Q before an intersection and the curve for Spring P above Spring Q after this intersection	**1 point**

Example Solution



Total for part (b) **6 points**

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(c)(i)	For correctly applying conservation of energy to solve for the velocity of Block A just before it collides with Block B	**1 point**

Example Response

$$m_A gH = \frac{1}{2} m_A v_A^2$$
$$v_A = \sqrt{2gH}$$
$$v_A = \sqrt{2\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)(0.75\text{ m})}$$
$$\therefore v_A = 3.8\text{ m/s}$$

For substituting the calculated value for v_A into a correct conservation of linear momentum expression and solving for v_f	**1 point**

Example Response

$$m_A v_A = (m_A + m_B)v_f$$
$$v_f = \frac{m_A v_A}{(m_A + m_B)}$$
$$v_f = \frac{(0.120\text{ kg})(3.8\text{ m/s})}{(0.120\text{ kg} + 0.070\text{ kg})}$$
$$\therefore v_f = 2.4\text{ m/s}$$

Example Solution

$$m_A gH = \frac{1}{2} m_A v_A^2$$
$$v_A = \sqrt{2gH}$$
$$v_A = \sqrt{2\left(9.8\,\frac{m}{s^2}\right)(0.75\,m)}$$
$$\therefore v_A = 3.8\text{ m/s}$$

$$m_A v_A = (m_A + m_B)v_f$$
$$v_f = \frac{m_A v_A}{(m_A + m_B)}$$
$$v_f = \frac{(0.120\text{ kg})(3.8\text{ m/s})}{(0.120\text{ kg} + 0.070\text{ kg})}$$
$$\therefore v_f = 2.4\text{ m/s}$$

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(c)(ii)	For equating kinetic energy of the two-block system and elastic potential energy of the spring	**1 point**

Example Response

$$\frac{1}{2}mv^2 = \int F(x)\,dx$$

For correctly integrating to determine the elastic potential energy at the spring's maximum compression **1 point**

Example Response

$$\frac{1}{2}mv^2 = \int_0^{x_{max}} Cx^{1/2}dx$$

$$\frac{1}{2}mv^2 = \frac{2}{3}Cx_{max}{}^{3/2}$$

For substituting the value for v_f from part (c)(i) into the integrated equation and solving for x_{max} **1 point**

Example Response

$$x_{max} = \left(\frac{3}{4}\frac{mv^2}{C}\right)^{2/3}$$

$$x_{max} = \left(\frac{3}{4}\frac{(0.190\text{ kg})(2.4\text{ m/s})^2}{20.0\text{ N/m}^{1/2}}\right)^{2/3}$$

Example Solution

$$K = U_s$$

$$\frac{1}{2}mv^2 = \int F(x)\,dx$$

$$\frac{1}{2}mv^2 = \int_0^{x_{max}} Cx^{1/2}dx$$

$$\frac{1}{2}mv^2 = \frac{2}{3}Cx_{max}{}^{3/2}$$

$$\rightarrow x_{max} = \left(\frac{1}{2}mv^2\frac{3}{2C}\right)^{2/3}$$

$$x_{max} = \left(\frac{3}{4}\frac{mv^2}{C}\right)^{2/3}$$

$$x_{max} = \left(\frac{3}{4}\frac{(0.190\text{ kg})(2.4\text{ m/s})^2}{20.0\text{ N/m}^{1/2}}\right)^{2/3}$$

$$\therefore x_{max} = 0.12\text{ m}$$

Total for part (c) 5 points

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(d)	For selecting $C > D$ with an attempt at a relevant justification	**1 point**
	For one of the following: • a statement that correctly relates the total energy of the system, the maximum compression distance of the springs, and the spring constant • a statement that correctly relates the force exerted on the blocks by the spring, the maximum compression distance of the springs, and the spring constant	**1 point**

Example Solutions

The energy of the two-block-spring system before the blocks compress the spring is the same in both procedures, so the total potential energy of both springs must be the same when each spring is at its maximum compression. Since Spring Q is compressed less than Spring R, C must be greater than D.

OR

The blocks are traveling at the same speed before colliding with the spring in each procedure. The maximum compression of Spring Q is less than Spring R, so the average force exerted on the blocks by Spring Q to stop the blocks must be greater than that of Spring R. Therefore, C must be greater than D.

Total for part (d) **2 points**

Total for question 1 **15 points**

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Question 2: Free-Response Question — 15 points

(a) For indicating the rotational inertia is the sum of the rotational inertia for all the stacked disks — **1 point**

Example Response

$$I_{eq} = \sum_{i=1}^{N} I_i$$

$$I_{eq} = N\left(\frac{1}{2}MR^2\right)$$

For an expression for the period consistent with the previous rotational inertia expression — **1 point**

Example Response

$$T = 2\pi\sqrt{\frac{N\left(\frac{1}{2}MR^2\right)}{\kappa}}$$

Example Solution

$$I_{eq} = \sum_{i=1}^{N} I_i$$

$$I_{eq} = N\left(\frac{1}{2}MR^2\right)$$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

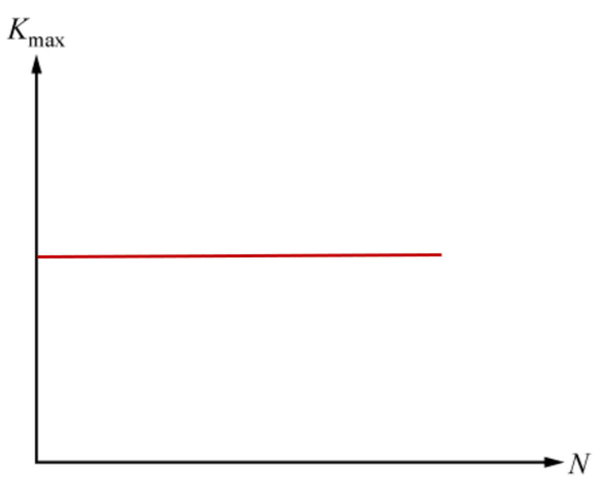
$$T = 2\pi\sqrt{\frac{N\left(MR^2\right)}{2\kappa}}$$

Total for part (a) — 2 points

(b) For a sketch that begins at a non-zero value — **1 point**

For a sketch that is constant with slope equal to zero — **1 point**

Example Solution



Total for part (b) — 2 points

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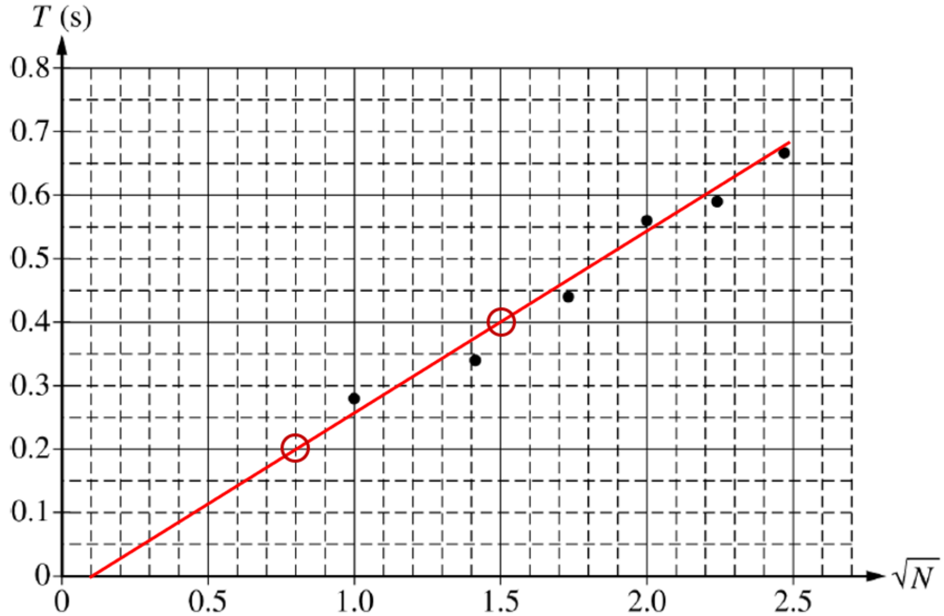
(c)(i)	For drawing an appropriate line or curve of best fit that approximates the data	1 point

Example Solution



(c)(ii)	For using two points on the line to calculate the slope	**1 point**

Example Response

$$\text{slope} = \frac{T_2 - T_1}{\sqrt{N_2} - \sqrt{N_1}}$$

$$\text{slope} = \frac{0.4\,\text{s} - 0.2\,\text{s}}{1.5 - 0.8} = 0.29\,\text{s}$$

Scoring Note: Slope values may range from .22 s to .33 s.

For correctly relating the slope to the period of the torsional pendulum consistent with part (a)	**1 point**

Example Response

$$T = 2\pi\sqrt{\frac{NI}{\kappa}}$$

$$\rightarrow \text{slope} = 2\pi\sqrt{\frac{I}{\kappa}}$$

For substituting the slope into the equation to determine the mass of the disk	**1 point**

Example Response

$$\rightarrow M = \frac{\kappa(\text{slope})^2}{2\pi^2 R^2}$$

Example Solution

$$T = 2\pi\sqrt{\frac{NI}{\kappa}}$$

$$\rightarrow \text{slope} = 2\pi\sqrt{\frac{I}{\kappa}}$$

$$(\text{slope})^2 = 4\pi^2\frac{I}{\kappa}$$

$$(\text{slope})^2 = 4\pi^2\frac{\left(\frac{1}{2}MR^2\right)}{\kappa}$$

$$\rightarrow M = \frac{\kappa(\text{slope})^2}{2\pi^2 R^2}$$

$$M = \frac{(1.6\,\text{N}\cdot\text{m})\left(\frac{0.4\,\text{s} - 0.2\,\text{s}}{1.5 - 0.8}\right)^2}{2\pi^2(0.2\,\text{m})^2}$$

$$\therefore M = 0.17\,\text{kg}$$

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(c)(iii)	For indicating a source of error that could correctly explain the observed difference with an attempt at a relevant justification	**1 point**
	For a correct justification that links the source of experimental error to the larger experimental value for M. Accept one of the following: • Experimental uncertainties for the period, for example the period is measured to be larger • The mass is more concentrated to the edge, or some other distribution that results in a larger rotational inertia **Scoring Note:** Responses that indicate the given κ is too large or the given radius is too small may earn the second point.	**1 point**
	Example Solution *The experimental value of the mass could be too large because the period measured by the student is too large.* **OR** *The experimental value of the mass could be too large because the mass of the disk is concentrated at the edge of the disk, causing the rotational inertia of the disk to be larger than that used to determine the experimental value of mass.*	
	Total for part (c)	**6 points**
(d)(i)	For indicating that the slope would be greater with an attempt at a relevant justification	**1 point**
	For indicating that using disks with densities that increase with r will increase the rotational inertia	**1 point**
	For indicating the functional relationship between slope and rotational inertia: $\sqrt{I} \propto slope$	**1 point**
	Example Solution *The disks with a density that increases towards the edge of the disk will have a greater proportion of their mass farther from the axis of rotation, so their rotational inertia will be larger than that of a uniform disk. Therefore, the slope of the line will be greater because the slope is proportional to* $\sqrt{I}$.	

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(d)(ii)	For selecting $\omega_U > \omega_{non\text{-}U}$ with an attempt at a relevant justification, or a selection consistent with part (d)(i)	**1 point**
	For using energy conservation to justify the relationship	**1 point**

Example Solution

Energy in a torsion pendulum is conserved, $\frac{1}{2}I\omega^2 = \frac{1}{2}\kappa\theta^2$*, where* $\frac{1}{2}\kappa\theta^2$ *is a constant.*

Therefore, since $I_U < I_{non\text{-}U}$, $\omega_U > \omega_{non\text{-}U}$ *to keep energy conserved.*

Total for part (d) 5 points

Total for question 2 15 points

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Question 3: Free-Response Question — 15 points

(a) For using a correct expression for conservation of energy of the rod-Earth system **1 point**

Example Response

$$\Delta U + \Delta K = 0$$

$$\left(0 - Mgh_{\text{CM}}\right) + \left(\frac{1}{2} I\omega_f^2 - 0\right) = 0$$

$$Mgh_{\text{cm}} = \frac{1}{2} I\omega_f^2$$

For correctly substituting h and I into the correct energy expression **1 point**

Example Response

$$Mg\frac{\ell}{2} = \frac{1}{2}\left(\frac{1}{3}M\ell^2\right)\omega_f^2$$

Example Solution

$$\Delta U + \Delta K = 0$$

$$\left(0 - Mgh_{\text{CM}}\right) + \left(\frac{1}{2} I\omega_f^2 - 0\right) = 0$$

$$Mgh_{\text{cm}} = \frac{1}{2} I\omega_f^2$$

$$Mg\frac{\ell}{2} = \frac{1}{2}\left(\frac{1}{3}M\ell^2\right)\omega_f^2$$

$$\therefore \omega_f = \sqrt{\frac{3g}{\ell}}$$

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Alternate Solution **1 point**

For using a correct expression of Newton's second law in rotational form

Alternate Example Response

$$\tau_{net} = I\alpha$$

For correctly substituting expressions for net torque and rotational inertia of the rod **1 point**

Alternate Example Response

$$\frac{\ell}{2}Mg\cos\theta = \frac{1}{3}M\ell^2\alpha$$

Alternate Example Solution

$$\tau_{net} = I\alpha$$

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \frac{d\omega}{d\theta}\omega$$

$$\frac{\ell}{2}Mg\cos\theta = \frac{1}{3}M\ell^2\alpha$$

$$\frac{\ell}{2}Mg\cos\theta = \frac{1}{3}M\ell^2\frac{d\omega}{d\theta}\omega$$

$$\frac{\ell}{2}Mg\cos\theta d\theta = \frac{1}{3}M\ell^2\omega d\omega$$

$$\int_0^{\frac{\pi}{2}}\frac{\ell}{2}Mg\cos\theta d\theta = \int_0^{\omega_f}\frac{1}{3}M\ell^2\omega d\omega$$

$$\frac{\ell}{2}Mg\left[\sin\frac{\pi}{2} - \sin 0\right] = \frac{1}{3}M\ell^2\left[\frac{1}{2}\omega_f^2 - 0\right]$$

$$3\frac{g}{\ell}[1-0] = \left[\omega_f^2\right]$$

$$\omega_f = \sqrt{\frac{3g}{\ell}}$$

Scoring Note: *The full integration is not needed to earn points but is presented for clarity.*

Total for part (a) 2 points

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(b)	For using a correct expression for conservation of angular momentum	**1 point**

Example Response

$$L_\text{i} = L_\text{f}$$
$$I\omega = mvr$$

For correctly substituting the expression for ω_f from part (a) **1 point**

Example Response

$$\frac{2}{3}m\ell^2\sqrt{\frac{3g}{\ell}} = mv_0\ell$$

Example Solution

$$L_\text{i} = L_\text{f}$$
$$I\omega = mvr$$
$$\frac{2}{3}m\ell^2\sqrt{\frac{3g}{\ell}} = mv_0\ell$$
$$\therefore v_0 = \sqrt{\frac{4}{3}g\ell}$$

Scoring Note: The last equation is not needed for scoring the item but is presented for clarity.

Total for part (b) 2 points

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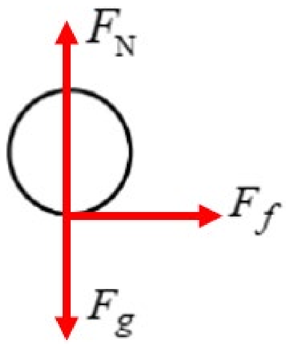
(c)	For correctly drawing and labeling the force due to gravity and the normal force on the sphere	**1 point**
	For drawing the frictional force horizontally to the right at the bottom of the sphere	**1 point**

Example Solution



Scoring note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force", "F Earth on sphere", "F on sphere by Earth", $F_{\text{Earth on sphere}}$, $F_{\text{E,Sphere}}$, $F_{\text{Sphere,E}}$. The labels G or g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force", "ground force", or similar labels may be used for the normal force.

Scoring Note: A response that includes extraneous vectors can earn a maximum of 1 point.

Scoring Note: Horizontally displacing the F_N and F_g vectors slightly is permitted in order to show the distinct points at which those forces are exerted on the sphere.

Total for part (c) 2 points

(d)(i)	For using Newton's second law in the horizontal direction	**1 point**

Example Response

$\Sigma F = ma$

$-\mu mg = ma$

For a correct derivation of acceleration	**1 point**

Example Response

$a = -\mu g$

For a correct expression for the velocity	**1 point**

Example Response

$\therefore v = v_0 - \mu gt$

Example Solution

$\Sigma F = ma$

$-\mu mg = ma$

$a = -\mu g$

$v = v_0 + at$

$\therefore v = v_0 - \mu gt$

Scoring Note: Only the final expression for velocity must have correct signs.

(d)(ii)	For using Newton's second law in rotational form with substitutions for rotational inertia and torque	**1 point**

Example Response

$\tau = I\alpha$

$F_f R = \frac{2}{5} m R^2 \alpha$

For correctly substituting for friction and solving for α — **1 point**

Example Response

$F_f = \mu m g$

$\alpha = \frac{5\mu g}{2R}$

For correctly substituting α into a rotational kinematic equation and solving for ω — **1 point**

Example Response

$\omega = \omega_0 + \alpha t$

$\therefore \omega = \frac{5\mu g}{2R} t$

Example Solution

$\tau = I\alpha$

$F_f R = \frac{2}{5} m R^2 \alpha$

$F_f = \mu m g$

$\alpha = \frac{5\mu g}{2R}$

$\omega = \omega_0 + \alpha t$

$\therefore \omega = \frac{5\mu g}{2R} t$

Total for part (d) 6 points

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(e)(i)	For indicating the linear speed is equal to $R\omega$ when slipping stops at Point B	**1 point**

Example Response

$v = R\omega$

For correctly substituting v and ω from parts (d)(i) and (d)(ii) **1 point**

Scoring Note: Substituting the acceleration from part (d)(i) into a valid kinematic equation that includes time can earn this point.

Example Response

$$v_0 - \mu g t = R\frac{5\mu g}{2R}t$$

Example Solution

$$v = R\omega$$

$$v_0 - \mu g t = R\frac{5\mu g}{2R}t$$

$$\therefore t = \frac{2v_0}{7\mu g}$$

(e)(ii)	For correctly substituting the expression for time from (e)(i) into the expression for velocity in (d)(i)	**1 point**

Example Solution

$$v = v_0 - \mu g t$$

$$v = v_0 - \mu g \frac{2v_0}{7\mu g}$$

$$\therefore v = \frac{5}{7}v_0$$

Scoring Note: The last equation is not needed for scoring the item but is presented for clarity.

Total for part (e) 3 points

Total for question 3 15 points