2023

AP® Physics C: Mechanics

Scoring Guidelines
Set 2

Question 1: Free-Response Question — 15 points

(a)(i)	For stating that the area bounded by the curve is the displacement	**1 point**
(a)(ii)	For indicating momentum is conserved	**1 point**

Example Response

$$\Sigma p_0 = \Sigma p_f$$

$$m_A v_{A0} + m_B v_{B0} = m_A v_{Af} + m_B v_{Bf}$$

$$m_A v_{A0} - m_A v_{Af} = m_B v_{Bf}$$

For the correct answer for speed with units (2 m/s) — **1 point**

Example Solution

$$m_A v_{A0} + m_B v_{B0} = m_A v_{Af} + m_B v_{Bf}$$

$$m_A v_{A0} - m_A v_{Af} = m_B v_{Bf}$$

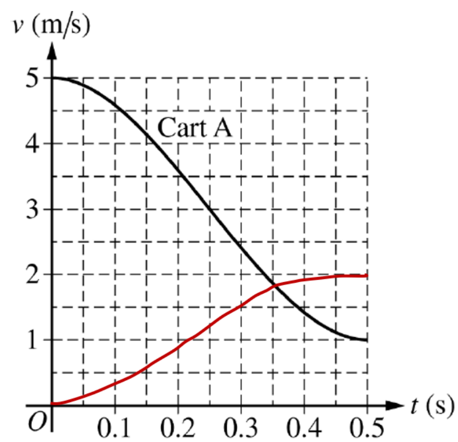
$$v_{Bf} = \frac{m_A}{m_B}(v_{A0} - v_{Af})$$

$$\rightarrow v_{Bf} = \frac{1000\text{ kg}}{2000\text{ kg}}(5\text{ m/s} - 1\text{ m/s})$$

$$\therefore v_{Bf} = 2\text{ m/s}$$

(a)(iii)	For a graph that starts at $(0,0)$ and ends at $(0.5, 2)$ or value consistent with (a)(ii)	**1 point**
	For a smooth, continuous curve that transitions from concave up to concave down	**1 point**

Example Solution



Total for part (a) — **5 points**

(b)(i)	For indicating the derivative of the velocity function is the acceleration of the cart	**1 point**

Example Response

$$a(t) = \frac{dv}{dt}$$

$$\rightarrow a(t) = \frac{d(64t^3 - 48t^2 + 5)}{dt}$$

$$\therefore a(t) = 192t^2 - 96t$$

For setting the derivative of the previously derived expression for acceleration equal to zero	**1 point**

Example Response

$$\frac{d}{dt}a(t) = 0$$

$$0 = 384t - 96$$

For indicating the maximum acceleration is $12\ \text{m/s}^2$ or maximum acceleration occurs at time $t = 0.25\ \text{s}$	**1 point**

Example Response

$$0 = 384t - 96$$

$$\therefore t_{max} = 0.25\ \text{s}$$

For substituting the time at which the acceleration is a maximum or the maximum acceleration into an expression of Newton's second law to calculate the value of the maximum force	**1 point**

Example Response

$$F(t) = ma(t)$$

$$F_{max} = ma(0.25\ \text{s})$$

Example Solution

$$a(t) = \frac{dv}{dt}$$

$$\rightarrow a(t) = \frac{d(64t^3 - 48t^2 + 5)}{dt}$$

$$\therefore a(t) = 192t^2 - 96t$$

$$\frac{d}{dt}a(t) = 0$$

$$\frac{d\left(192t^2 - 96t\right)}{dt} = 0$$

$$\rightarrow 0 = 384t - 96$$

$$\therefore t_{\text{max}} = 0.25 \text{ s}$$

$$F(t) = ma(t)$$

$$F_{\text{max}} = ma(0.25 \text{ s})$$

$$F_{\text{max}} = (1000 \text{ kg})\left(192(0.25 \text{ s})^2 - 96(0.25 \text{ s})\right)$$

$$\therefore \left|F_{\text{max}}\right| = 12{,}000 \text{ N}$$

Alternate Solution

For indicating the derivative of the momentum function is the force exerted on the cart	***1 point***

Alternate Example Response

$$F(t) = \frac{dp}{dt}$$

$$F(t) = \frac{d}{dt}(64000t^3 - 48000t^2 + 5000)$$

For setting the derivative of the previously derived expression for force equal to zero	***1 point***

Alternate Example Response

$$\frac{d}{dt}F(t) = 0$$

$$\frac{d}{dt}\left(192000t^2 - 96000t\right) = 0$$

For indicating the maximum force occurs at time $t = 0.25\text{ s}$	***1 point***

Alternate Example Response

$$0 = 384000t - 96000$$

$$\therefore t_{\text{max}} = 0.25\text{ s}$$

For substituting the time at which the force is at a maximum into an expression for momentum to calculate the value of the maximum force	***1 point***

Alternate Example Response

$$F(t) = \frac{d}{dt}p(t)$$

$$F_{\text{max}} = \frac{d}{dt}p(0.25\text{ s})$$

Alternate Example Solution

$p(t) = mv(t)$

$\rightarrow\ p(t) = (1000\ \text{kg})(64t^3 - 48t^2 + 5)$

$\therefore\ p(t) = 64000t^3 - 48000t^2 + 5000$

$F(t) = \dfrac{dp}{dt}$

$\rightarrow\ F(t) = \dfrac{d}{dt}(64000t^3 - 48000t^2 + 5000)$

$\therefore\ F(t) = 192000t^2 - 96000t$

$\dfrac{d}{dt}F(t) = 0$

$\dfrac{d}{dt}\left(192000t^2 - 96000t\right) = 0$

$\rightarrow\ 0 = 384000t - 96000$

$\therefore\ t_{\text{max}} = 0.25\ \text{s}$

$F(t) = \dfrac{d}{dt}\,p(t)$

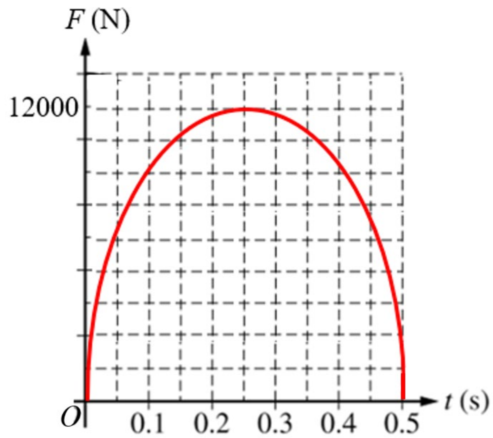
$F_{\text{max}} = \dfrac{d}{dt}\,p(0.25\ \text{s})$

$F_{\text{max}} = 192000(0.25\ \text{s})^2 - 96000(0.25\ \text{s})$

$\therefore\ |F_{\text{max}}| = 12{,}000\ \text{N}$

(b)(ii)	For a curve that increases and then decreases in value that is only concave down	**1 point**
	For a labeled maximum value consistent with the calculated value in part (b)(i)	**1 point**
	For the graph having values of 0 N at $t = 0\text{ s}$ and $t = 0.5\text{ s}$	**1 point**

Example Solution



	Total for part (b)	**7 points**
(c)	For selecting $F_1 < F_2$ with an attempt at a relevant justification	**1 point**
	For indicating that the impulse or change in momentum of each cart in both collisions is the same	**1 point**
	For indicating that decreasing the time of collision means the average force must be greater	**1 point**

Example Solution

Since the initial and final velocities are the same for both collisions, Δp is the same for both collisions; as a result, the impulse is the same for both collisions. So, if Δt is smaller, F_{avg} is larger.

Total for part (c)	**3 points**
Total for question 1	**15 points**

Question 2: Free-Response Question — 15 points

(a) For indicating the equivalent spring constant k_{eq} is the sum of the spring constants of all the springs arranged in parallel — **1 point**

Example Response

$k_{eq} = Nk$

For a correct expression for period consistent with k_{eq} above — **1 point**

Example Response

$T = 2\pi\sqrt{\frac{m}{Nk}}$

Example Solution

$$k_{eq} = \sum_{i=1}^{N} k_i$$

$$k_{eq} = Nk$$

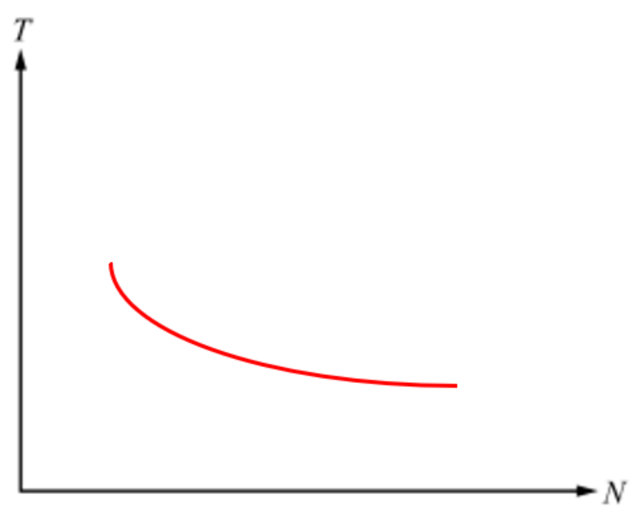
$$T = 2\pi\sqrt{\frac{m}{Nk}}$$

Total for part (a) — 2 points

(b) For a graph that increases or decreases with N consistent with the expression derived in part (a) — **1 point**

For a concavity that is consistent with the expression derived in part (a) — **1 point**

Example Solution



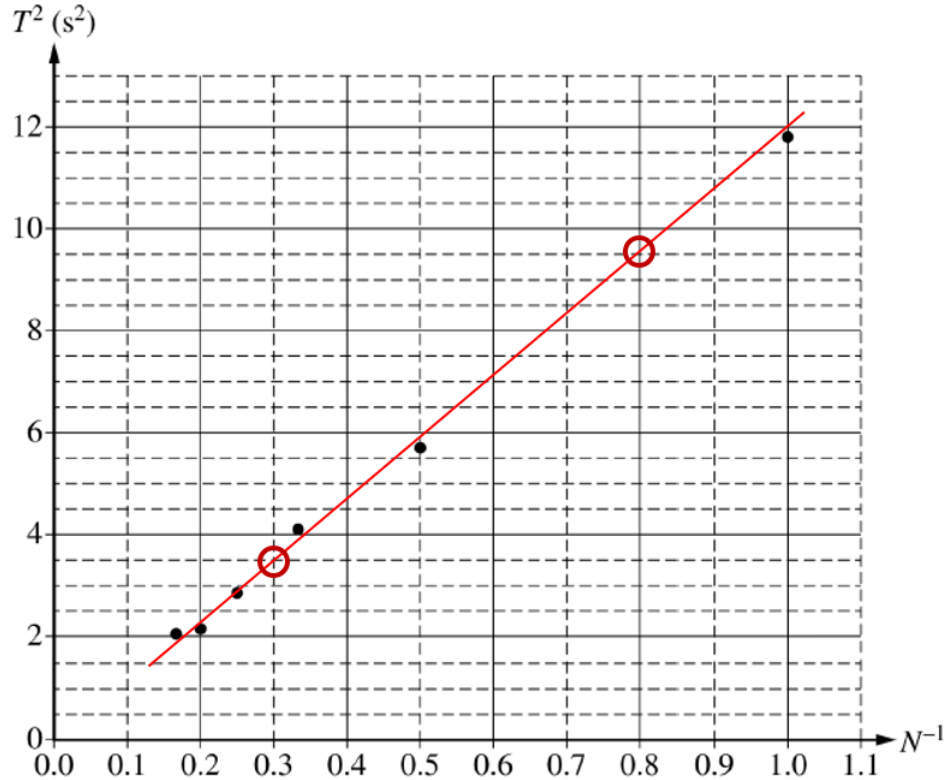
Scoring Note: The student is not required to label $N = 1$ on the horizontal axis, so the curve can start at the vertical axis, implying that $N = 1$ is at the origin.

Scoring Note: Discrete points following the appropriate curve will earn this point.

Total for part (b) — 2 points

(c)(i)	For drawing an appropriate line of best fit that approximates the data	**1 point**

Example Solution



(c)(ii) For using two points from the best-fit line to calculate slope **1 point**

Scoring Note: Using data points that fall on the best-fit line is acceptable.

Example Response

$$\text{slope} = \frac{\Delta\left(T^2\right)}{\Delta\left(N^{-1}\right)}$$

$$\text{slope} = \frac{(9.5\ \text{s}^2 - 3.5\ \text{s}^2)}{(0.8 - 0.3)}$$

$$\therefore \text{slope} = 12\ \text{s}^2$$

For relating the slope of the line of the T^2 vs. N^{-1} graph consistent with the expression derived in part (a) **1 point**

Example Response

$$T^2 = 4\pi^2 \frac{m}{Nk}$$

$$\text{slope} = T^2 N$$

$$\text{slope} = 4\pi^2 \frac{m}{k}$$

For a calculated answer that has units of N/m **1 point**

Example Response

$k = 4.93$ N/m

Example Solution

$$T^2 = 4\pi^2 \frac{m}{Nk}$$

$$\text{slope} = 4\pi^2 \frac{m}{k}$$

$$k = \frac{4\pi^2 m}{\text{slope}}$$

$$k = \frac{4\pi^2 (1.5\ \text{kg})}{\frac{(9.5\ \text{s}^2 - 3.5\ \text{s}^2)}{(0.8 - 0.3)}}$$

$$\rightarrow k = \frac{4\pi^2 (1.5\ \text{kg})}{12\ \text{s}^2}$$

$k = 4.93$ N/m

(c)(iii)	For indicating a source of error that could result in the observed difference with an attempt at a relevant justification Examples include: • Experimental uncertainties in the mass of the system • Motion detector miscalibration • Timing error due to reaction time	**1 point**
	For a correct justification that links the source of experimental error to the smaller experimental value of k Examples include: • Experimental uncertainties in the mass of the system (e.g., mass of block is too small, not accounting for the mass of the spring) • Calibration of the motion sensor produces a graph from which too large a period is measured • A larger period due to measurement error will result in a smaller value of k	**1 point**
	Total for part (c)	**6 points**
(d)(i)	For indicating that the slope would not change with an attempt at a relevant justification	**1 point**
	For a justification that the period depends on the mass and spring constant which remain unchanged **OR** For a justification that indicates that the period is independent of gravitational force	**1 point**

Example Solution

The period of oscillation of the spring-block system depends on the mass and the effective spring constant, $T = 2\pi\sqrt{\frac{m}{Nk}}$ *. The slope is equal to the square of the period over the inverse number of identical springs,* $\text{Slope} = \frac{T^2}{N^{-1}}$ *. This means that the slope is proportional to the mass divided by the spring constant. Since neither the mass nor the spring constant change, the slope will remain unchanged.*

OR

The period of oscillation of the spring-block system depends on the mass and the effective spring constant, $T = 2\pi\sqrt{\frac{m}{Nk}}$ *. The slope is equal to the square of the period over the inverse number of identical springs,* $\text{Slope} = \frac{T^2}{N^{-1}}$ *. Since the period is independent of the gravitational force, and the only change was arranging the block-spring system horizontally instead of vertically, the period is not affected by the change in the orientation of the system. Therefore, the slope will remain unchanged.*

(d)(ii)	For a relationship between v_{max} and N that is consistent with the expression from part (a) with an attempt at relevant justification	**1 point**
	For using energy conservation to justify the relationship **OR** For using the relationship between v_{max} and ω to justify the relationship	**1 point**
	For indicating that the effective spring constant changes the elastic potential energy of the spring block system U_s therefore changing v_{max} in a manner consistent with the expression from part (a) **OR** For an inverse relationship between ω and period, indicating a change in v_{max} consistent with the expression from part (a)	**1 point**

Example Solution

An increase in the number of identical springs attached in parallel to each other causes the effective spring constant to increase, $k_{eff} = \sum_{i=1}^{N} k_i$ *. Since the effective spring constant is proportional to potential energy of the block-spring system,* $U_s = \frac{1}{2}k_{eff}x^2$ *, an increase in the effective spring constant causes the potential energy of the block-spring system to increase for a given displacement. Therefore, based on conservation of energy,* $\frac{1}{2}k_{eff}x^2 = \frac{1}{2}mv_{max}^2$ *, an increase in the potential energy will cause an increase in the kinetic energy, resulting in a greater maximum velocity. Therefore, the maximum velocity will increase with an increase in the number of springs added.*

OR

An increase in the number of identical springs attached in parallel to each other causes the effective spring constant to increase, $k_{eff} = \sum_{i=1}^{N} k_i$ *. Since the effective spring constant is inversely proportional to the period of oscillation of the block-spring system,* $T = 2\pi\sqrt{\frac{m}{k_{eff}}}$ *, an increase in the effective spring constant causes the period to decrease. Since the period is inversely proportional to the angular frequency of the block-spring system,* $\omega = \frac{2\pi}{T}$ *, the angular frequency increases. Since the maximum velocity is proportional to the angular frequency,* $v_{max} = x_0\omega$ *, this results in an increase in the maximum velocity. Therefore, the maximum velocity will increase with an increase in the number of springs added.*

Total for part (d) 5 points

Total for question 2 15 points

Question 3: Free-Response Question — 15 points

(a) For stating the parallel axis theorem — **1 point**

Example Response

$$I_{\text{blade}} = I_{\text{CM}} + Md^2$$

For using correct substitutions of the rotational inertia of one blade about its center of mass and substituting the distance from the center of mass — **1 point**

Example Response

$$I_{\text{blade}} = \frac{1}{18}ML^2 + M\left(\frac{L}{3}\right)^2$$

For multiplying the rotational inertia of one blade by 3 — **1 point**

Example Response

$$I_{\text{rotor}} = 3I_{\text{blade}} = \frac{1}{2}ML^2$$

Example Solution

$$I_{\text{blade}} = I_{\text{CM}} + Md^2$$

$$I_{\text{blade}} = \frac{1}{18}ML^2 + M\left(\frac{L}{3}\right)^2$$

$$I_{\text{blade}} = \frac{1}{6}ML^2$$

$$I_{\text{rotor}} = 3I_{\text{blade}} = \frac{1}{2}ML^2$$

Total for part (a) — 3 points

(b) For calculating the correct answer with correct units $(2.4\ \text{s})$ — **1 point**

Example Solution

$$v = r\omega$$

$$v = L\omega_0$$

$$\frac{d}{t} = L\omega_0$$

$$t = \frac{d}{L\omega_0} = \frac{2\pi L}{L\omega_0} = \frac{2\pi}{\omega_0}$$

$$t = \frac{2\pi}{(2.6\ \text{rad/s})}$$

$$\therefore t = 2.4\ \text{s}$$

Total for part (b) — 1 point

(c)(i)	For indicating that the total initial rotational kinetic energy is dissipated	**1 point**

Example Response

$$\Delta K_{\text{rot}} = E_{\text{dis}}$$

$$0 - \frac{1}{2} I \omega_0^{\ 2} = E_{\text{dis}}$$

For substituting correct values for the rotational inertia and initial angular speed of the system	**1 point**

Example Response

$$E_{\text{dis}} = -\frac{1}{2}\left(6.7 \times 10^6 \text{ kg} \cdot \text{m}^2\right)\left(2.6 \text{ rad/s}\right)^2$$

For an answer consistent with substitutions above and with correct units	**1 point**

Example Response

$$E_{\text{dis}} = -2.3 \times 10^7 \text{ J}$$

Example Solution

$$\Delta K_{\text{rot}} = E_{\text{dis}}$$

$$0 - \frac{1}{2} I \omega_0^{\ 2} = E_{\text{dis}}$$

$$E_{\text{dis}} = -\frac{1}{2}\left(6.7 \times 10^6 \text{ kg} \cdot \text{m}^2\right)\left(2.6 \text{ rad/s}\right)^2$$

$$E_{\text{dis}} = -2.3 \times 10^7 \text{ J}$$

Scoring Note: A response may earn full credit for positive or negative values of dissipated energy.

(c)(ii)	For using Newton's second law in rotational form	**1 point**
	For attempting to differentiate the equation for ω	**1 point**

Example Response

$$\tau = I_{\text{sys}} \frac{d}{dt}\left(\omega_0 e^{-\beta_0 t}\right)$$

	For a correct expression for the torque on the system	**1 point**

Example Response

$$\tau = -\beta_0 I_{\text{sys}} \omega_0 e^{-\beta_0 t}$$

Example Solution

$$\tau = I\alpha$$

$$\tau = I_{\text{sys}} \frac{d\omega}{dt}$$

$$\tau = I_{\text{sys}} \frac{d}{dt}\left(\omega_0 e^{-\beta_0 t}\right)$$

$$\therefore \tau = -\beta_0 I_{\text{sys}} \omega_0 e^{-\beta_0 t}$$

(c)(iii)	For attempting to integrate the expression for angular speed	**1 point**

Example Response

$$\Delta\theta = \int \omega(t)\,dt$$

For using the correct limits of integration	**1 point**

Example Response

$$\Delta\theta = \int_0^t \omega_0 e^{-\beta_0 t}\,dt$$

For a correct expression for angular displacement	**1 point**

Example Response

$$\Delta\theta = \frac{\omega_0}{\beta_0}\left(1 - e^{-\beta_0 t}\right)$$

Example Solution

$$\Delta\theta = \int \omega(t)\,dt$$

$$\Delta\theta = \int_0^t \omega_0 e^{-\beta_0 t}\,dt$$

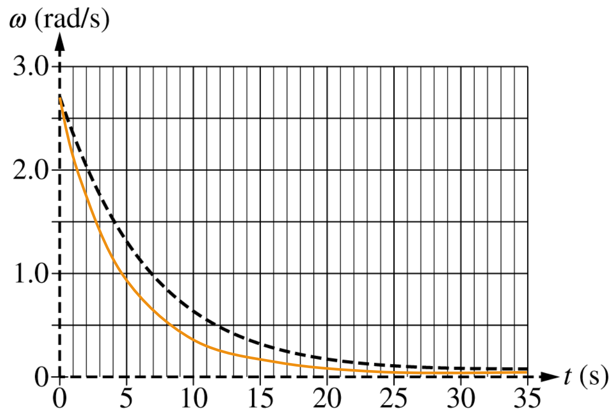
$$\Delta\theta = \left(-\frac{\omega_0}{\beta_0} e^{-\beta_0 t}\right)\Bigg|_0^t = -\frac{\omega_0}{\beta_0} e^{-\beta_0 t} + \frac{\omega_0}{\beta_0}(1)$$

$$\therefore \Delta\theta = \frac{\omega_0}{\beta_0}\left(1 - e^{-\beta_0 t}\right)$$

Total for part (c) 9 points

(d)	For drawing a continuous curve showing an exponential decay	**1 point**
	For starting at $\omega = 2.6\ \text{rad/s}$ and drawing a curve below the original curve	**1 point**

Example Solution



Total for part (d) 2 points

Total for question 3 15 points