2024

AP® Physics C: Mechanics

Free-Response Questions

Set 1

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit, $1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\ \text{MeV}/c^2$

Planck's constant, $h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$

$hc = 1.99 \times 10^{-25}\ \text{J}\cdot\text{m} = 1.24 \times 10^3\ \text{eV}\cdot\text{nm}$

Vacuum permittivity, $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$

Coulomb's law constant, $k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^9\ (\text{N}\cdot\text{m}^2)/\text{C}^2$

Vacuum permeability, $\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$

Magnetic constant, $k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$

1 atmosphere pressure, $1\text{ atm} = 1.0 \times 10^5\ \text{N/m}^2 = 1.0 \times 10^5$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi \sqrt{\frac{m}{k}}$

$T_p = 2\pi \sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0} \left| \frac{q_1 q_2}{r^2} \right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r \ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

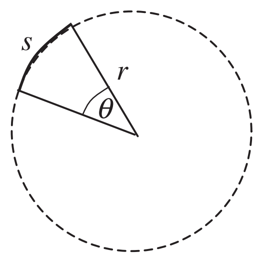
$a^2 + b^2 = c^2$

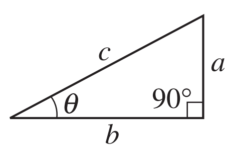
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta$$

Begin your response to **QUESTION 1** on this page.

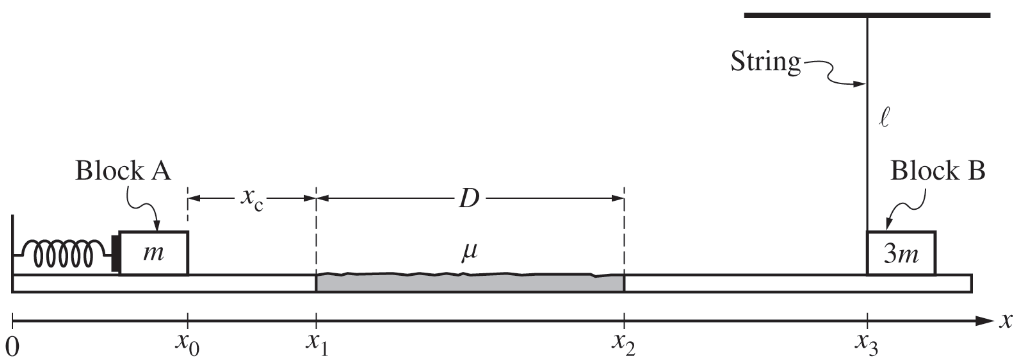
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

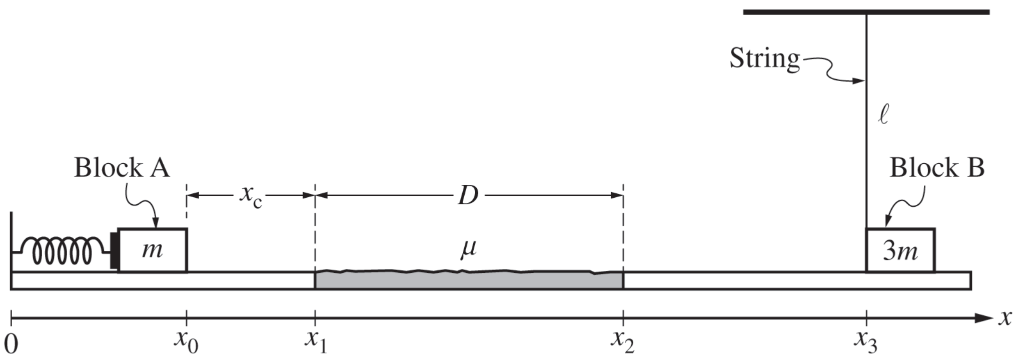
(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

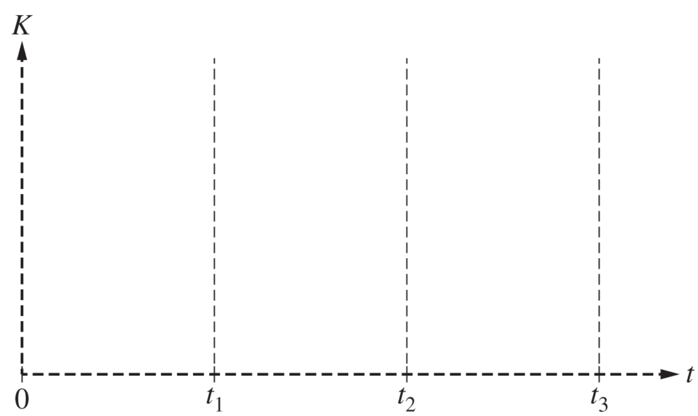


Note: Figure not drawn to scale.

Figure 1

(b)

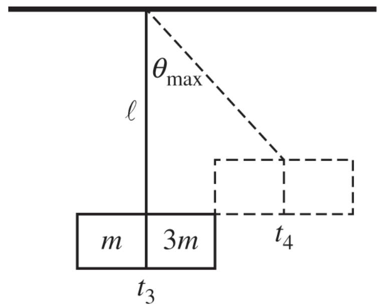
i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3.



ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.



Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ.

(c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

____ $f_{2\ell} > f_\ell$ ____ $f_{2\ell} < f_\ell$ ____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 2** on this page.

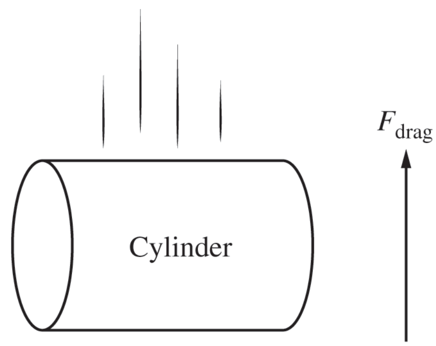


Figure 1

2. A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

   (a) **Derive**, but do NOT solve, a differential equation that could be used to determine the speed v of the cylinder as a function of time t. Express your answer in terms of given quantities and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

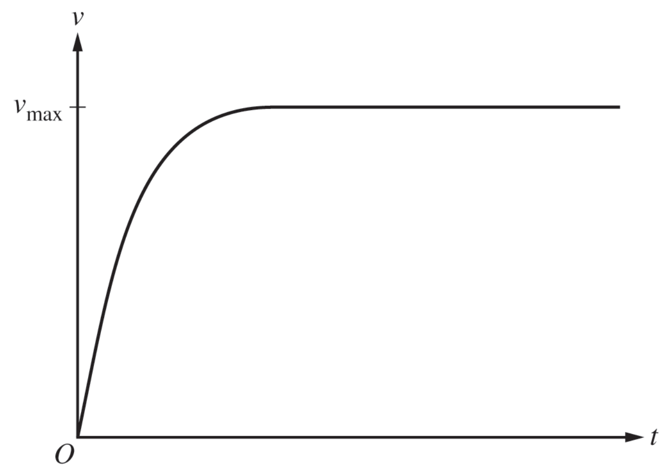


Figure 2

(b) The student correctly sketches the speed v of the cylinder as a function of time t, as shown in Figure 2.

i. **Draw** a vertical line on the sketch in Figure 2 to indicate the earliest time at which F_{drag} on the cylinder is equal to the magnitude of the weight of the cylinder. Label this time as t_1 on the time axis.

ii. **Justify** the location of t_1. Explicitly reference appropriate features of the sketch in Figure 2.

(c) Rather than dropping the cylinder from rest, the student throws the cylinder upward with a nonzero initial speed. The cylinder is in the same orientation as when the cylinder was previously dropped. The student allows the cylinder to fall toward the ground.

Indicate whether the magnitude of the cylinder's maximum downward speed after being thrown upward would be greater than, less than, or equal to the maximum speed v_{max} in Figure 2.

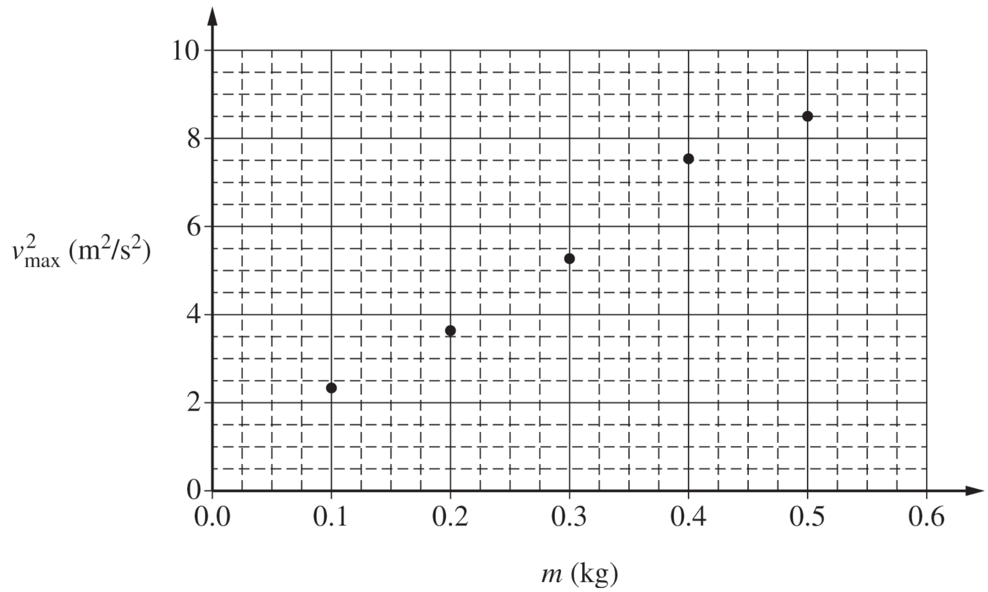
_____ Greater than _____ Less than _____ Equal to

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The student conducts an experiment to better understand the relationship between maximum speed v_{max} and mass. The student collects data to determine the maximum speed for cylinders dropped from rest, each with the same physical size and shape but a different mass m. The student then graphs v_{max}^2 as a function of mass.



i. **Draw** the best-fit line for the data.

ii. Use the best-fit line to **calculate** an experimental value for b.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

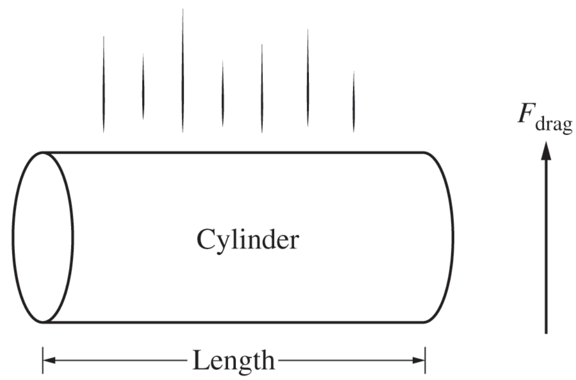


Figure 3

A student claims that the magnitude of the maximum speed of a cylinder dropped from rest depends on the length of the cylinder. The student designs an experiment to collect data that can be used to provide evidence to support the claim. The student drops cylinders with the orientation shown in Figure 3.

(e) The student has access to but does not have to use all of the following equipment.

- Cylinder Set 1: cylinders of the same known length with different known masses
- Cylinder Set 2: cylinders of the same known mass with different known lengths
- A motion detector that can measure velocity as a function of time

i. **Indicate** two quantities that when graphed could be used to determine whether the length of the cylinder affects the maximum speed.

Vertical axis: _______________ Horizontal axis: ________________

ii. Briefly **describe** how the quantities graphed could be used to determine the relationship between cylinder length and maximum speed.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 3** on this page.

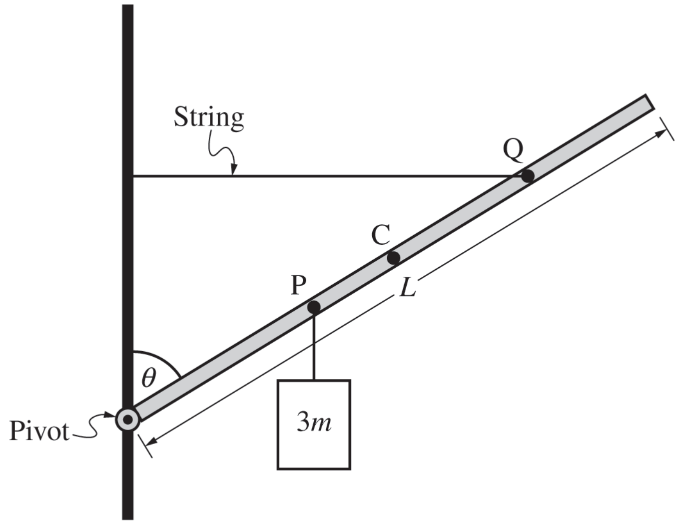
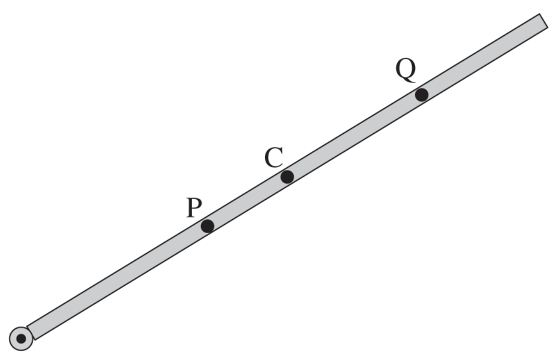


Figure 1

Note: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

   (a) On the following representation of the rod, **draw** and **label** the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot. **Derive** an equation for the tension F_T in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

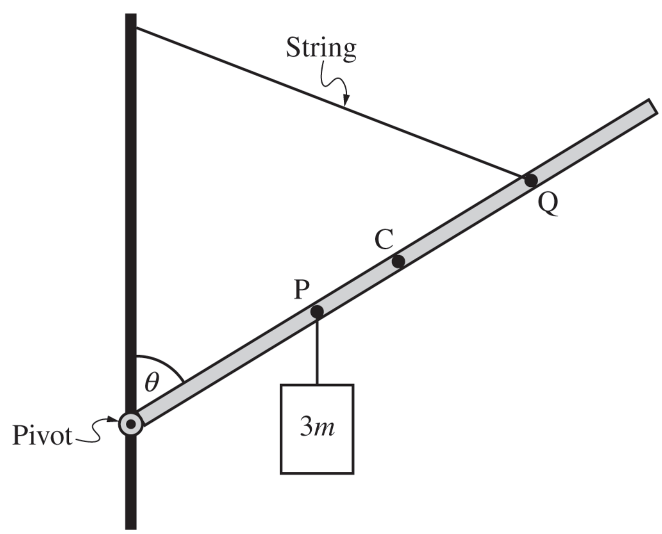


Figure 2

Note: Figure not drawn to scale.

(c) The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,\,new}$ compare with the original tension F_T from part (b) ? **Justify** your reasoning.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

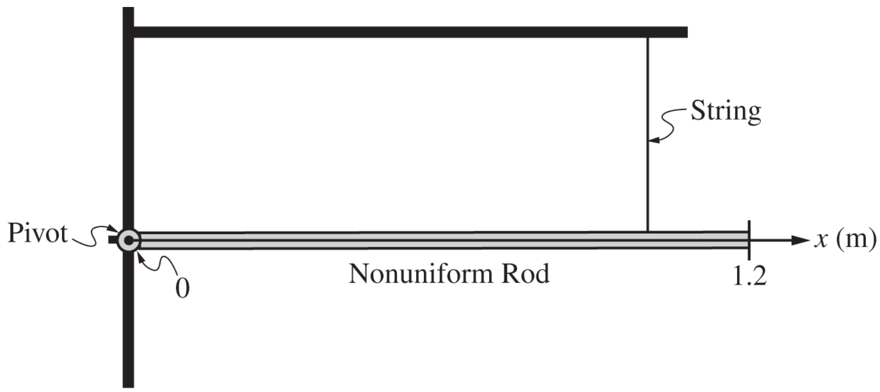


Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0$ kg/m, and $B = 10.0$ kg/m^2.

i. **Calculate** the mass of the rod.

ii. **Calculate** the rotational inertia of the rod about the pivot.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM