Question 2

12 points total | **Distribution of points**

(a)

i) 3 points

For indicating that R_{eq} of the entire circuit or the combination of bulbs 2 and 3 decreases	1 point
For indicating a change in I_{tot} or the potential difference across bulb 1 consistent with Ohm's law and the change in R_{eq} stated in the response	1 point
For indicating a change in brightness consistent with the current or potential difference change stated in the response	1 point

ii) 3 points

For indicating that $P_1 = \frac{1}{4}\left(\mathcal{E}^2/R\right)$	1 point
For indicating that the new equivalent resistance of the circuit is $R_{eq,new} = (3/2)R$ Note: Credit is earned if calculation is done in part (i) and used here.	1 point
For manipulating equations to show that the power expended by bulb 1 is $P_{new} = \frac{16}{9}P_1$	1 point

iii) 1 point

For using or referring to the expression from part (a)(ii) to support the claim made in (a)(i) regarding the brightness of bulb 1: e.g., $16/9 > 1$, and indicating an understanding that brightness is related to power consumption	1 point

(b)

i) 1 point

For explaining that the brightness of bulb 2 decreases after the switch is closed because it expends less power (or the current through bulb 2 decreases, or the potential difference across bulb 2 decreases)	1 point

ii) 1 point

For a calculation that supports the reasoning in part i	1 point

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Question 2 (continued)

Distribution of points

(c) 3 points

For indicating in either part (c)i or part (c)ii that brightness is dependent on potential difference across the bulb OR on current through the bulb — 1 point

i)

For a reasonable explanation for why bulb 1 is brighter than bulb 2 — 1 point

Example: Immediately after the switch is closed, the potential difference across the capacitor will be zero (like a short in the circuit), so the current through bulb 2 would be zero, which is less than the current through bulb 1.

Note: No points will be awarded for indicating that bulb1 is brighter than bulb 2 with no justification.

ii)

For a reasonable explanation for why bulb 1 is the same brightness as bulb 2 — 1 point

Example: The current through bulb 2 increases as the potential difference across the capacitor increases (becomes like an open circuit), so a long time after the switch is closed, the current through bulb 2 will be equal to the full current through bulb 1.

Note: No points will be awarded for indicating that bulb 1 is the same brightness as bulb 2 without a justification.

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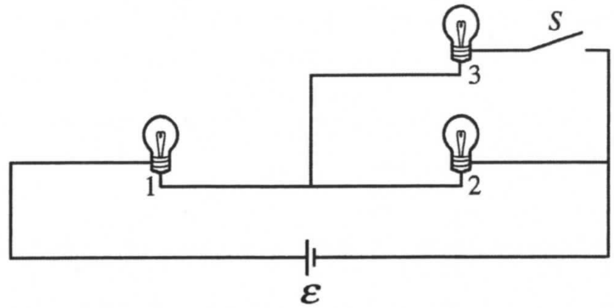


2. (12 points, suggested time 25 minutes)

A battery of emf $\mathcal{E}$ and negligible internal resistance, three identical incandescent lightbulbs, and a switch S that is initially open are connected in the circuit shown above. The bulbs each have resistance R. Students make predictions about what happens to the brightness of the bulbs after the switch is closed.

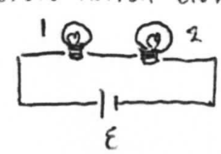
(a) A student makes the following prediction about bulb 1: "Bulb 1 will decrease in brightness when the switch is closed."

i. Do you agree or disagree with the student's prediction about bulb 1 ? Qualitatively explain your reasoning.

I disagree with the student's prediction. ~~The Bulb~~ Bulb 1 will increase in brightness. Adding Bulb 3 in parallel with Bulb 2 (by ~~to~~ closing the switch) will decrease the total resistance in the circuit, thus increasing the current through the battery. Since the current through the battery is equivalent to that of ~~the bulb,~~ ~~Bulb~~ Bulb 1, Bulb 1 will increase in brightness.

ii. Before the switch is closed, the power expended by bulb 1 is P_1. Derive an expression for the power P_{new} expended by bulb 1 after the switch is closed in terms of P_1.

Initial Circuit: (Before switch closed)



Before switch is closed:

$P = I^2 R$

$R_{tot} = R_1 + R_2$, $V = IR \rightarrow \mathcal{E} = IR_{tot}$, $P = IV$

$\mathcal{E} = I(R_1 + R_2) = I(2R) = 2IR$, $I = \frac{\mathcal{E}}{2R}$

$P_1 = \left(\frac{\mathcal{E}}{2R}\right)^2 R = \frac{\mathcal{E}^2}{4R}$

After switch is closed:

Bulb 3 & Bulb 2 in ||

$R_{tot} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = \frac{3R}{2}$

$\mathcal{E} = I\left(R_1 + \frac{R_2 R_3}{R_2 + R_3}\right) = I\frac{3R}{2} \rightarrow I = \frac{2\mathcal{E}}{3R}$

$P_{new} = \left(\frac{2\mathcal{E}}{3R}\right)^2 R = \frac{4\mathcal{E}^2}{9R}$

$P_{new} = \frac{16}{9}\left(\frac{\mathcal{E}^2}{4R}\right) = \frac{16}{9} P_1$

iii. How does the result of your derivation in part (a)ii relate to your explanation in part (a)i?

From a.ii), $P_{new} > P_1$. Since $P \propto I^2$, $I_{new} > I_1$.

~~Since support~~ This confirms my explanation in part (a)i, as an ~~increased~~ greater current through the bulb means ~~an increased~~ a greater brightness.

~~Thus, $P_{new} > P_1$ implies Brightness of Bulb I_{new} >~~

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(b) A student makes the following prediction about bulb 2: "Bulb 2 will decrease in brightness after the switch is closed."

i. Do you agree or disagree with the student's prediction about bulb 2 ? Explain your reasoning in words.

I agree, Bulb 2 will ~~decrease~~ decrease in brightness. Since I_1 increases in brightness, the voltage through Bulb will also increase. Since the voltage ~~of~~ through Bulb 2 is still the difference between ε and V_1 (they form closed loop), the voltage through Bulb 2 decreases. Thus, the current and power through Bulb 2 will also decrease, and ~~the~~ Bulb will decrease in brightness.

ii. Justify your explanation with a calculation.

(From (a) ii) $I_{\varepsilon,new} = \frac{2\varepsilon}{3R}$ & $I_{\varepsilon,0} = \frac{\varepsilon}{2R}$

$V_{1,new} = IR = \frac{2\varepsilon}{3R} \cdot R = \frac{2\varepsilon}{3}$, $V_{1,0} = \frac{\varepsilon}{2R} \cdot R = \frac{\varepsilon}{2}$

$V_2 = \varepsilon - V_1$, $V_{2,new} = \frac{\varepsilon}{3}$, $V_{2,0} = \frac{\varepsilon}{2}$

$I_{2,new} = \frac{V_{2,new}}{R} = \frac{\varepsilon}{3R}$, $I_{2,0} = \frac{\varepsilon}{2R}$

$I_{2,new} < I_{2,old}$
Bulb 2 decreases in brightness
($P_{2,new} < P_{2,old}$ since $P \propto I^2$)

(c) While the switch is open, bulb 3 is replaced with an uncharged capacitor. The switch is then closed.

i. How does the brightness of bulb 1 compare to the brightness of bulb 2 immediately after the switch is closed? Justify your answer.

~~No change occurs. Initially~~ ~~Initially~~ Immediately after the switch is closed, the capacitor has 0 resistance. Thus, the current through Bulb 2 will equal 0 (all the current will go through the capacitor instead) and the ~~overall~~ total resistance will decrease (from $R_1 + R_2$ to R_1). Thus, the current, voltage, and power through bulb 1 will increase, while bulb 2 will produce no light. In other words, the brightness of Bulb 1 will be greater than that of Bulb 2.

ii. How does the brightness of bulb 1 compare to the brightness of bulb 2 a long time after the switch is closed? Justify your answer.

They will be equal. I through capacitor = 0, so circuit will be equivalent to:

$R_1 = R_2 = R$ bulb 1 & 2 in series

$I_1 = I_2 = I$

$V_1 = V_2$ $(V = IR)$

$P_1 = P_2$ $(P = IV)$

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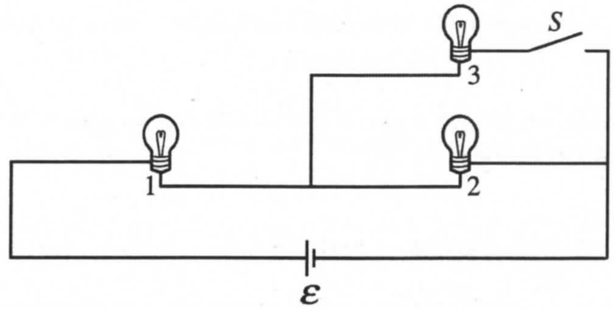
$\frac{(2U_1)^2}{2R} = 2P_1$

$\frac{(2U_1)^2}{\frac{3}{2}R_1} = \frac{8}{3}P_1$



2. (12 points, suggested time 25 minutes)

A battery of emf $\mathcal{E}$ and negligible internal resistance, three identical incandescent lightbulbs, and a switch S that is initially open are connected in the circuit shown above. The bulbs each have resistance R. Students make predictions about what happens to the brightness of the bulbs after the switch is closed.

(a) A student makes the following prediction about bulb 1: "Bulb 1 will decrease in brightness when the switch is closed."

i. Do you agree or disagree with the student's prediction about bulb 1 ? Qualitatively explain your reasoning.

Disagree
Because if the switch is closed, the combined resistance of the parallel bulbs 2 and 3 is smaller than that of a single bulb ~~so the portion~~ of voltage on bulb 1 increases and according to $P = \frac{U^2}{R}$ power will increase

ii. Before the switch is closed, the power expended by bulb 1 is P_1. Derive an expression for the power P_{new} expended by bulb 1 after the switch is closed in terms of P_1.

$P_1 = I_1 U_1$
$= I_1^2 R_1$

$R_P = \frac{1}{\frac{2}{R_1}} = \frac{R_1}{2}$

$\frac{R_1}{\frac{R_1}{2} + R_1} \times 2U_1 = \frac{4}{3}U_1$

So $P_{new} = \frac{U^2}{R}$
$= \frac{\frac{16}{9}U_1^2}{R_1}$
$= \frac{16}{9}P_1$

iii. How does the result of your derivation in part (a)ii relate to your explanation in part (a)i?

derivation in part (a) ii confirms my explanation in part (a) i.

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(b) A student makes the following prediction about bulb 2: "Bulb 2 will decrease in brightness after the switch is closed."

i. Do you agree or disagree with the student's prediction about bulb 2 ? Explain your reasoning in words.

Agree

Because the combined resistance is smaller than that of a single bulb so the portion of voltage on bulb 2 decreases According to $P = \frac{U^2}{R}$. power decreases

ii. Justify your explanation with a calculation.

$P_1 = I_1 U_1$

$R_P = \frac{1}{\frac{2}{R_1}} = \frac{R_1}{2}$

$\frac{\frac{1}{2}R_1}{\frac{R_1}{2} + R_1} \times 2U_1 = \frac{2}{3}U_1$

$P = \frac{U^2}{R}$

$= \frac{\frac{4}{9}(U_1)^2}{R_1}$

$= \frac{4}{9}P_1$

So the brightness decreases

(c) While the switch is open, bulb 3 is replaced with an uncharged capacitor. The switch is then closed.

i. How does the brightness of bulb 1 compare to the brightness of bulb 2 immediately after the switch is closed? Justify your answer.

Immediately after the switch is closed, the voltage on the capacitance is 0V, so bulb 1 is very bright and bulb 2 is dark

ii. How does the brightness of bulb 1 compare to the brightness of bulb 2 a long time after the switch is closed? Justify your answer.

A long time after the switch is closed, the voltage on the capacitance is ε so the bulb 2 is very bright and bulb 1 is dark

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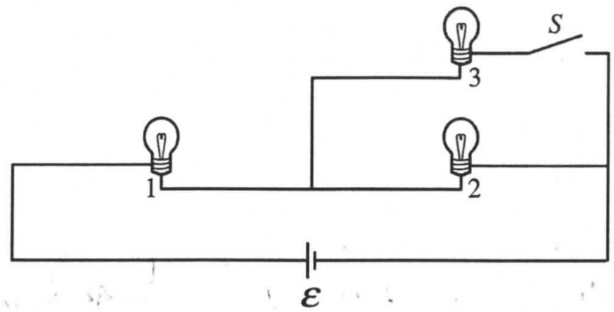

2. (12 points, suggested time 25 minutes)

A battery of emf $\mathcal{E}$ and negligible internal resistance, three identical incandescent lightbulbs, and a switch S that is initially open are connected in the circuit shown above. The bulbs each have resistance R. Students make predictions about what happens to the brightness of the bulbs after the switch is closed.

(a) A student makes the following prediction about bulb 1: "Bulb 1 will decrease in brightness when the switch is closed."

i. Do you agree or disagree with the student's prediction about bulb 1 ? Qualitatively explain your reasoning.

I disagree with the student's logic because I believe the bulb will have the same brightness. Because bulb 1 is on the same leg as the battery, it is unaffected by whether or not bulb 3 gets any power. The current will split, and bulbs 2 and 3 will be dimmer, but bulb one has unsplit current and is unaffected. $P_1 = I\Delta V$

ii. Before the switch is closed, the power expended by bulb 1 is P_1. Derive an expression for the power P_{new} expended by bulb 1 after the switch is closed in terms of P_1.

$P = I\Delta V$

$$\mathcal{E} - I_2R_2 - I_1R_1 = 0$$
$$\mathcal{E} - I_2R_2 = I_1R_1$$
$$\frac{\mathcal{E} - I_2R_2}{R_1} = I_1$$

$$P_{new} = \left(\frac{\mathcal{E} - I_2R_2}{R_1}\right)\Delta V$$

iii. How does the result of your derivation in part (a)ii relate to your explanation in part (a)i?

My derivation relies on the fact that bulb 1 and bulb 2 receive different currents when the switch is closed. This relates to part i because bulb 1 keeps the same current regardless of the switch (keeping it's brightness stagnant) while the brightness of bulb 2 relies on the switch to determine it's current.

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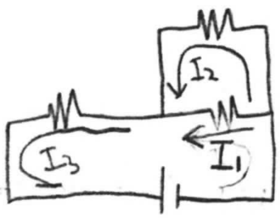
(b) A student makes the following prediction about bulb 2: "Bulb 2 will decrease in brightness after the switch is closed."

i. Do you agree or disagree with the student's prediction about bulb 2 ? Explain your reasoning in words.

Yes, I agree with his statement because the current that would be going to bulb 2 is split in half once the switch is closed. Once you start to lose current, you start to lose brightness.

ii. Justify your explanation with a calculation.



$$I_1 + I_2 = I_3$$

(c) While the switch is open, bulb 3 is replaced with an uncharged capacitor. The switch is then closed.

i. How does the brightness of bulb 1 compare to the brightness of bulb 2 immediately after the switch is closed? Justify your answer.

Bulb 1 is very bright while bulb 2 is completely dark. Because a capacitor acts as a wire for the starting instant, the current will choose the path of least resistance and bypass bulb 2 while bulb 1 gets all the current.

ii. How does the brightness of bulb 1 compare to the brightness of bulb 2 a long time after the switch is closed? Justify your answer.

After a long time bulb 1 and bulb 2 will have the same brightness because the capacitor will eventually be fully charged and will block all current through it's leg, leaving bulbs 1 and 2 with the same current and thus the same brightness.

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Question 2

Overview

The intent of the question was to assess student understanding of basic parallel and series circuits including resistors and capacitors. An understanding of Ohm's law and the relationship between current, voltage, power, and brightness of lightbulbs was being tested. Students were asked to work qualitatively and quantitatively, and to be able to relate the two representations.

Sample: P2Q2 A
Score: 12

This full-credit paper is well-written and clearly organized. Details of the student's reasoning are fully described.

Sample: P2Q2 B
Score: 8

Part (a)(i) earned 2 points, because there is no indication that bulb 1 becomes brighter. Part (a)(ii) earned 3 points for full credit, and (a)(iii) earned no credit. Part (b) earned 2 points for full credit. Part (c) just earned 1 point for (c)(i). There is no indication in either part (c)(i) or (c)(ii) that brightness is dependent on the potential difference across a bulb.

Sample: P2Q2 C
Score: 5

Part (a)(i) earned 1 point for correctly relating brightness to current. Parts (a)(ii) and (a)(iii) earned no credit. Part (b)(i) earned 1 point for explaining the brightness of bulb 2 based on a change in current. Part (b)(ii) earned no credit. The single junction rule equation does not justify a decrease in current when the switch is closed. Part (c) earned full credit.

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