2019

AP® CollegeBoard

AP® Physics 2: Algebra-Based

Sample Student Responses and Scoring Commentary

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AP® PHYSICS
2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

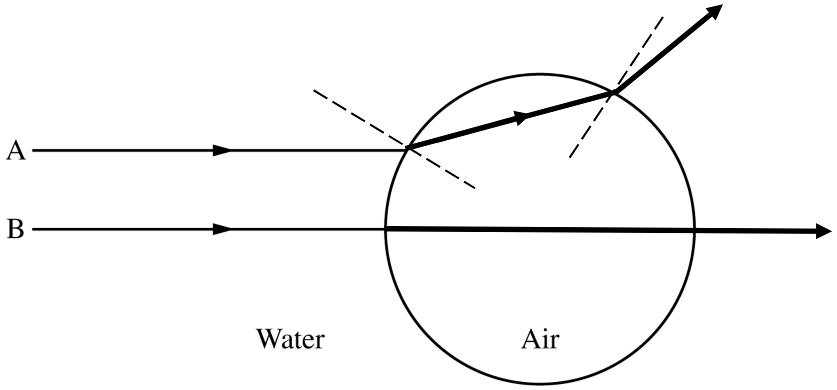
Question 4

10 points

A student notices many air bubbles rising through the water in a large fish tank at an aquarium.

(a) LO 6.E.3.1, SP 1.1, 1.4
3 points

In the figure below, the circle represents one such air bubble, and two incoming rays of light, A and B, are shown. Ray B points toward the center of the circle. On the diagram, draw the paths of rays A and B as they go through the bubble and back into the water. Your diagram should clearly show what happens to the rays at each interface.



For ray B going straight through		1 point
For ray A bending away from the normal as it enters the air from the water		1 point
For ray A bending the opposite direction in relationship to the normal as it exits the air and enters the water compared to the refraction entering the air from the water		1 point
Note: The normals need not be shown.		

(b) LO 5.B.4.1, SP 6.4, 7.2; LO 5.B.4.2, SP 1.4, 7.2; LO 5.B.5.4, SP 6.4, 2.2; LO 5.B.5.5, SP 2.2, 6.4
3 points

The bubble has a volume V_1, the air inside it has density ρ_A, and the water around it has density ρ_W. The bubble starts at rest and has a speed v_f when it has risen a height h. Assume that the change in the bubble's volume is negligible. Derive an expression for the mechanical energy dissipated by drag forces as the bubble rises this distance. Express your answer in terms of the given quantities and fundamental constants, as appropriate.

For a valid application of the work-energy theorem		1 point
$\Delta K = W_{\text{net}} = \lvert W_b \rvert - \lvert W_g \rvert - \lvert W_{\text{diss}} \rvert$		
For finding the work done by the buoyant force		1 point
$W_b = \rho_w V_1 g h$		
For correct substitutions into an equation with consistent relative signs for the terms		1 point
$\frac{1}{2}\rho_A V_1 v_f^2 = \rho_W V_1 g h - \rho_A V_1 g h - \lvert W_{\text{diss}} \rvert$		
$\lvert W_{\text{diss}} \rvert = \rho_W V_1 g h - \rho_A V_1 g h - \frac{1}{2}\rho_A V_1 v_f^2$		

Question 4 (continued)

(c)

At a particular instant, one bubble is 4.5 m below the water's surface. The surface of the water is at sea level, and the density of the water is $1000\ \text{kg/m}^3$.

i. LO 5.B.10.1, SP 2.2
1 point

Determine the absolute pressure in the bubble at this location.

$P_{4.5\text{m}} = P_{atm} + \rho_w g d$		
$P_{4.5\text{m}} = \left(1.0 \times 10^5 \text{Pa}\right) + \left(1000\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)(4.5\ \text{m})$		
For a correct answer with units		1 point
$P_{4.5\text{m}} = 1.44 \times 10^5\ \text{Pa}$ (or $1.45 \times 10^5\ \text{Pa}$ using $g = 10\ \text{m/s}^2$)		

ii. LO 7.A.3.3, SP 5.1
2 points

The bubble has a volume V_1 when it is 4.5 m below the water's surface. Assume that the temperature of the air in the bubble remains constant as it rises. In terms of V_1, calculate the volume of the bubble when it is just below the surface of the water.

For applying the ideal gas law at two locations in an attempt to determine the new bubble volume		1 point
$P_{4.5\text{m}} V_1 = P_{atm} V_{surface}$		
$V_{surface} = P_{4.5\text{m}} V_1 / P_{atm}$		
For substituting pressures consistent with part (i)		1 point
$V_{surface} = \left(1.44 \times 10^5\ \text{Pa}\right) V_1 / \left(1 \times 10^5\ \text{Pa}\right)$		
$V_{surface} = 1.44 V_1$ (or $1.45 V_1$ using $g = 10\ \text{m/s}^2$)		

iii. LO 7.A.3.3, SP 5.1
1 point

If the air in the bubble cooled as it rose, the volume of the bubble would be less than the value calculated in part (c)(ii). Use physics principles to briefly explain why.

For a correct explanation		1 point
<u>Note:</u> The explanation may be qualitative or quantitative. The explanation may also be macroscopic or microscopic.		

Question 4 (continued)

(c) (continued)
iii. (continued)

Example 1: By the ideal gas law, $P_{4.5\text{m}}V_1/T_1 = P_{atm}V_{surface}/T_{surface}$, so $V_{surface} = P_{4.5\text{m}}V_1T_{surface}/P_{atm}T_1$. The two pressures still have their previous values. $T_{surface} < T_1$, so the volume at the surface will be smaller.		
Example 2: As the bubble cools, the air molecules move slower. Slower molecules exert less force on the inner surface of the bubble. The unbalanced force, due to the difference in the forces on the inside and outside of the bubble, causes the bubble to expand less than it did in the constant temperature situation or contract.		
Claim (given): The volume of the bubble will decrease Example 1 evidence: $P_{4.5\text{m}}V_1/T_1 = P_{atm}V_{surface}/T_{surface}$, so $V_{surface} = P_{4.5\text{m}}V_1T_{surface}/P_{atm}T_1$ Example 1 reasoning: The two pressures still have their previous values. $T_{surface} < T_1$, so the volume at the surface will be smaller. Example 2 evidence: As the bubble cools, the air molecules move slower. Slower molecules exert less force on the inner surface of the bubble. Example 2 reasoning: The unbalanced force, due to the difference in the forces on the inside and outside of the bubble, causes the bubble to contract.		

Learning Objectives:

LO 5.B.4.1: The student is able to describe and make predictions about the internal energy of systems. [See Science Practices 6.4, 7.2]

LO 5.B.4.2: The student is able to calculate changes in kinetic energy and potential energy of a system using information from representations of that system. [See Science Practices 1.4, 2.1, 2.2]

LO 5.B.5.4: The student is able to make claims about the interaction between a system and its environment in which the environment exerts a force on the system, thus doing work on the system and changing the energy of the system (kinetic energy plus potential energy). [See Science Practices 6.4, 7.2]

LO 5.B.5.5: The student is able to predict and calculate the energy transfer to (i.e., the work done on) an object or system from information about a force exerted on the object or system through a distance. [See Science Practices 2.2, 6.4]

LO 5.B.10.1: The student is able to make calculations related to a moving fluid using Bernoulli's equation. [See Science Practices 2.2]

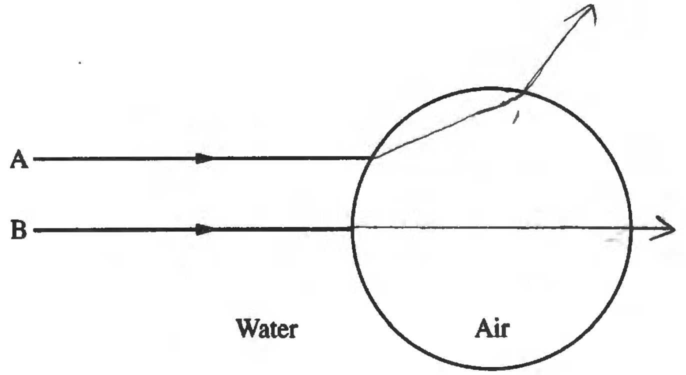
LO 6.E.3.1: The student is able to describe models of light traveling across a boundary from one transparent material to another when the speed of propagation changes, causing a change in the path of the light ray at the boundary of the two media. [See Science Practices 1.1, 1.4]

LO 7.A.3.3: The student is able to analyze graphical representations of macroscopic variables for an ideal gas to determine the relationships between these variables and to ultimately determine the ideal gas law $PV = nRT$. [See Science Practices 5.1]

4. (10 points, suggested time 20 minutes)

A student notices many air bubbles rising through the water in a large fish tank at an aquarium.

(a) In the figure below, the circle represents one such air bubble, and two incoming rays of light, A and B, are shown. Ray B points toward the center of the circle. On the diagram, draw the paths of rays A and B as they go through the bubble and back into the water. Your diagram should clearly show what happens to the rays at each interface.



(b) The bubble has a volume V_1, the air inside it has density ρ_A, and the water around it has density ρ_W. The bubble starts at rest and has a speed v_f when it has risen a height h. Assume that the change in the bubble's volume is negligible. Derive an expression for the mechanical energy dissipated by drag forces as the bubble rises this distance. Express your answer in terms of the given quantities and fundamental constants, as appropriate.

buoyant force = $\rho_W V_1 \cdot g$

gravitational force = $\rho_A V_1 \cdot g$

net upward force = $V_1 g (\rho_W - \rho_A)$

work done by net force = change in KE + change in PE + energy lost to drag

$W = Fd \Rightarrow \quad V_1 g (\rho_W - \rho_A) \cdot h = \frac{1}{2} \rho_A V_1 \cdot v_f^2 + \rho_A V_1 \cdot g \cdot h +$ energy lost to drag

(c) At a particular instant, one bubble is 4.5 m below the water's surface. The surface of the water is at sea level, and the density of the water is $1000\ \text{kg/m}^3$.

i. Determine the absolute pressure in the bubble at this location.

$P_{abs} = \rho g h + P_{atm}$

$P_{abs} = 1000 \cdot 9.8 \cdot 4.5 + 1 \cdot 10^5\ Pa$

$= \boxed{1.4 \cdot 10^5\ Pa}$

ii. The bubble has a volume V_1 when it is 4.5 m below the water's surface. Assume that the temperature of the air in the bubble remains constant as it rises. In terms of V_1, calculate the volume of the bubble when it is just below the surface of the water.

$PV = nRT \Rightarrow P_1 V_1 = P_2 V_2$ (constant moles and temperature)

$1.4 \cdot 10^5 \cdot V_1 = 1 \cdot 10^5 \cdot V_2$ $V_2 = 1.4 V_1$

iii. If the air in the bubble cooled as it rose, the volume of the bubble would be less than the value calculated in part (c)(ii). Use physics principles to briefly explain why.

$PV = nRT \Rightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$ if temperature varies

if $T_2 < T_1$, and the ratio of $\frac{P_1}{P_2}$ is the same, V_2 must be less than V_1 for the expression to stay true.

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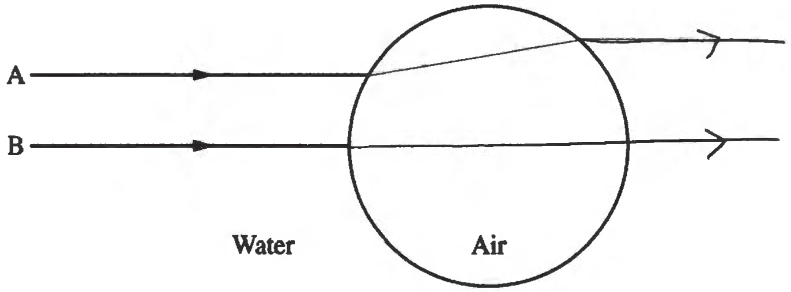
A student notices many air bubbles rising through the water in a large fish tank at an aquarium.

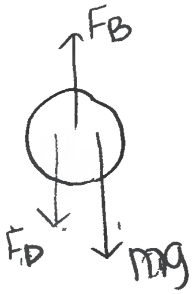
(a) In the figure below, the circle represents one such air bubble, and two incoming rays of light, A and B, are shown. Ray B points toward the center of the circle. On the diagram, draw the paths of rays A and B as they go through the bubble and back into the water. Your diagram should clearly show what happens to the rays at each interface.



(b) The bubble has a volume V_1, the air inside it has density ρ_A, and the water around it has density ρ_W. The bubble starts at rest and has a speed v_f when it has risen a height h. Assume that the change in the bubble's volume is negligible. Derive an expression for the mechanical energy dissipated by drag forces as the bubble rises this distance. Express your answer in terms of the given quantities and fundamental constants, as appropriate.

F_D = drag

$\rho = \frac{m}{V}$ $m = \rho V$



$F_B = F_D + mg$ $\rho_W V_1 g = F_D + \rho_A V_1 g$

$F_D = V_1 g(\rho_A - \rho_W)$ $\Delta E = W = F \cdot d$ $\theta = 0$

$\cos\theta = 1$

$$\boxed{\Delta E = h[V_1 g(\rho_A - \rho_W)]}$$

GO ON TO THE NEXT PAGE.

(c) At a particular instant, one bubble is 4.5 m below the water's surface. The surface of the water is at sea level, and the density of the water is $1000\ \text{kg/m}^3$.

i. Determine the absolute pressure in the bubble at this location.

$P = P_0 + \rho g h$

$P = 100000 + 1000(10)(4.5)$

$\boxed{P = 145000\ \frac{N}{m^2}}$

ii. The bubble has a volume V_1 when it is 4.5 m below the water's surface. Assume that the temperature of the air in the bubble remains constant as it rises. In terms of V_1, calculate the volume of the bubble when it is just below the surface of the water.

$PV = nRT \quad \frac{PV}{T} = \frac{PV}{T} \quad \Delta y = 4.5\text{m} \quad P_1V_1 = P_2V_2$

$P\downarrow \quad V\uparrow \quad V_2 > V_1 \quad \Delta P = 45000\ \frac{N}{m^2}$

iii. If the air in the bubble cooled as it rose, the volume of the bubble would be less than the value calculated in part (c)(ii). Use physics principles to briefly explain why.

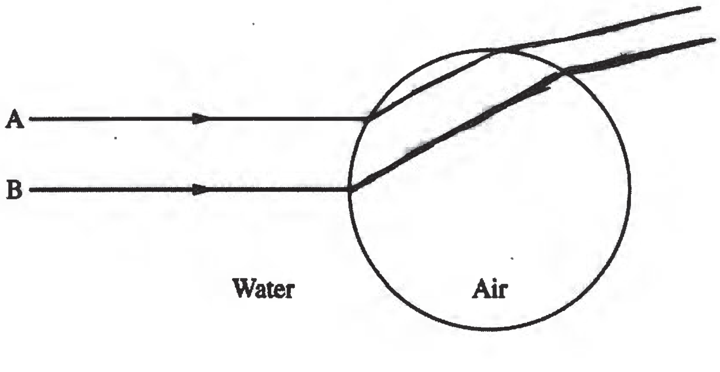
according to the ideal gas law, PV=nRT, volume and temperature are directly proportional. a decrease in temperature of the bubble would correspond to a decrease in volume as well

low to high, bend towards
high to low, bend away

4. (10 points, suggested time 20 minutes)

A student notices many air bubbles rising through the water in a large fish tank at an aquarium.

(a) In the figure below, the circle represents one such air bubble, and two incoming rays of light, A and B, are shown. Ray B points toward the center of the circle. On the diagram, draw the paths of rays A and B as they go through the bubble and back into the water. Your diagram should clearly show what happens to the rays at each interface.



(b) The bubble has a volume V_1, the air inside it has density ρ_A, and the water around it has density ρ_W. The bubble starts at rest and has a speed v_f when it has risen a height h. Assume that the change in the bubble's volume is negligible. Derive an expression for the mechanical energy dissipated by drag forces as the bubble rises this distance. Express your answer in terms of the given quantities and fundamental constants, as appropriate.

$\rho g h$

$\rho_a V_1 g +$ [illegible] $(V_1 \rho_A g)$

$P_1 + \rho g h + \frac{1}{2}\rho v^2$

$P_2 + \rho g h + \frac{1}{2}\rho v^2$

GO ON TO THE NEXT PAGE.

(c) At a particular instant, one bubble is 4.5 m below the water's surface. The surface of the water is at sea level, and the density of the water is 1000 kg/m^3.

i. Determine the absolute pressure in the bubble at this location.

$$P_{\text{absolute}} = P_0 + \rho g h \qquad g = 10$$

$$1 \times 10^5 + (1000)(10)(4.5)$$

$$P_{\text{absolute}} = 1.45 \times 10^5$$

ii. The bubble has a volume V_1 when it is 4.5 m below the water's surface. Assume that the temperature of the air in the bubble remains constant as it rises. In terms of V_1, calculate the volume of the bubble when it is just below the surface of the water.

$$PV = nRT$$

$$\frac{PV}{T} = \frac{PV}{T}$$

$$P = \rho g h$$

$$\frac{P_1 V_1}{\cancel{T_1}} = \frac{P_2 V_2}{\cancel{T_2}}$$

$$P_1 V_1 = P_2 V_2$$

$$\cancel{\rho g} h V_1 = \cancel{\rho g} h_2 V_2$$

$$\frac{4.5 V_1 \cancel{h_2}}{h_2} = V_2$$

$$V_{\text{below water surface}} = \frac{4.5 V_1}{h_2}$$

iii. If the air in the bubble cooled as it rose, the volume of the bubble would be less than the value calculated in part (c)(ii). Use physics principles to briefly explain why.

As the temperature cools and the bubble rises, PV=nRT makes the relationship between P and V.

If temperature decereases and so does pressure due to less height from the surface of the water, then the volume should be less than originally

Question 4

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

Part (a) of this question assessed student understanding of refraction. Part (b) required students to understand work, buoyant force, fluids, and potential and kinetic energies. Part (c) required students to understand the ideal gas law and pressure.

Sample: P2 Q4 A
Score: 9

In part (a) full credit was earned for drawing ray B straight through the air bubble and ray A refracting away from the normal as it enters the air bubble and toward the normal as it exits the air bubble. Two points were earned in part (b) for applying the work-energy theorem and deriving the work done by the buoyant force. No point was earned for a correct equation because $\rho_A V_1 gh$ appears on both sides of the equation and when simplified would result in a factor of 2 multiplying that term. In part (c) full credit was earned for finding the correct pressure with units in part (c)(i), finding the relationship between the volumes at two depths in part (c)(ii), and explaining how both temperature and pressure would affect the volume of a bubble as it rises in part (c)(iii).

Sample: P2 Q4 B
Score: 6

In part (a) 2 of 3 points were earned for correctly showing ray B moving straight through the air bubble and ray A refracting away from the normal as it enters the air bubble, but no point was earned for the refraction of ray A as it exits the air bubble. In part (b) 2 of 3 points were earned for applying the work-energy theorem and finding the work done by the buoyant force, but no point was earned for a correct equation. In part (c) 2 of 4 points were earned by calculating the pressure with units in (c)(i) and applying the ideal gas law at two positions in (c)(ii), but no points were earned in part (c)(iii) because although the explanation is based on the ideal gas law, the explanation does not address pressure.

Sample: P2 Q4 C
Score: 3

In part (a) 1 point was earned because the initial refraction of ray A is correct, but no points were earned for the second refraction of ray A or for ray B. Part (b) earned no points because the work-energy theorem is not addressed, the work done by the buoyant force is not provided, and there is no correct equation. Part (c)(i) earned no points because the answer does not include units. In part (c)(ii) 1 point was earned because the ideal gas law is applied at two positions, but no point was earned for a correct substitution. In part (c)(iii) 1 point was earned because the explanation is based on the ideal gas law and includes a discussion of the effect of both temperature and pressure on volume.