2021

AP CollegeBoard

AP® Physics 2: Algebra-Based

Sample Student Responses and Scoring Commentary

Inside:

Free Response Question 2

☑ Scoring Guideline

☑ Student Samples

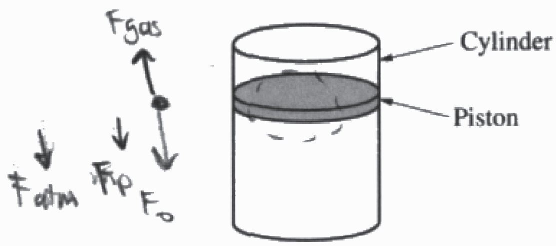
☑ Scoring Commentary

Question 2: Experimental Design — 12 points

(a)	**i.**	For describing a valid method for keeping the temperature constant	**1 point**
		For describing a valid use of the objects of known mass to affect the pressure	**1 point**
		For explicitly measuring the height h and the radius r (or diameter) of the piston	**1 point**
		For explicitly obtaining more than two data points	**1 point**
		Example response for part (a)(i) *Place the container in an ice bath, so the part below the piston is submerged. Measure the radius and height of the piston. For eight different objects of known mass, add each object on the piston and measure the height of the piston for each object.*	
	ii.	For an equation that correctly relates pressure to measured quantities consistent with the procedure in (a)(i)	**1 point**
		Example response for part (a)(ii) $P_{tot} = P_{atm} + \left(m_p + Nm_o\right)g/A = P_{atm} + \left(m_p + Nm_o\right)g/\left(\pi r^2\right)$, *where N is the number of objects on the piston and r is the radius of the piston.*	
	iii.	For an equation that correctly relates the density of the gas to measured quantities	**1 point**
		Example response for part (a)(iii) $\rho = M_g/V = M_g/\left(\pi r^2 h\right)$	
	iv.	For referring to the ideal gas law or Boyle's Law and using the equation to show pressure and volume are inversely proportional	**1 point**
		For a conclusion based on an analysis of the slope of the graph and a correct relationship between pressure and density	**1 point**
		Example response for part (a)(iv) *According to the ideal gas law, pressure is proportional to 1/V. Because the mass of this gas is constant, pressure is, therefore, directly proportional to density. The graph does not show a linear relationship between density and pressure, so the gas is not ideal.*	
		Total for part (a)	**8 points**
(b)		For indicating that the water pressure, and thus the pressure on the balloon and of the gas, increases as the depth is increased	**1 point**
		Example response for part (b) *When the balloon goes deeper in the fluid, the pressure increases. This will cause the volume of the balloon to decrease.*	

(c)	For correctly applying Newton's second law with some specific elements of the problem, including one of the two weights	**1 point**
	For a correct expression for the weight of the balloon and gas	**1 point**
	For a correct expression for the buoyant force	**1 point**
	Example response for part (c) $\sum F = 0 = F_B - W_{\text{balloon}} - W_{\text{gas}} - F_{\text{student}}$ $F_B = \rho_w V_b g$ $W = \rho_g V_b g + m_b g$ $F_{\text{student}} = \rho_w V_b g - \left(\rho_g V_b g + m_b g\right)$	
	Total for part (c)	**3 points**
	Total for question 2	**12 points**

Begin your response to **QUESTION 2** on this page.



2. (12 points, suggested time 25 minutes)

A group of students design an experiment to investigate the relationship between the density and pressure of a sample of gas at a constant temperature. The gas may or may not be ideal. They will create a graph of density as a function of pressure. They have the following materials and equipment.

- A sample of the gas of known mass M_g in a sealed, clear, cylindrical container, as shown above, with a movable piston of known mass m_p
- A collection of objects each of known mass m_o
- A meterstick

(a)

i. Describe the measurements the students should take and a procedure they could use to collect the data needed to create the graph. Specifically indicate how the students could keep the temperature constant. Include enough detail that another student could follow the procedure and obtain similar data.

The temperature should remain constant if the container remains sealed and away from direct sunlight. Place objects onto the piston one at a time. Density will be M_g divided by the current volume of the gas ($\pi r^2 h$, r and h measured by meter stick). Pressure is equal to the added weight of the piston and objects divided by the area of the piston. Graph Density as a function of pressure and repeat with different numbers of objects.

ii. Determine an expression for the absolute pressure of the gas in terms of measured quantities, given quantities, and physical constants, as appropriate. Define any symbols used that are not already defined.

$P_0 = \frac{F_{atm}}{A}$

$F_{atm} = P_0 A$

$P = \frac{F}{A} = \frac{F_p + nF_o + P_0 A}{\pi r^2}$

$$\boxed{\frac{g(m_p + m_o) + P_0 \pi r^2}{\pi r^2} = P}$$

r = radius of piston
P_0 = atm pressure

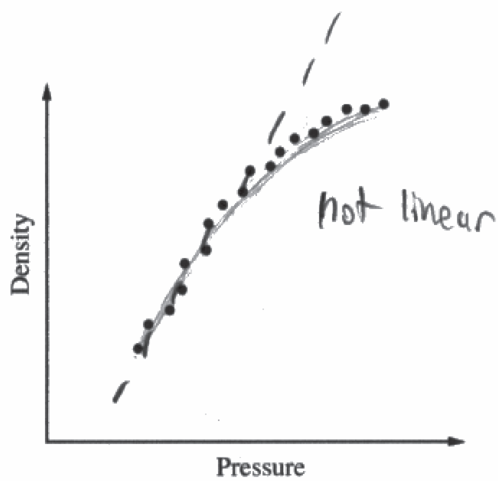
Continue your response to **QUESTION 2** on this page.

iii. Determine an expression for the density of the gas in terms of measured quantities, given quantities, and physical constants, as appropriate. Define any symbols used that are not already defined.

$$\rho = \frac{m}{V}$$

$$\boxed{\rho = \frac{Mg}{\pi r^2 h}}$$

r = radius of piston
h = height of piston



PV = nRT

iv. The graph above represents the students' data. Does the data indicate that the gas is ideal? Describe the application of physics principles in an analysis of the graph that can be used to arrive at your answer.

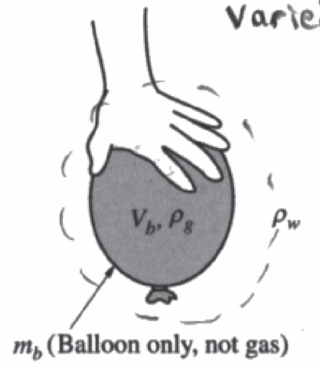
The graph shows that Density^2 is proportional to Pressure. In an ideal gas, Pressure in inversly proportional to volume, and volume is inversley proportional to density, therefore in an ideal gas, density should be directly proportional to pressure. Since this is not the case for this gas, this gas is not an ideal gas.

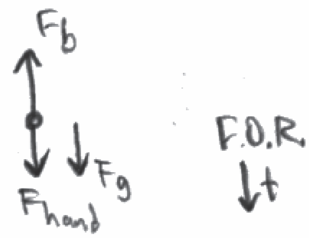
Continue your response to **QUESTION 2** on this page.

Another group of students propose that the relationship between density and pressure could also be obtained by filling a balloon with the gas and submerging it to increasing depths in a deep pool of water.

(b) Why could submerging the balloon to increasing depths be useful for determining the relationship between the density and pressure of the gas?

The pressure of the water increases with depth, which would decrease the volume (increasing the density since density is inversly proportional to volume) because of the pressure difference. This allows volume and density to be varied and graphed.





(c) The balloon is kept underwater in the deep pool by a student pushing down on the balloon, as shown above. Let V_b represent the volume of the inflated balloon, m_b represent the mass of just the balloon (not including the mass of the gas), ρ_g represent the density of the gas in the balloon, and ρ_w represent the density of the water. Derive an expression for the force the student must exert to hold the balloon at rest under the water, in terms of the quantities given in this part and physical constants, as appropriate.

$\rho = \frac{m}{V}$

$m = \rho V$

$$\cancel{F_{net}}^{0} = F_{hand} + F_g - F_b$$

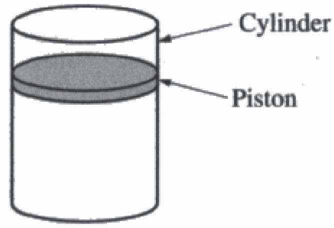
$$F_{hand} + F_g = F_b$$

$$F_{hand} + g(m_b + M_g) = \rho V g$$

$$F_{hand} + g(m_b + \rho_g V_b) = \rho_w V_b g$$

$$\boxed{F_{hand} = \rho_w V_b g - g(m_b + \rho_g V_b)}$$

Begin your response to **QUESTION 2** on this page.



2. (12 points, suggested time 25 minutes)

A group of students design an experiment to investigate the relationship between the density and pressure of a sample of gas at a constant temperature. The gas may or may not be ideal. They will create a graph of density as a function of pressure. They have the following materials and equipment.

- A sample of the gas of known mass M_g in a sealed, clear, cylindrical container, as shown above, with a movable piston of known mass m_p
- A collection of objects each of known mass m_o
- A meterstick

(a)

i. Describe the measurements the students should take and a procedure they could use to collect the data needed to create the graph. Specifically indicate how the students could keep the temperature constant. Include enough detail that another student could follow the procedure and obtain similar data.

Firstly, use the meterstick to measure the radii of the disk and get the area from that ($A = \pi r^2$). Next, multiply the known mass of the piston (m_p) with acceleration due to gravity (g) to get its force. Using $P = \frac{F}{A}$, calculate pressure. For volume of gas, measure the height ~~the cylinder and~~ of the cylinder with air in it with the area of the piston (πr_p^2). To keep temperature constant, DO NOT press on the piston and keep in a constant temperature room.

ii. Determine an expression for the absolute pressure of the gas in terms of measured quantities, given quantities, and physical constants, as appropriate. Define any symbols used that are not already defined.

$$P = P_0 + \left(\frac{m_p g}{A_p}\right)$$

P_0 = Atmospheric pressure (10^5 Pa)

m_p = mass of piston

g - gravity (acceleration)

A_p = area of piston (πr_p^2)

P = absolute pressure

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

iii. Determine an expression for the density of the gas in terms of measured quantities, given quantities, and physical constants, as appropriate. Define any symbols used that are not already defined.

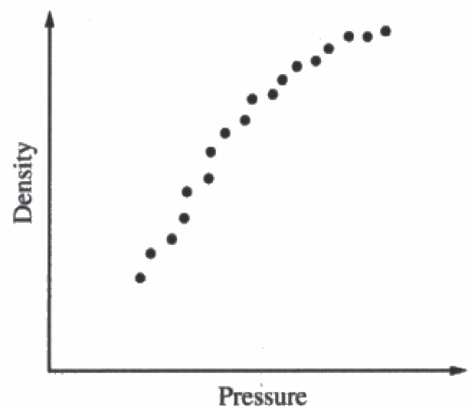
$$\rho = \frac{M_g}{(\pi r_p^2 l_c)}$$

ρ = density

M_g = mass of gas

r_p = radii of piston

l_c = height of cylinder up to the piston



$\frac{\rho}{P} \rightarrow \frac{\frac{m}{V}}{P}$

$\frac{m}{VP}$

$\frac{1}{V} \propto P$

iv. The graph above represents the students' data. Does the data indicate that the gas is ideal? Describe the application of physics principles in an analysis of the graph that can be used to arrive at your answer.

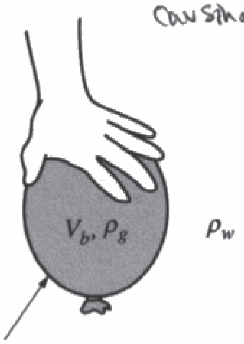
Due to the ideal gas law, $PV = nRT$, and because number of moles and gas constant doesn't change, PV is proportional to ~~temp~~ temperature. An ideal gas is ~~sg~~ a gas that follows this law, and as the graph shows the relationship of the density of the gas to its pressure, it is essentially showing that pressure is inversely proportional to density (as mass does not chang), which follows the ideal gas law. So therefore, yes, the gas is ideal.

Continue your response to **QUESTION 2** on this page.

Another group of students propose that the relationship between density and pressure could also be obtained by filling a balloon with the gas and submerging it to increasing depths in a deep pool of water.

(b) Why could submerging the balloon to increasing depths be useful for determining the relationship between the density and pressure of the gas?

Due to the relationship, $P=\rho gh$, pressure is directly proportional to depth in water due to gravity and fluid density being unchanging. This is ~~[illegible]~~ useful because as the balloon submerges deeper and deeper more force from pressure will be put onto it, potentially causing its volume, and thus density, to change proportionally.



m_b (Balloon only, not gas)

(c) The balloon is kept underwater in the deep pool by a student pushing down on the balloon, as shown above. Let V_b represent the volume of the inflated balloon, m_b represent the mass of just the balloon (not including the mass of the gas), ρ_g represent the density of the gas in the balloon, and ρ_w represent the density of the water. Derive an expression for the force the student must exert to hold the balloon at rest under the water, in terms of the quantities given in this part and physical constants, as appropriate.

$F_{hand} = F_b$

$= \rho V g$

$= \rho_w V_b g \rightarrow F_{hand} = \rho_w V_b g$

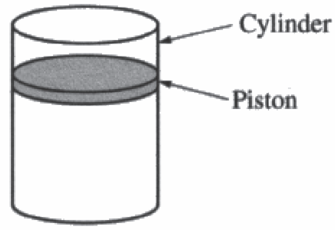
F_{hand} = the force the student is exerting with

ρ_w = density of the water

V_b = volume of the ball

g = gravity

Begin your response to **QUESTION 2** on this page.



2. (12 points, suggested time 25 minutes)

A group of students design an experiment to investigate the relationship between the density and pressure of a sample of gas at a constant temperature. The gas may or may not be ideal. They will create a graph of density as a function of pressure. They have the following materials and equipment.

$PV = nRT$

- A sample of the gas of known mass M_g in a sealed, clear, cylindrical container, as shown above, with a movable piston of known mass m_p
- A collection of objects each of known mass m_o
- A meterstick

(a)

i. Describe the measurements the students should take and a procedure they could use to collect the data needed to create the graph. Specifically indicate how the students could keep the temperature constant. Include enough detail that another student could follow the procedure and obtain similar data.

Use the meterstick to measure the radius and height of the gas in the container, and use $V = \pi r^2 h$ to find the Volume of the gas. Use M_g and V to find density ρ, and keep the gas at room temperature. Vary the volume, which will vary the density, to create the experiment data.

ii. Determine an expression for the absolute pressure of the gas in terms of measured quantities, given quantities, and physical constants, as appropriate. Define any symbols used that are not already defined.

$P_{Abs} = P_{gauge} + P_{Atm}$

$P_{Absolute} = \frac{nRT}{V} + \rho g h$

GO ON TO THE NEXT PAGE.

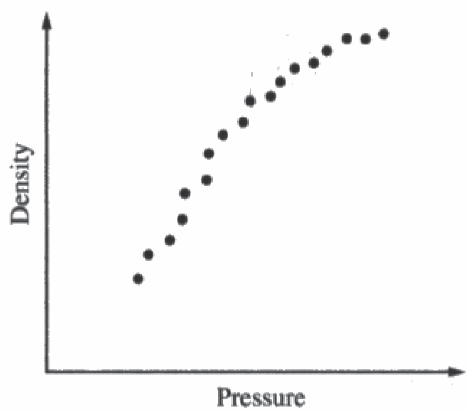
Continue your response to **QUESTION 2** on this page.

iii. Determine an expression for the density of the gas in terms of measured quantities, given quantities, and physical constants, as appropriate. Define any symbols used that are not already defined.

r = radius of cylinder

h = height of piston

$$\rho = \frac{M_g}{\pi r^2 h}$$



iv. The graph above represents the students' data. Does the data indicate that the gas is ideal? Describe the application of physics principles in an analysis of the graph that can be used to arrive at your answer.

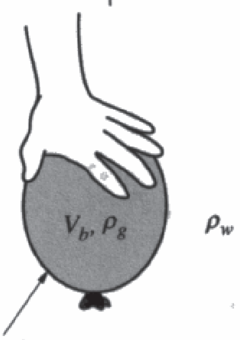
Yes because the graph is showing an increase in density as volume increases, which proves that it's an ideal gas.

Continue your response to **QUESTION 2** on this page.

Another group of students propose that the relationship between density and pressure could also be obtained by filling a balloon with the gas and submerging it to increasing depths in a deep pool of water.

(b) Why could submerging the balloon to increasing depths be useful for determining the relationship between the density and pressure of the gas?

Because submerging the balloon will change the pressure on the balloon, and then you'll be able to see the relationship between density and pressure.



m_b (Balloon only, not gas)

(c) The balloon is kept underwater in the deep pool by a student pushing down on the balloon, as shown above. Let V_b represent the volume of the inflated balloon, m_b represent the mass of just the balloon (not including the mass of the gas), ρ_g represent the density of the gas in the balloon, and ρ_w represent the density of the water. Derive an expression for the force the student must exert to hold the balloon at rest under the water, in terms of the quantities given in this part and physical constants, as appropriate.

$$F_B - F_g = F_N$$

$$\rho V g - mg = mg$$

Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses to this question were expected to demonstrate the following:

- Describe a procedure to investigate the relationship between the density and pressure of a sample of an ideal gas
- Derive equations for the density and pressure of the gas using measured quantities
- Analyze a density vs. pressure graph to determine whether the sample of gas is indeed ideal
- Compare the first experiment to a new experiment involving a balloon that is submerged underwater
- Derive an equation for the applied force necessary to hold the balloon stationary underwater

Sample: P2 Q2 A
Score: 11

Part (a)(i) earned 3 points. One point was earned for indicating a valid method for affecting the pressure by placing objects on the piston. One point was earned for mentioning that taking measurements of the height and radius of the cylinder. One point was earned for indicating a valid method for generating more than two experimental data points. The response does not state a valid experimental method for maintaining a constant temperature of the gas. Part (a)(ii) earned 1 point for indicating a valid expression that is in terms of measured quantities from part (a)(i). Part (a)(iii) earned 1 point for indicating a valid expression that is in terms of measured quantities from part (a)(i). Part (a)(iv) earned 2 points. One point was earned for referring to the ideal gas law to state an inversely proportional relationship between pressure and volume—note the expression written to the side of the graph. One point was earned for clearly establishing a linear relationship between pressure and density and clearly interpreting the shape or slope of the graph in a manner that validates the nonideal behavior of the gas. Part (b) earned 1 point for clearly indicating that the water pressure increases as the depth increases. Part (c) earned 3 points. One point was earned by indicating a valid expression for the buoyant force in terms of the quantities given. One point was earned by indicating a valid expression for the weights of the gas and balloon in terms of the quantities given. One point was earned by deriving and establishing a relationship between the buoyant force, the force of the student, and at least one of the weights via Newton's second law.

Sample: P2 Q2 B
Score: 6

Part (a)(i) earned 1 point for mentioning the measurements of the height of the cylinder and the radius/diameter of the piston. No points were earned for not stating a valid experimental method for maintaining a constant temperature of the gas, not mentioning a valid use of the objects that would affect the pressure of the system during the experiment, and not indicating a valid method for generating more than two experimental data points. Part (a)(ii) earned 1 point for providing an equation that is consistent with the procedure provided in part (a)(i) even though it does not incorporate the objects. Part (a)(iii) earned 1 point for indicating a valid expression that is in terms of measured quantities from part (a)(i). Part (a)(iv) earned 1 point for referring to the ideal gas law to state an inversely proportional relationship between pressure and volume. No point was earned for not clearly interpreting the shape or slope of the graph and the response does not clearly establish a linear relationship between pressure and density. Part (b) earned 1 point by connecting the increasing force on the balloon as coming from pressure, the response clearly indicates that the water pressure increases as the depth increases. Part (c) earned 1 point for indicating a valid expression for the buoyant force in terms of the quantities given. No other points were earned because the response does not indicate an expression for the combined weights of the balloon and gas and does not derive and establish a relationship between the buoyant force, either the weight of the balloon or gas, and the force of the student via Newton's second law.

Question 2 (continued)

Sample: P2 Q2 C
Score: 2

Part (a)(i) earned 1 point for indicating the measurement of the height of the piston from the base of the cylinder and the radius/diameter of the piston. No points were awarded because the response does not state a valid experimental method for maintaining a constant temperature of the gas, does not mention any use of the objects during the experiment, and does not elaborate on generating more than two experimental data points. Part (a)(ii) earned no points for not indicating a valid expression that is in terms of measured quantities from part (a)(i). Part (a)(iii) earned 1 point for indicating a valid expression that is in terms of measured quantities from part (a)(i). Part (a)(iv) earned no points. The response does not refer to the ideal gas law to state an inversely proportional relationship between pressure and volume, does not clearly interpret the shape or slope of the graph, and does not clearly establish a linear relationship between pressure and density. Part (b) earned no points because the response indicates that the water pressure changes with depth but does not establish that the pressure would increase as the depth increases. Part (c) earned no points for not indicating a valid expression for the buoyant force in terms of the quantities given, not developing a valid expression for the combined weights of the balloon and gas, and not deriving and establishing a relationship between the weights, buoyant force, and the force of the student via Newton's second law.