2023

AP®

AP® Physics 2: Algebra-Based

Sample Student Responses and Scoring Commentary

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Free-Response Question 3

☑ Scoring Guidelines

☑ Student Samples

☑ Scoring Commentary

Question 3: Quantitative/Qualitative Translation — 12 points

(a)(i) For a statement that collisions from the water particles exert upward forces on the block and collisions from the air particles exert downward forces on the block — **1 point**

For a statement indicating that the force from the water is greater than the force from the air — **1 point**

Example Response

The air particles collide with the top of the block and exert downward forces on the block. The water particles collide with the bottom of the block and exert upward forces on the block. The force exerted by the water particles is greater than the force exerted by the air particles. Therefore, the result of these forces is an upward buoyant force from the particles.

(a)(ii) For indicating that Block A has a greater density than Block B because Block A displaces a larger volume of water, thus the buoyant force on Block A is greater than the buoyant force on Block B — **1 point**

Example Response

Because Block A *displaces a greater volume of fluid, the buoyant force on Block* A *is greater than the buoyant force on Block* B. *Because the buoyant force and gravitational force are balanced for both blocks, Block* A *must weigh more than Block* B. *Because the blocks have the same volume, Block* A *is more dense than Block* B.

Total for part (a) — 3 points

(b)(i) For using Bernoulli's equation to derive the relationship between v_p and h — **1 point**

For indicating that $P_2 = P_1$ — **1 point**

For correct substitutions of the heights and speeds — **1 point**

Example Solution

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho v_s^2 + \rho g h = \rho g(0) + \frac{1}{2}\rho v_p^2$$

$$v_p^2 = v_s^2 + 2gh$$

$$v_p = \sqrt{v_s^2 + 2gh}$$

(b)(ii) For using the continuity equation to derive the relationship between v_p and R — **1 point**

For correct substitutions for the expressions of areas and speeds — **1 point**

Example Solution

$A_1 v_1 = A_2 v_2$

$\pi R^2 v_s = \pi r^2 v_p$

$v_p = \dfrac{R^2}{r^2} v_s$

(b)(iii)	For using conservation principles to justify that when $R \gg r$, then $v_s \ll v_p$	**1 point**
	For indicating a very small value of v_s will have a negligible effect on v_p	**1 point**

Example Response

When the cross sectional area of the tank is very large compared to the cross sectional area of the pipe, the speed v_s of the surface of the water is much less than the speed of the water v_p exiting the pipe due to the constant volume flow rate. As a result, the speed of the surface of the water can be approximated as zero, so the speed of the water exiting the pipe can be approximated as $v_p = \sqrt{2gh}$.

Total for part (b) 7 points

(c)	For correctly relating the decrease in v_p to the decrease in the height of the surface of the water h	**1 point**
	For correctly relating the decrease in v_s to the increase in radius R	**1 point**

Example Response

According to the equation in part (b)(i), $v_p = \sqrt{v_s^2 + 2gh}$. As h decreases, v_p decreases. When solving the equation in part (b)(ii) for v_s, it can be shown that $v_s = \dfrac{r^2}{R^2} v_p$. Therefore, an increase in R results in a decrease in v_s. Because v_p decreases with decreasing h, by using the same expression from part (b)(ii) in the case in which v_p decreases and R increases, it can be shown that the speed v_s of the water at the surface decreases.

OR

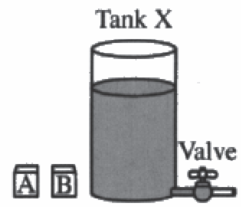
If the two equations from parts (b)(i) and (b)(ii) are solved simultaneously for v_s as a function of h and R, it can be shown that $v_s = \sqrt{\dfrac{2gh}{\dfrac{R^4}{r^4} - 1}}$. Therefore, as h decreases and R increases, v_s decreases.

Total for part (c) 2 points

Total for question 3 12 points

Question 3

Begin your response to QUESTION 3 on this page.



Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

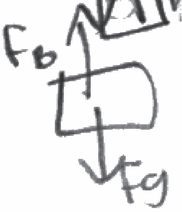
(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

The air particles exert a downwards force on the blocks, but the water particles under it exert upwards forces. The stronger force of the water particles below (at a greater pressure) lead to an upwards buoyant force.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

A has a greater density.



$\rho = \frac{m}{V}$

$mg = \rho_w V g$

$m = \rho_w V$

$V_A > V_B$ because more has to be submerged to touch bottom first.

If $V_A > V_B$, $m_A > m_B$

$\rho = \frac{m}{V}$

If $m_A > m_b$, $\rho_A > \rho_b$

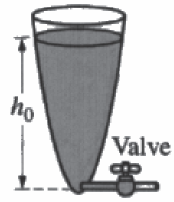
$F_g - F_B = 0$

$mg - \rho V g = 0$

Question 3

Continue your response to **QUESTION 3** on this page.

Tank Y



Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

$h = h_0 - V_s t$

$\cancel{P_1} + \rho g h_1 + \frac{1}{2}\rho V_s^2 = \cancel{P_1} + \rho g h_2 + \frac{1}{2}\rho V_p^2$

$\cancel{\rho} g h_1 + \frac{1}{2}\cancel{\rho} V_s^2 = \frac{1}{2}\cancel{\rho} V_p^2$

$2gh + V_s^2 = V_p^2$

$\boxed{V_p = \sqrt{V_s^2 + 2gh}}$

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \frac{R^2}{r^2} v_s$.

$A_1 V_1 = A_2 V_2$

$\cancel{\pi} R^2 V_s = \cancel{\pi} r^2 V_p$

$\boxed{\frac{R^2}{r^2} V_s = V_p}$

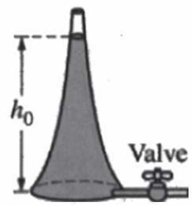
iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

when R is much greater than r, $\frac{r^2}{R^2}$ is very small, $V_p\left(\frac{r^2}{R^2}\right) = V_s$, so V_s is very small (close to 0). $V_p = \sqrt{V_s^2 + 2gh}$, but becaus V_s is very small, it is negligible and $V_p = \sqrt{2gh}$

Question 3

Continue your response to **QUESTION 3** on this page.

Tank Z



Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

~~As the water goes down, R gets much larger than r. If~~ $\cancel{\frac{R^2}{r^2}V_s = V_p}$, $\cancel{V_s = V_p\left(\frac{r^2}{R^2}\right)}$, ~~and~~ ~~$V_s$ will get smaller.~~

$\cancel{V_p = \sqrt{V_s^2+2gh}}$ ~~shows that V_p lessens when h decreases, so as the water level goes down, V_p will decrease.~~

$$\frac{R^2}{r^2}V_s = V_p \rightarrow \frac{R^2}{r^2}V_s = \sqrt{V_s^2+2gh}$$

$$V_p = \sqrt{V_s^2+2gh}$$

$$\frac{R^4}{r^4}V_s^2 = V_s^2+2gh$$

$$\left(\frac{R^4}{r^4}-1\right)V_s^2 = 2gh$$

$$\boxed{V_s = \sqrt{\frac{2gh}{\left(\frac{R^4}{r^4}-1\right)}}}$$

Because h↓ and R↑, V_s will decrease over time

Question 3

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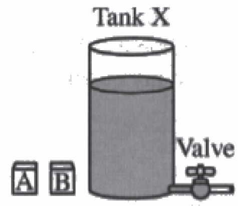

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Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

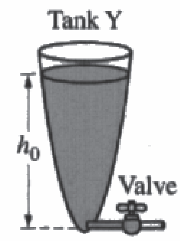
The air and water molecules collide with each other and momentum is conserved. Since momentum is conserved, the air molecules are pushing the denser water molecules up.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

Block A is denser than Block B. The blocks have the buoyant force and F_g exerted on them. Since they have the same volume, F_B will be the same for both blocks since they occupy the same volume of water. F_g will be greater for the denser material since $m = \rho V$ and mass is directly proportional to density when V is constant. Therefore, the denser block will feel a greater downward net force since $\Sigma F = F_B - F_g$, and Block A is denser than Block B since it accelerated to the bottom of the tank faster.

Question 3

Continue your response to QUESTION 3 on this page.



Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

$\cancel{P_1} + \cancel{\rho} g y_1 + \frac{1}{2}\cancel{\rho} v_1^2 = \cancel{P_2} + \cancel{\rho g y_2} + \frac{1}{2}\cancel{\rho} v_2^2$

$P_1 = P_2$

$\rho_1 = \rho_2$

$y_2 = 0$

$g y_1 + \frac{1}{2} v_1^2 = g y_2 + \frac{1}{2} v_2^2$

$\frac{1}{2} v_1^2 + g y_1 = \frac{1}{2} v_2^2$

$v_1^2 + 2 g y_1 = v_2^2$

$v_s^2 + 2gh = v_p^2$

$\boxed{v_p = \sqrt{v_s^2 + 2gh}}$

$v_2 = v_p$

$v_1 = v_s$

$y_1 = h$

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \dfrac{R^2}{r^2} v_s$.

$A_1 = R^2 \pi$

$A_2 = r^2 \pi$

$v_1 = v_s$

$v_2 = v_p$

$A_1 v_1 = A_2 v_2$

$R^2 \cancel{\pi} \cdot v_s = r^2 \cancel{\pi} \cdot v_p$

$R^2 v_s = r^2 v_p$

$\boxed{v_p = \frac{R^2}{r^2} \cdot v_s}$

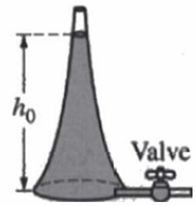
iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

When R is suffuently greater than r, the speed v_s can be considered to be 0 m/s since it moves so slowly relative to the speed v_p. Therefore, because $A_{top} > A_{bottom}$ by such a large factor; $v_s = 0$ compared to the enormous speed of v_p that is much greater than v_s. from the equation in part (ii), $\frac{R^2}{r^2}$ is so great that v_s is minuscule to v_p.

Question 3

Continue your response to QUESTION 3 on this page.

Tank Z



Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

$V_p = \sqrt{V_s^2 + 2gh}$ $\quad V_s = \sqrt{V_p^2 - 2gh}$

At time $t = 0$, the valve of Tank Z is opened.

$V_p = \frac{R^2}{r^2} V_s$ $\quad V_s = \frac{r^2}{R^2} V_p$

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

V_s increases over time.

When rearranged to be a function V_s, the equations from part b.i and b.ii are $V_s = \sqrt{V_p^2 - 2gh_0}$ and $V_s = \frac{r^2}{R^2} V_p$

As time increases, h_0 approaches 0, so V_s must increase since it is directly proportional to V_p. Additionally, the top surface area of the water increases over time as water exits the tank, so the ratio of $\frac{r^2}{R^2}$ becomes closer to 1, increasing the value of V_s when V_p is constant or increasing.

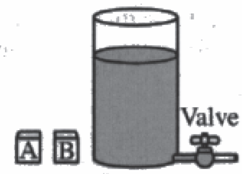
Therefore, since the height of the water decreases and area of the water surface increases, the speed V_s increases based on $V_s = \sqrt{V_p^2 - 2gh_0}$ and $V_s = \frac{r^2}{R^2} V_p$

$t\uparrow \quad h_0\downarrow \quad V_s = \sqrt{V_p^2 - (0)} \Rightarrow V_s = V_p$ and $V_s\uparrow$

$t\uparrow \quad R\uparrow$

Question 3

Begin your response to QUESTION 3 on this page.



Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

$F_b = \rho V g$. There is an upward buoyant force because the denser water particle's are pushing up with a greater force than the combination of air particles and block and gravity pushing down.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

Block A has greater density. $F_b = \rho V g$. Gravity and volume are the same for both A and B so density must be higher in the block that hits the bottom first.

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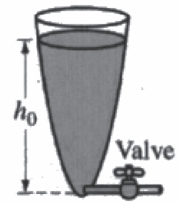
Question 3

Continue your response to QUESTION 3 on this page.



Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

$V_p < \Delta h$

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \sqrt{\frac{R^2}{r^2}}\, v_s$.

$V_p > R \longrightarrow R = \frac{\sqrt{V_p r^2}}{v^2}$

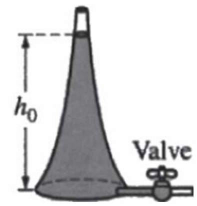
iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

When R is a lot greater than r, $\sqrt{V_s^2}$ in the equation $v_p = \sqrt{vs^2 + 2gh}$ is negligible because the radius R is so large.

Question 3

Continue your response to **QUESTION 3** on this page.

Tank Z



Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

As the water comes out, its speed will decrease. At first, $v_p = \sqrt{2gh}$ can be used, but as the radius increases, $v_p = \sqrt{v_s^2 + 2gh}$ must be used and this equation results in a larger amount.

GO ON TO THE NEXT PAGE.

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Describe the microscopic cause of the buoyant force in terms of particle collisions on the upper and lower surfaces of a partially submerged object.
- Analyze the buoyant forces on partially submerged objects to compare the density of objects.
- Derive mathematical expressions that model the behavior of flowing fluids from Bernoulli's equation and the continuity equation.
- Analyze given mathematical expressions to determine the functional dependence between changing variables such as surface height, radius, and speed of water as it flows from a tank.

Sample: 3A
Score: 12

Part (a) earned 3 points. The first point was earned for correctly indicating the air particles exert downward forces on the block and the water particles exert upward forces on the block. The second point was earned for indicating that the force exerted from the water is greater than the force exerted from the air. The third point was earned for claiming that Block A has a greater density than Block B and uses a correct analysis of the forces exerted on the block to show the greater volume of water displaced by Block A means that Block A has a greater mass and, therefore, greater density. Part (b) earned 7 points. The first point was earned for using Bernoulli's equation to attempt to derive the relationship between v_p and h. The second point was earned for showing that the pressures at the two locations (surface and pipe exit) are the same by eliminating the terms. The third point was earned for correctly substituting the speeds and heights into the expression. The fourth point was earned for using the continuity equation to derive the relationship between v_p and R. The fifth point was earned for correctly substituting values for areas and speeds. The sixth point was earned for referencing conservation principles, the equation in part (b)(ii), to show that surface speed is very small. The seventh point was earned for indicating the term for v_s is not needed in the expression because v_s is negligible. Part (c) earned 2 points. The first point was earned for deriving a mathematical expression for v_s as a function of R and h based on the expressions from parts (b)(i) and (b)(ii) that includes the correct functional dependence between v_p and h. The second point was earned for deriving a mathematical expression for v_s as a function of R and h based on the expressions from parts (b)(i) and (b)(ii) that includes the correct functional dependence between v_s and R.

Question 3 (continued)

Sample: 3B
Score: 7

Part (a) earned no points. The first point was not earned because the response does not indicate that the water pushes up on the block or that the air pushes down. The second point was not earned because response does not indicate that the force exerted from the water is greater than the force exerted from the air. The third point was not earned because, although the response does claim Block A has a greater density than Block B, the response incorrectly claims that the buoyant force on each block is the same. Part (b) earned 7 points. The first point was earned for using Bernoulli's equation to attempt to derive the relationship between v_p and h. The second point was earned for showing that the pressures at the two locations (surface and pipe exit) are the same by eliminating the terms. The third point was earned for correctly substituting the speeds and the heights into the expression. The fourth point was earned for using the continuity equation to derive the relationship between v_p and R. The fifth point was earned for correctly substituting values for areas and speeds. The sixth point was earned for referencing conservation principles (comparing areas to compare speeds) to show that surface speed is small (zero) compared to the speed exiting the pipe. The seventh point was earned for indicating that v_s is not part of the equation because v_s is "miniscule" compared to v_p. Part (c) earned no points. The first point was not earned because, although the response references the expression from part (b)(i), the response reaches an incorrect conclusion that the speed of the water exiting the pipe must increase. The second point was not earned because, although the response references the equation from part (b)(ii), the response claims that v_s increases when R increases, rather than v_s decreasing when R increases.

Sample: 3C
Score: 2

Part (a) earned 1 point for correctly indicating that the air particles push down on the block and the water particles push up on the block. The second point was not earned because the response indicates that the force exerted by the water is greater than the forces exerted by the air and gravity, rather than the force exerted by the water being greater than the force exerted by the air. The third point was not earned because, although the response claims that Block A has the greater density, the response does not correctly connect this claim to the buoyant force or the volume of the displaced fluid. Part (b) earned 1 point for correctly indicating that v_s is a negligible value. The second point was not earned because the response does not use Bernoulli's equation to derive the relationship between v_p and h. The third point was not earned because the response does not correctly substitute the speeds and the heights into an expression. The fourth point was not earned because the response does not show that the pressures at the two locations (surface and pipe exit) are the same. The fifth point was not earned because the response does not use the continuity equation to derive the relationship between v_p and R. The sixth point was not earned because the response does not substitute values for areas and speeds in the expression for v_p. The seventh point was not earned because the response does not reference conservation principles to explain why v_s is much smaller than v_p. Part (c) earned no points. The first point was not earned because the response does not connect the decreasing height to the decreasing v_p as the water exits the pipe. The second point was not earned because the response does not connect the increasing radius to the decreasing surface speed of the water.